The Interaction of Actual and Fundamental House Prices: A General Model with an Application to Denmark and Sweden

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Abstract

The paper presents a general method for estimating a country's level of fundamental house prices and its interaction with actual house prices. We set up a unified empirical model which can be used to analyze the time-series behavior of the fundamental house price and to test various hypotheses regarding its relation to the actual house price. To illustrate how the method works, we apply it to data for Denmark and Sweden. We find a tendency for actual house prices to converge on fundamental prices, albeit rather slowly.

JEL: R31, G12, C31

KEY WORDS: FUNDAMENTAL HOUSE PRICES, HOUSE PRICE DYNAMICS, HOUSING BUBBLES.

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1 Introduction

Housing bubbles and their bursting have played a key role in many financial crises. Analytical tools that may help policy makers to spot a housing bubble before it grows dangerously big should therefore improve the basis for macroeconomic stabilization policy. This paper presents an empirical methodology for estimating whether a country's level of house prices is out of line with fundamentals. To illustrate its practical use, we apply the method to data for Denmark and Sweden.

Our method for estimating fundamental house prices broadly follows the one developed by Hott and Monnin (2008) which is based on the dividend-discount model of asset prices pioneered by Campbell and Shiller (1988a,1988c). This model has been used recently and in different contexts by, i.a., Campbell et al. (2009), Costello et.al. (2011), Hiebert and Sydow (2011), Hott and Jokipii (2012), the European Commission (2012), Ambrose et al. (2013), Fairchild et.al. (2015) and Kishor and Morley (2015). Our main contribution is to set up a unified empirical model which can be used to analyze the time-series behavior of the estimated fundamental house price and to test various hypotheses regarding its relation to the actual house price. For example, we can investigate whether actual house prices converge on fundamental prices and how long it takes before a gap between the actual and the fundamental price is closed, identify permanent and temporary shocks to house prices and generate confidence bands for our estimates of fundamental house prices. As a further extension of previous work in this area, we use a more general specification of the housing demand function which allows us to analyze the sensitivity of the estimates of fundamental house prices to the long-run price and income elasticities of housing demand. We also propose a new method to calibrate the link between the imputed rent on owner-occupied housing and the fundamental house price.

The paper is structured as follows: Section 2 presents the theoretical model of fundamental house prices and shows how it may be linearized to facilitate empirical estimation. Section 3 describes the empirical strategy for modeling the forward-looking expectations determining fundamental house prices, and section 4 explains how the fundamental house price may be calculated, once the formation of expectations has been modeled. Section 5 presents our data sets and 6 applies the general methodology laid out in sections 2 through 4 to estimate fundamental house prices in Denmark and Sweden. In section 7 we set up a unified econometric model allowing an analysis of the interaction between actual and fundamental house prices, and section 8 illustrates how our model may be used to analyze the effects of policy-induced shocks to the housing market. Section 9 summarizes our main findings.

2 The theoretical model

2.1 Deriving the fundamental house price

We use the following notation:

$P =$ real price of a unit of owner-occupied housing,

$R^H =$ imputed rent on a unit of owner-occupied housing,

$R =$ real rent on a unit of rental housing,

$Y =$ aggregate real disposable household income,

$H =$ aggregate real housing stock,

$i =$ nominal mortgage interest rate,

$\pi =$ expected rate of consumer price inflation,

$\tau^i =$ capital income tax rate,

$\tau =$ effective property tax rate,

$\eta =$ user cost premium for risk and credit constraints (constant),

$\delta =$ rate of depreciation of the real housing stock (constant),

$t =$ subscript for time period t,

$E_t [X_{t+i}] =$ expectation held at time t regarding the value of variable X at time $t+i$

The imputed rent on a unit of owner-occupied housing is the amount the consumer is willing to pay for the housing service, i.e., the marginal rate of substitution between the housing service and all other goods. Standard consumer theory implies that, in a utility maximum, this marginal rate of substitution is

$$R_t^H = \left[i_t \left(1 - \tau_t^i\right) - \pi_t + \tau + \delta + \eta\right] P_t - \{E_t [P_{t+1}] - P_t\} . \tag{1}$$

If the housing investment is fully debt-financed, the term $[i_t (1 - \tau_t^i) + \tau + \delta + \eta] P_t$ is the home-owner's nominal cash expenses, including expenses on repair and maintenance (δ), and the term $\pi_t P_t + \{E_t [P_{t+1}] - P_t\}$ is the expected nominal capital gain. If the housing investment is equity-financed, the home-owner forgoes the after-tax interest income he could have earned by investing his wealth in the capital market, so $i_t (1 - \tau_t^i) P_t$ is still part of the (opportunity) cost of housing. The term ηP_t — which is unimportant for our present purpose — is a premium reflecting risk and possible credit constraints.[1] Rearranging (1), we get an expression for the house price in period t:

$$P_t = \frac{R_t^H + E_t\left[P_{t+1}\right]}{1+\gamma_t}, \qquad \gamma_t \equiv i_t\left(1-\tau_t^i\right) - \pi_t + \tau_t + \delta + \eta. \tag{2}$$

The variable γ_t is the user cost of owner-occupied housing, *excluding* the expected capital gain. If agents are rational, they realize that the link between prices and imputed rents is given by (2). By forward iteration one finds that (2) implies

$$P_t = E_t\left[\sum_{i=0}^{\infty} \frac{R_{t+i}^H}{\prod_{j=0}^{i}\left(1+\gamma_{t+j}\right)}\right]. \tag{3}$$

Equation (3) shows that the fundamental house price is the discounted value of expected future imputed rents, where the period-by-period discount rate is given by γ_{t+j}. The discounted sum on the right-hand side of (3) will be finite if the real imputed rent grows at an average rate lower than the average value of γ_{t+j}. We assume that this condition is met. This is equivalent to ruling out bubbles in the housing market.

The expected future imputed rents in (3) are not directly observable. Following Hott and Monnin (2008), we consider two alternative ways of pinning down the expected future values of R_t^H. Our first model, denoted the "Rent" model, is based on the notion that a housing market equilibrium requires a stable relationship between the cost of rental housing and the imputed rent on owner-occupied housing of similar quality, that is:

$$R_t^H = \omega R_t\ , \qquad 0 < \omega \leq 1. \tag{4}$$

If the market for rental housing is free and the two forms of housing are perfect substitutes, arbitrage between them will ensure that $\omega = 1$. If rent controls keep the cost of rental housing below the free-market level so that access to this form of housing is rationed, we have $\omega < 1$. For tractability, we make the bold assumption that the degree of rationing and therefore ω is roughly constant over the long run. In that case we can use (4) to infer the level of imputed rents by observing the level of rents in the market for rental housing.

However, since the assumption of a constant value of ω may be problematic, we will also consider a "Supply-and-Demand model" (S-D model) in which imputed rents are assumed to adjust so as to equilibrate the supply of and demand for housing services. Specifically, suppose the aggregate long-run demand for housing services (D) varies positively with aggregate real disposable income (Y) and negatively with imputed rents so that

$$D_t = BY_t^{\varepsilon_Y}\left(R_t^H\right)^{-\varepsilon_R}, \tag{5}$$

[1] The determinants of this term (which we treat as an exogenous constant in the present paper) are explained in detail in Sørensen (2012, Appendix).

where B is a constant, ε_Y is the long-run income elasticity of housing demand, and ε_R is a price elasticity measuring the numerical long-run elasticity of housing demand with respect to the imputed rent.[2] The aggregate supply of housing services is proportional to the aggregate housing stock (H), and the proportionality factor may be normalized at unity by appropriate choice of units. In a housing market equilibrium we thus have $H_t = D_t$. From (5) this implies

$$R_t^H = B^{1/\varepsilon_R} Y_t^{\varepsilon_Y/\varepsilon_R} H_t^{-1/\varepsilon_R}. \tag{6}$$

Equations (3) and (6) can be used to estimate the fundamental house price, provided data on real disposable income and the total real housing stock are available.

2.2 A linearized version of the house price model

To make the house price model presented above more tractable for empirical analysis, it will be convenient to respecify it in terms of the price-to-imputed-rent ratio, defined as

$$X_t \equiv P_t / R_t^H. \tag{7}$$

With this definition, and introducing the simplifying notation $P_{t+1}^e \equiv E_t\left[P_{t+1}\right]$, $R_{t+1}^{He} \equiv E_t\left[R_{t+1}^H\right]$ and $X_{t+1}^e \equiv P_{t+1}^e / R_{t+1}^{He}$, we can rewrite equation (2) as[3]

$$(1+\gamma_t) P_t = R_t^H + P_{t+1}^e \quad \Rightarrow$$

$$(1+\gamma_t) X_t = 1 + X_{t+1}^e \frac{R_{t+1}^{He}}{R_t^H} \quad \Rightarrow$$

$$\begin{aligned} x_t &= \ln\left(1+\exp\left(x_{t+1}^e + \Delta r_{t+1}^{He}\right)\right) - \ln\left(1+\gamma_t\right), \\ x_t &\equiv \ln X_t, \qquad x_{t+1}^e \equiv \ln X_{t+1}^e, \qquad \Delta r_{t+1}^{He} \equiv \ln R_{t+1}^{He} - \ln R_t^H. \end{aligned} \tag{8}$$

Let

$$\bar{m} \equiv \bar{x}^e + \Delta \bar{r}^{He} \tag{9}$$

denote the mean value of the term $x_{t+1}^e + \Delta r_{t+1}^{He}$ in (8) over the sample period considered. Taking a first-order Taylor approximation of (8) around $x_{t+1}^e + \Delta r_{t+1}^{He} = \bar{m}$ and $\gamma_t = 0$, we get

$$x_t \approx \ln\left(1+\exp\left(\bar{m}\right)\right) + \left(\frac{\exp\left(\bar{m}\right)}{1+\exp\left(\bar{m}\right)}\right)\left(x_{t+1}^e + \Delta r_{t+1}^{He} - \bar{m}\right) - \gamma_t. \tag{10}$$

Defining

$$\phi \equiv \frac{\exp\left(\bar{m}\right)}{1+\exp\left(\bar{m}\right)}, \tag{11}$$

[2] This specification of housing demand is more general than the one used by Hott and Monnin (2008). They assume that $\varepsilon_Y = \varepsilon_R = 1$, as would be the case if consumers have Cobb-Douglas utility functions and the savings rate is constant.

[3] This normalization procedure follows the one suggested by Hott and Monnin (2008).

we can restate (10) as[4]

$$x_t = \kappa + \phi\left(x^e_{t+1} + \Delta r^{He}_{t+1}\right) - \gamma_t \ , \qquad \kappa \equiv -\phi \ln \phi - (1-\phi)\ln(1-\phi)\,. \tag{12}$$

Assuming that agents are forward-looking, and defining $\Delta r^{He}_{t+j} \equiv E_t\left[\Delta r^H_{t+j}\right]$, we find by forward iteration of (12) that

$$x_t = c + \sum_{j=1}^{\infty} \phi^j E_t\left[\Delta r^H_{t+j} - \gamma_{t+j}\right] - \gamma_t \ , \qquad c \equiv \frac{\kappa}{1-\phi}. \tag{13}$$

In our rent model of fundamental house prices, we have $R^H_t = \omega R_t$ so that $\Delta r^H_{t+j} = \Delta r_{t+j}$ where $\Delta r_{t+j} \equiv \ln r_{t+j} - \ln r_{t+j-1}$. Inserting this into (13), we get the linearized version of the *Rent model*:

$$x_t = c + \sum_{j=1}^{\infty} \phi^j E_t\left[\Delta r_{t+j} - \gamma_{t+j}\right] - \gamma_t. \tag{14}$$

In the S-D model it follows from (6) that

$$\Delta r^H_{t+j} = (\varepsilon_Y/\varepsilon_R)\,\Delta y_{t+j} - (1/\varepsilon_R)\,\Delta h_{t+j}, \tag{15}$$

where $\Delta y_{t+j} \equiv \ln Y_{t+j} - \ln Y_{t+j-1}$ and $\Delta h_{t+j} \equiv \ln H_{t+j} - \ln H_{t+j-1}$. Substituting (15) into (13), we obtain the linear version of the *Supply-and-Demand model*:

$$x_t = c + \sum_{j=1}^{\infty} \phi^j E_t\left[(\varepsilon_Y/\varepsilon_R)\,\Delta y_{t+j} - (1/\varepsilon_R)\,\Delta h_{t+j} - \gamma_{t+j}\right] - \gamma_t. \tag{16}$$

3 Modeling expectations

To apply the formulas (14) and (16) for estimation purposes, we must model the way expectations are formed. Following Hott and Monnin (2008), Campbell et al. (2009), Ambrose et al. (2012) and others, we do so by assuming that agents base their forecasts on a VAR model. The VAR model should as a minimum include the variables which determine fundamental house prices according to our theoretical models, i.e., Δr_t, Δy_t, Δh_t and γ_t. For consistency, we also include the change in the actual house price, Δp^a_t, where p^a_t is the log of the actual house price, since knowledge of the current house price is included in the agent's information set in equation (1) which was used to derive the fundamental house price. This is similar to the procedure followed by Campbell and Ammer (1993) and Engsted et al. (2012) who emphasize the need to include the actual current stock price in a VAR-model of the stock market.

[4] In deriving (12), we use the facts that $\ln(1+\exp(\bar{m})) = -\ln(1-\phi)$ and $\ln(\phi) = \bar{m} - \ln(1+\exp(\bar{m})) = \bar{m} + \ln(1-\phi)$.

When forming expectations about future rents, user-costs, disposable income and the stock of housing, rational agents might consider more variables than those included in our VAR model, but for reasons of parsimony we assume that they base their forecasts on a VAR model comprising all observables in our two models of the fundamental house price plus the change in the log of the actual house price level. In matrix form the VAR forecasting model is

$$b_t = \Phi_0 + \Phi_1 b_{t-1} + \Phi_2 b_{t-2} + ... + \Phi_n b_{t-n} + \varepsilon_t, \tag{17}$$

where the time series vector b_t is defined as

$$b_t \equiv \begin{bmatrix} \Delta p_t^a \\ \Delta r_t \\ \gamma_t \\ \Delta y_t \\ \Delta h_t \end{bmatrix}, \tag{18}$$

and where Φ_j is a 5×5 matrix of coefficients on the lagged endogenous variables with lag length j, ε_t is a 5×1 column vector of white noise processes, and n is the total number of lags.

Defining the column vector z_t as

$$z_t \equiv \begin{bmatrix} b_t - \mu \\ b_{t-1} - \mu \\ \cdot \\ \cdot \\ b_{t-n+1} - \mu \end{bmatrix}, \qquad \mu = (I_5 - \Phi_1 - \ldots - \Phi_n)^{-1} \Phi_0, \tag{19}$$

we can rewrite the VAR(n) model in (17) in the following VAR(1) form

$$z_t = A z_{t-1} + \xi_t, \tag{20}$$

where

$$A \equiv \begin{bmatrix} \Phi_1 & \Phi_2 & \cdot & \Phi_{n-1} & \Phi_n \\ I_5 & 0 & \cdot & 0 & 0 \\ 0 & I_5 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & I_5 & 0 \end{bmatrix}, \qquad \xi_t \equiv \begin{bmatrix} \varepsilon_t \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}. \tag{21}$$

Our VAR(1)-model (20) will be covariance stationary if all eigenvalues of the A matrix in (21) are less than one in absolute value. We assume this condition to be met.

Using the VAR(1) in (20) and (21), we can now calculate the expected future values of the variables in the VAR(n) model (17) from the relationship

$$E_t \left[z_{t+i} \right] = A^i z_t. \tag{22}$$

4 Calculating the fundamental house price

Next we note from the definition of b_t stated in (18) that — abstracting from an unimportant constant which will depend on c and the elements of the column vector μ — our Rent model in (14) can be written as

$$x_t = \sum_{j=1}^{\infty} \phi^j g_1 E_t \left[z_{t+j} \right] + g_2 z_t, \tag{23}$$

where the $1 \times 5n$ vectors g_1 and g_2 are defined as

$$g_1 \equiv \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & \cdot & 0 \end{bmatrix}, \qquad g_2 \equiv \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & \cdot & 0 \end{bmatrix}. \tag{24}$$

Like the Rent model, our S-D model (16) may be written in the general form (23), but now the row vectors g_1 and g_2 must be specified as

$$g_1 \equiv \begin{bmatrix} 0 & 0 & -1 & \varepsilon_Y/\varepsilon_R & -1/\varepsilon_R & 0 & \cdot & 0 \end{bmatrix}, \qquad g_2 \equiv \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & \cdot & 0 \end{bmatrix}. \tag{25}$$

Inserting (22) into (23) and solving for x_t, we obtain for both theoretical models

$$x_t = \sum_{j=1}^{\infty} \phi^j g_1 A^j z_t + g_2 z_t \quad \Rightarrow$$

$$x_t = \left[g_2 + \phi g_1 A \left(I - \phi A \right)^{-1} \right] z_t. \tag{26}$$

Once we have estimated the coefficients in the VAR(n) model in (17), we can use (26) along with the definitions of A, g_1 and g_2 given in (21), (24) and (25) to compute an estimate of the fundamental price-to-imputed-rent ratio.

In the rent model we have assumed that $R_t^H = \omega R_t$. Recalling that $X_t \equiv P_t / R_t^H$, our estimate of the log of the fundamental house price, $\hat{p}_t$, may then be calculated as

$$\hat{p}_t = x_t + r_t + \alpha_0, \tag{27}$$

where the constant α_0 partly captures the (log of the) parameter ω, and partly picks up the conversion factor needed to convert our data for P_t and R_t into comparable units. We choose the value of α_0 that minimizes the sum Σ of the squared deviations between the actual house prices and the estimated fundamental house prices over the sample period:

$$\Sigma = \min_{\alpha_0} \sum_{t=1}^{T} (p_t^a - \hat{p}_t)^2 = \min_{\alpha_0} \sum_{t=1}^{T} \left[p_t^a - (\alpha_0 + x_t + r_t) \right]^2 .$$

The solution to this minimization problem gives

$$\alpha_0 = \frac{1}{T} \sum_{t=1}^{T} \left(p_t^a - x_t - r_t \right) = \bar{p}^a - \bar{x} - \bar{r}, \tag{28}$$

where $\bar{p}^a$, $\bar{x}$ and $\bar{r}$ denote the sample averages of the three variables. Inserting (28) into (27), we end up with the following expression for the estimated fundamental house price:

$$\hat{p}_t = \bar{p}^a + (x_t - \bar{x}) + (r_t - \bar{r}) \,. \tag{29}$$

Note that the estimate of the fundamental house price is derived from observable variables, i.e., the actual house price and the actual rent, and the fundamental price-to-imputed rent ratio which is computed from observables using (26) plus an estimate of ϕ (which is likewise derived from observables, cf. below). As shown by (29) our estimation procedure also implies that, on average over the sample period, the level of fundamental house prices equals the actual house price level.

In our demand-and-supply model of the fundamental house price we follow a similar procedure. Using (25) and (26) and an estimate of ϕ, we obtain the estimate for the fundamental price-to-imputed-rent ratio x_t implied by the S-D model. According to (6) and (7) we can then back out the estimate of the fundamental house price from the relationship

$$\hat{p}_t = x_t + (\varepsilon_Y/\varepsilon_R)\, y_t - (1/\varepsilon_R)\, h_t + \beta_0, \tag{30}$$

where β_0 captures the constant term in (6) plus the conversion factor needed to transform our data into comparable units. Again, we calibrate β_0 so as to minimize the sum of the squared deviations of the (log of the) actual house prices from the estimated (log of the) fundamental house prices. This procedure yields

$$\beta_0 = \frac{1}{T}\sum_{t=1}^{T}\left[p_t^a - x_t - (\varepsilon_Y/\varepsilon_R)\, y_t + (1/\varepsilon_R)\, h_t\right] = \bar{p}^a - \bar{x} - (\varepsilon_Y/\varepsilon_R)\, \bar{y} + (1/\varepsilon_R)\, \bar{h}, \tag{31}$$

which may be inserted in (30) to give

$$\hat{p}_t = \bar{p}^a + (x_t - \bar{x}) + (\varepsilon_Y/\varepsilon_R)\,(y_t - \bar{y}) - (1/\varepsilon_R)\left(h_t - \bar{h}\right). \tag{32}$$

When applying equation (32), we use prior knowledge on the size of the long-run elasticities ε_Y and ε_R.[5] By varying these parameters, we can analyze the sensitivity of the estimated fundamental house price to the income and price elasticities of housing demand.

In order to estimate x_t from (26), we need an estimate of the parameter ϕ. In the previous literature it has been common to simply postulate a plausible value of ϕ, but from (9) and (11) it follows that a model-consistent estimate of this parameter is given by

$$\phi \equiv \frac{\exp\left(\bar{x}^e + \Delta\bar{r}^H\right)}{1 + \exp\left(\bar{x}^e + \Delta\bar{r}^H\right)}, \tag{33}$$

[5] We might also calibrate the elasticities ε_Y and ε_R so as to minimize the sum of the squared deviations between actual house prices and fundamental house prices. This would be equivalent to running an OLS regression of p_t^a on y_t and h_t. However, since all of these variables are likely to form part of a larger simultaneous system, the resulting OLS estimates of ε_Y and ε_R would probably be biased. Moreover, we prefer to be able to vary ε_Y and ε_R to check the sensitivity of our results to these parameters.

where $\bar{x}^e$ is the expected mean value of x_t which is not observed. However, since we have already assumed that the mean value of the fundamental house price equals the mean of the actual house price, $\bar{p}^a$, it is natural to assume that

$$\bar{x}^e = \bar{p}^a - \bar{r}^H, \tag{34}$$

where $\bar{r}^H$ is the mean value of the level of imputed rent. Moreover, from the assumption (4) underlying our Rent model it follows that

$$\Delta \bar{r}^H = \Delta \bar{r}. \tag{35}$$

Alternatively, in our Supply-and-Demand model we see from (15) that

$$\Delta \bar{r}^H = (\varepsilon_Y / \varepsilon_R) \, \Delta \bar{y} - (1/\varepsilon_R) \, \Delta \bar{h}. \tag{36}$$

The mean values $\bar{p}^a$, $\Delta \bar{r}$, $\Delta \bar{y}$ and $\Delta \bar{h}$ are directly observable, but to obtain an estimate for ϕ from (33), (34) and (35) or (36) we also need an estimate for $\bar{r}^H$. Since we do not observe the initial value of the *level* of imputed rent, we cannot calculate $\bar{r}^H$ from our estimate of $\Delta \bar{r}^H$. However, since we do observe $\bar{p}^a$, and since we assume that $\bar{p}^a = \bar{p}$, we can use the relationship between R^H and P stated in (1) to infer what a plausible average level of R^H would be, given the observed average house price level. In particular, when measured relative to the average level of house prices, the average level of imputed rent should not deviate too much from an average long-term real interest rate. In both our theoretical models we have therefore calibrated $\bar{r}^H$ so as to imply that the ratio of the average imputed rent to the average house price level is equal to the average after-tax real mortgage interest rate.[6] Numerical experiments reveal that our estimate for ϕ is not very sensitive to reasonable variations in the assumed magnitude of the real interest rate.

5 Data

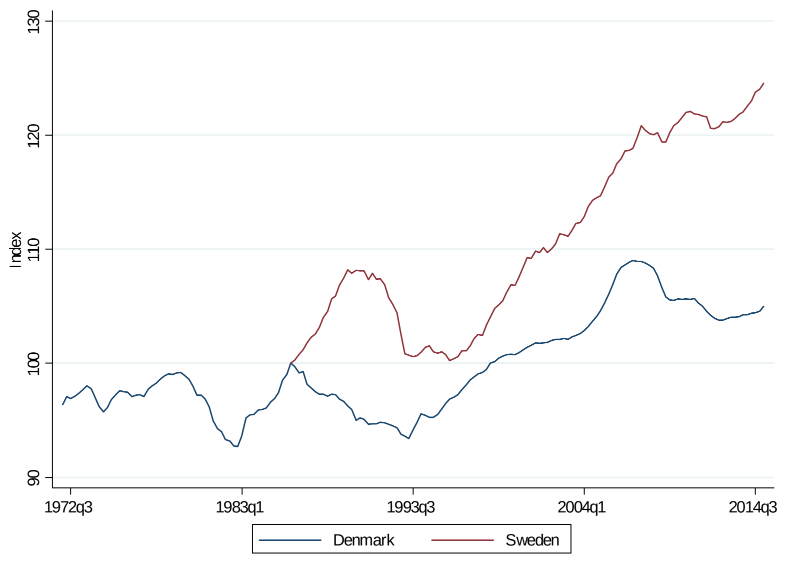
In our empirical analysis we use the same five variable VAR model for estimating both theoretical house price models. As mentioned above, it is necessary to include the actual house price in the model. Even though our Rent model summarized in eq. (14) implies that the fundamental house price depends only on the expected future changes in rents in the rental housing sector and on expected future user costs, we assume that agents form their expectations regarding these variables "as if" they were using a VAR model

[6] In other words, we calibrate $\bar{r}^H$ so as to satisfy the equation

$$\bar{r}^H = \bar{p}^a + \log \left[i \left(1 - \tau^i \right) - \pi \right]$$

where $i \left(1 - \tau^i \right) - \pi$ is the mean value of the after-tax real interest rate observed over the sample period.

Figure 1: Real house prices in Denmark and Sweden (Natural logarithms, 1986Q1=100).



which also includes changes in real disposable incomes and changes in the real housing stock. The motivation is that there are good theoretical reasons to believe that the latter two variables interact in a systematic manner with user costs and rents. Although the fundamental house price in our Supply-and-Demand model does not directly depend on changes in rents, the variables which do determine the fundamental price will surely interact with rents, so this variable should be included in the VAR forecasting model.

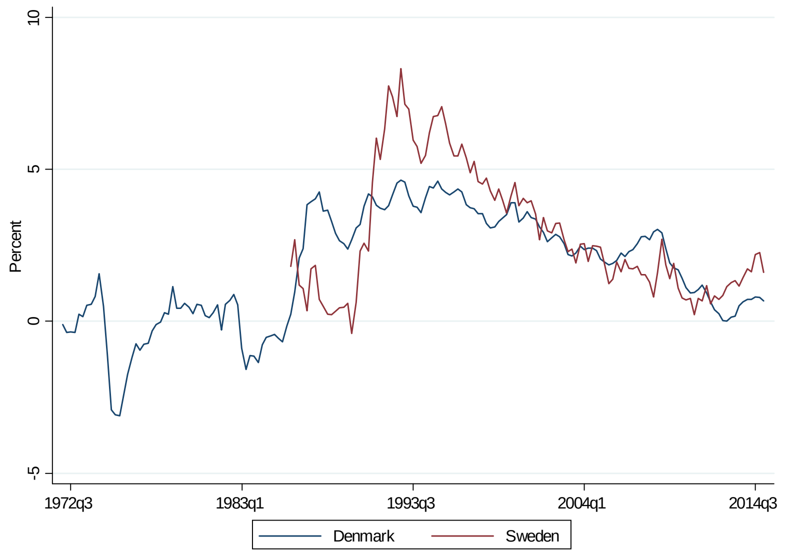
For these reasons we use a VAR model comprising the change in the actual real house price (Δp_t^a), the change in the level of real rents (Δr_t), the change in real household disposable income (Δy_t), the change in the real housing stock (Δh_t), and the expected real interest rate adjusted for capital income tax and property tax which is used as a proxy for our user cost variable γ_t in the theoretical models.

We have two datasets, one for Denmark and one for Sweden. All the Danish data are taken from the Mona databank provided by the Danish central bank and cover the sample period 1974Q1 until 2015Q1. The Swedish data set covers the period from 1986Q1 to 2015Q1 and is collected from various sources listed in Appendix A.

In Figure 1 we show how (the log of) real house prices have evolved over time in Denmark and Sweden. The graph suggests that Sweden experienced a housing bubble in the beginning of the 1990s and that Denmark went through a bubble in the mid-2000s. In general, house prices have increased more rapidly in Sweden, in particular during the most recent 5 years.

The user cost of owner-occupied housing $\gamma_t \equiv i_t \left(1 - \tau_t^i\right) - \pi_t + \tau_t + \delta + \eta$ plays a central role in our theoretical model. Figure 2 shows the evolution of user costs in Denmark and Sweden, where we have normalized the constant term $\delta + \eta$ to zero. From the mid 1990s

Figure 2: Real user cost of owner-occupied housing in Denmark and Sweden.



the user cost declined in both countries from a relatively high level. Currently, the Swedish user cost has returned to a level comparable to the level before the banking crisis of the early 1990s, whereas the Danish user cost is back to the historically low level prevailing in the 1970s.

The remaining three variables used in our estimations are shown in Figure B.1 in Appendix B.

6 Estimating the fundamental house price

6.1 Estimating the VAR forecasting model

We will now apply the method laid out above to estimate fundamental house prices in Denmark and Sweden. The first step is to estimate the VAR model described in section 3 which is used to forecast the variables determining expected future imputed rents. To determine the lag length in the VAR model we use the Schwarz Bayesian information criteria with a maximum of 12 lags allowed and then we test for autocorrelation, heteroscedasticity and normality in the residuals. If any of the tests suggests a rejection of the null hypothesis that there is no remaining autocorrelation or heteroscedasticity, we add one lag and repeat the tests. We stop at the lag length where we cannot reject the null. This procedure leads us to set the lag length equal to three quarters in the VAR model for Denmark and four quarters in the model for Sweden.

Panel A of Table 1 shows in the first five rows the p-values of tests of conditional autoregressive heteroskedasticity and normality in the residuals from each of the five equations in the VAR(n) model specified in (20) and (21). The last row shows the p-values of a multivariate LM test for autocorrelation (using six lags). It is clear from this table that the VAR model fits Swedish data better than Danish data. We can always reject the null hypothesis that the residuals in the Danish model are normally distributed and there are indications of ARCH in two of the equations. For the Swedish data we can never reject the null of no ARCH and normality in two equations. Experimenting with the specification of the Danish VAR model reveals that these symptoms do not disappear when adding lags to the VAR model. Furthermore, estimating the Danish model for subsamples also suggests that there will still be problems with ARCH effects regardless of sample.

One underlying assumption in our theoretical model and the procedure by which we intend to estimate fundamental house prices is that the VAR model is stable. The first row of Panel B in Table 1 reports the maximum eigenvalue of the VAR models. As can be seen, there seems to be a unit root in the Danish data but maybe not when using Swedish data. To formally test for cointegration we apply the standard Johansen trace test. Panel B reports the trace test statistics which reject that there are up to 2 cointegration relations in the VAR model for Danish data and up to 3 cointegration vectors when using Swedish data.

These results do not change when we add or subtract lags. Testing for unit roots in the data reveals that there is a potential unit root in the user cost γ and housing investment Δh. Judging from the graphs in Figures 1, 2 and B.1 the user cost and housing investment move along broken linear trends. The Danish user cost remains fairly stable until the mid 1980s when it increased sharply and remained at a higher level until the mid 1990s when it started to slowly fall over time. The Swedish user cost increased sharply during the banking crisis in the early 1990s and reached its peak in 1993. From then on it fell to a level comparable to the level in the early 1990s. A similar pattern is also noted for housing investment in Sweden where investment fell substantially during the banking crisis and is now settled on a very low level. Danish housing investments seem more stable in comparison. It may be that the cointegration tests pick up these secular changes and identify them as unit roots. In our empirical VAR model we therefore include a linear trend in order to capture these secular movements.

In principle, our method would work even if the VAR model is non-stationary. What we need to do in that case is to estimate the Vector Error Correction representation and then compute the companion matrix in equation (21). Estimating the VEC representation under the assumption that there are three cointegration vectors in the Danish model, we can then estimate the fundamental house price. Comparing this estimate to the one we obtain under the assumption of full rank, we find very similar results and that the quantitative conclusions drawn from both models are consistent. Therefore we decided to continue under the assumption that the VAR model for Danish data is stable. Similarly,

Table 1: VAR model diagnostics.

Panel A				
	Denmark		Sweden	
Lags in VAR	3		4	
Equation	ARCH	Jarque-Bera	ARCH	Jarque-Bera
Δp_t^a	0.002	0.021	0.114	0.812
Δr_t	0.001	0.000	0.304	0.000
γ_t	0.107	0.000	0.719	0.373
Δy_t	0.002	0.000	0.542	0.825
Δh_t	0.056	0.000	0.172	0.006
LM-test(6)	0.628		0.700	

Panel B		
	Denmark	Sweden
Max(modulus)	0.956	0.920
Hypothesis	Trace test	Trace test
r=0	158.1***	150.0***
r=1	105.0***	96.3***
r=2	54.6***	60.1***
r=3	12.7	26.2**
r=4	4.2	7.5

Note: Panel A: Only p-values are shown in the table. ARCH refers to an LM-test for autoregressive conditional heteroskedasticity, Jarque-Bera is a test for normality and LM-test refers to a multivariate test for autocorrelation using 6 lags. Panel B: Max(modulus) is the maximum eigenvalue of the VAR model and Trace test refers to the standard Johansen trace test. All tests are based on VAR models with 3 (4) lags for Denmark (Sweden) data with a constant and a linear trend.

since we found no strong evidence suggesting that the Swedish VAR model is unstable, we will continue under the assumption that our VAR model is stationary.

Having estimated the VAR(n) model, we can transform it into the VAR(1) model (20) by using (21). We may then apply (22) to forecast the relevant variables in our two theoretical house price models and plug the resulting forecasts into (26) to obtain an estimated time series for the ratio of the fundamental house price to the imputed rent. As a final step, we can use these estimates to derive a time series for the fundamental house price from (29) or (32). For brevity, our analysis below will focus on the Supply-and-Demand model.[7]

6.2 Fundamental versus actual house prices

Figures 3 and 4 display the actual house price along with the estimates of the fundamental house price implied by the Supply-and-Demand model, computed from (32) plus (33), (34) and (36), for Denmark and Sweden. As a baseline case the left graph assumes that $\varepsilon_Y = \varepsilon_R = 1$ while the right graph shows estimates for the case when $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$ as a robustness check. The dotted lines in the figures are 90 percent confidence bands around the fundamental house price computed using a non-parametric bootstrap with 999 trials, see Appendix C for details.

The Danish house price bubble in the mid 2000s stands out clearly in Figure 3. Irrespective of our assumptions regarding the price elasticity ε_R we find that actual house prices exceeded the fundamental price level significantly during this period. We can also see signs of overpricing in the Danish housing market in the latter part of the 1980s, but the Danish bubble in the mid 2000s was much more severe and shows a typical bubble phenomenon with a sharp increase in house prices during the boom followed by a sudden bust where prices fell substantially during a very short period of time. However, the recent (2015:1) house price is very close to the estimated fundamental price when we use our benchmark assumptions about the elasticities and almost spot on for the alternative assumption that the price elasticity is equal to -0.5 while the income elasticity is 1.

The Swedish case is shown in Figure 4. The house price bubble during the Swedish banking and currency crisis is clearly illustrated in these graphs as is the current overpricing. Irrespective of our assumptions about the elasticities, we find significant overpricing on the Swedish housing market since the beginning of 2010. The amount of overpricing during the most recent period mimics the overpricing during the banking crisis and this has been recognized as a potential threat to the Swedish economy.[8]

Figures 3 and 4 indicate that deviations of actual house prices from their fundamental

[7] The estimates using the rent model are available from the authors upon request .

[8] Our conclusions for Denmark and for Sweden are robust to alternative assumptions about the elasticities. These graphs are not shown here for brevity but are available upon request from the authors.

Figure 3: Actual and fundamental house prices in Denmark ($\varepsilon_Y = \varepsilon_R = 1$ in the LHS graph and $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$ in the RHS graph).

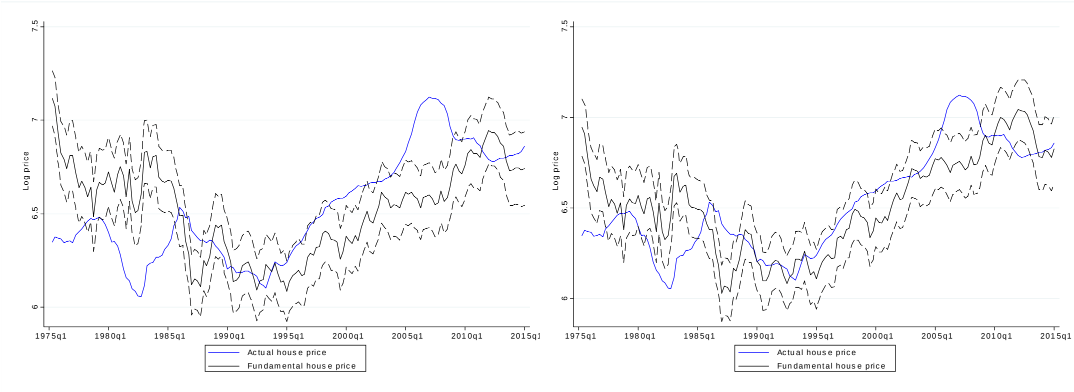
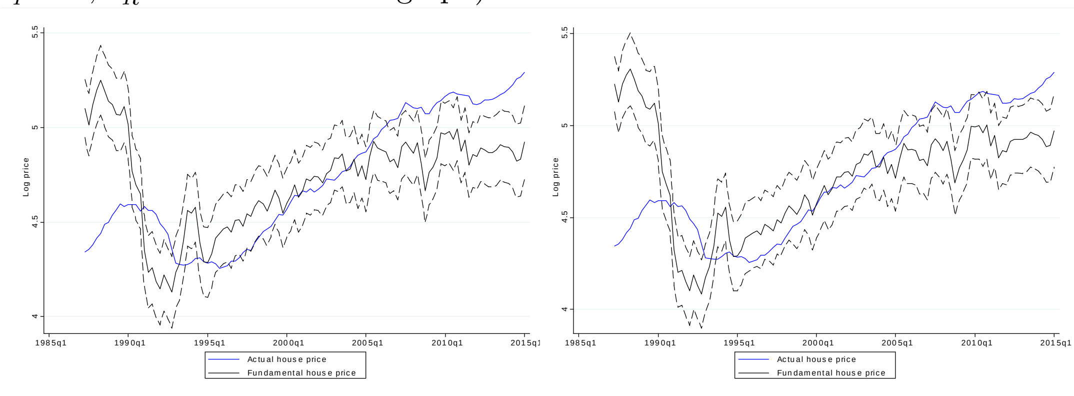


Figure 4: Actual and fundamental house prices in Sweden ($\varepsilon_Y = \varepsilon_R = 1$ in the LHS graph and $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$ in the RHS graph).



level may be quite persistent. Below we will provide measures of the speed of adjustment of actual house prices towards the fundamental level.

7 Analyzing the interaction of actual and fundamental house prices

7.1 Modeling the gap between actual and fundamental house prices

Having computed the fundamental house price, we can now analyze the behavior of the gap between the actual and the fundamental price. One interesting question is whether the actual price tends to converge towards the fundamental price? In the short and medium term actual house prices may deviate from the fundamental price level due to various frictions in the housing market and due to temporary house price bubbles, but in the long run one would expect actual prices to converge on fundamental prices. Empirically this would imply that the gap between actual and fundamental prices is mean-reverting. At the same time we would expect actual prices to react to changes in fundamental prices, but not necessarily the opposite since the fundamental price does not depend directly on the actual price according to eq. (3).

All of these hypotheses can be analyzed in terms of a bivariate VAR model comprising the actual and the fundamental house price. Define $q_t \equiv \begin{bmatrix} p_t & p_t^a \end{bmatrix}'$, where we recall that p_t^a is the log of the actual real house price whereas p_t is the log of the fundamental real house price. Further, assume that q_t is generated by the following Vector Error Correction Model (VECM),

$$\Delta q_t = \widetilde{\mu} - \alpha\beta' q_{t-1} + \sum_{i=1}^{n-1} \Gamma_i \Delta q_{t-i} + \varepsilon_t, \tag{37}$$

where $\widetilde{\mu}$ is a 1×2 vector of constants (which may be zero) and ε_t is a 1×2 vector of shocks. The hypothesis that the gap between the actual and the fundamental house price is mean-reverting implies that these two variables must be cointegrated with the cointegration vector $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}$. Moreover, we expect that the actual house price reacts negatively to a positive deviation from the fundamental price, and vice versa. This implies that the adjustment coefficient α_2 in the α-vector in (37) should be negative and significantly different from zero. Our theoretical model also suggests that the fundamental house price should not respond to any gap between actual and fundamental prices, i.e., $\alpha_1 = 0$.

The VECM stated above also allows us to analyze the effects of a shock to the fundamental price or a shock to the actual price. Provided the there exists one cointegration vector in the bivariate system, we can identify one common stochastic shock and one transitory shock in this system. The common stochastic shock (or stochastic trend) will have a permanent effect on both variables but no long-run effect on the gap, given that the gap is stationary. The transitory shock on the other hand can only have short-run effects on the variables and therefore also on the gap. In Appendix D we show how the procedure suggested by Bergman et al. (2011) may be applied to the VEC system (37) to identify permanent and transitory shocks to the housing market.

Having identified the two structural shocks, we can estimate the impulse response functions associated with each structural shock. These impulse responses can then be used to analyze the speed of adjustment in each variable, including the gap. In this manner our VECM allows us to measure the speed of convergence of actual house prices towards their fundamental level following a permanent change in the fundamental house price. Earlier studies of fundamental house prices such as Hott and Monnin (2008) and Hott and Jokipii (2012) do not examine these issues.[9]

[9] Note that the fundamental house price in the bivariate VAR model (37) is computed from estimates of

7.2 Does the house price gap behave in accordance with theory?

We will now use our two datasets to estimate the VECM in equation (37) and investigate whether house prices do in fact behave as predicted by theory. Based on model specification tests prior to the final estimation, we incorporate 6 lags for Denmark and 4 lags for Sweden. The cointegration vector includes a linear trend, i.e., we have a restricted trend in both models.

Our first step is to test for the number of cointegration vectors. Next we test whether the null that the gap between the actual and the fundamental house price is stationary can be rejected. Table 2 reports the Johansen trace tests for cointegration, the LR-test of the null that the gap is stationary, and estimates of the adjustment coefficients. Overall we find some support for our hypothesis that the models contain one cointegration vector. For the Danish data we find large p-values (exceeding 0.5) for the hypothesis that the rank is 1, indicating that we cannot reject the null that the rank is equal to 1. For the Swedish data we find strong rejection of the null that the rank is 0 whereas the p-values for the null that the rank is 1 is above the 10 percent level for our benchmark model with $\varepsilon_Y = \varepsilon_R = 1$ but only above the 5 percent level for our alternative model with $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$. Comparing the eigenvalues used in the Johansen test reported in the first two columns of Table 2 shows that the second eigenvalue is substantially smaller than the first eigenvalue, indicating that it is likely that the rank is one for both models.

Assuming there is one cointegration vector present, we then perform tests of the hypothesis that the gap between actual and fundamental house prices is stationary. The results are reported in the third column of Table 2. The p-values indicate that we cannot reject the null that the gap is stationary at conventional significance levels for any of the cases. Continuing under the assumption that the gap is indeed stationary, we estimate the adjustment coefficients. As mentioned above, we expect that the actual house price should respond negatively to a widening of the gap. Therefore we expect to find that $\alpha_2 < 0$. At the same time we expect that the fundamental house price should be independent of the gap, deviations of the actual house price from the fundamental price should have no effect on the fundamental house price. Therefore, we expect to find that $\alpha_1 = 0$. The estimates shown in Table 2 suggest that the actual house price is responding to the gap. The coefficient is negative, implying that when actual house prices exceed the fundamental price, actual house prices will fall, and vice versa. This holds for all four models. The hypothesis that the fundamental house price should not adjust to the gap

the VAR model in (17) and therefore includes measurement errors. In order to take this uncertainty into account when computing the impulse responses we apply a bootstrap approach. As was mentioned above, we use a non-parametric bootstrap to generate confidence bands around our estimate of the fundamental house price. For each trial we also set up the bivariate VAR model, identify the structural shocks and compute the implied impulse responses. It is then straightforward to construct confidence bands for the impulse responses. Appendix C provides a detailed description of the bootstrap we use.

seems to be consistent with the Swedish data but not with the Danish data. As can be seen in the first two rows of the table, we find that α_1 is significant in the Danish case. This is an inconsistency with the underlying assumption of our theoretical model. Our conclusion then is that the model for Sweden satisfies the underlying assumptions of our theoretical model whereas the model for Denmark does not fully comply with all these assumptions.

Table 2: Johansen LR-trace tests, tests of the null that the gap between actual and fundamental house prices is stationary and estimates of the adjustment coefficients in VECM.

			LR-trace test				
	λ_1	λ_2	$r=0$	$r=1$	$\beta' = \begin{bmatrix} 1 & -1 \end{bmatrix}$	α_1	α_2
			Denmark				
$\varepsilon_Y = 1$, $\varepsilon_R = 1$	0.096	0.034	20.86	5.26	1.775	0.062	-0.019
			[0.144]	[0.535]	[0.183]	[2.540]	[-2.211]
$\varepsilon_Y = 1$, $\varepsilon_R = 0.5$	0.112	0.034	23.59	5.35	1.77	0.068	-0.025
			[0.093]	[0.556]	[0.184]	[2.404]	[-2.454]
			Sweden				
$\varepsilon_Y = 1$, $\varepsilon_R = 1$	0.216	0.098	33.69	10.34	0.24	0.076	-0.063
			[0.003]	[0.114]	[0.622]	[1.382]	[-4.794]
$\varepsilon_Y = 1$, $\varepsilon_R = 0.5$	0.199	0.106	32.37	11.32	0.68	0.075	-0.049
			[0.005]	[0.078]	[0.408]	[1.630]	[-4.183]

Note: The first two columns report the eigenvalues of the Johansen test. The next two columns report the Bartlett corrected Johansen trace tests. The fifth column reports LR test of the null that the gap between actual and fundamental house price is stationary. P-values are shown within parentheses below each test statistic. The last two columns report the estimated adjustment coefficients under the assumption that the gap is stationary. The number of lags in the model for Denmark is 6 and we use 4 lags in the model for Sweden. Both models are estimated with a restricted trend.

7.3 Impulse responses: How long does it take for the housing market to adjust to a shock?

The VECM in (37) can also be used to estimate the impulse responses of the variables to structural shocks. The empirical results discussed in the previous subsection suggest that there is one cointegration vector in the system while the tests for stationarity of the house price gap suggest that this hypothesis cannot be rejected. As long as there exists one cointegration vector in the system, the variables are affected by one common trend shock and one transitory shock. If the gap is stationary, the effects of the common trend shock on each variable cancel out such that the long-run effect on the gap is zero. We will now assume that the gap is stationary and estimate the impulse responses using the method of Bergman et al. (2011) explained in detail in Appendix D. In addition, we will use the estimated impulse responses to calculate the implied half-life of a shock (the time it takes until half the initial effect of the shock disappears).

We associate the common trend shock with a shock to fundamental house prices which should have a permanent effect on the fundamental house price as well as on the actual house price. Since we also assume that the gap is stationary, this assumption imposes an identifying restriction on the impulse responses such that the difference between the actual and the fundamental house price approaches zero when the horizon goes to infinity. The transitory shock is associated with shocks to the actual house price. Based on our theoretical observations, we identify this shock by assuming it has no long-run effects on either variable in the system.

Figures 5 and 6 show the effects of the two structural shocks in the S-D model with $\varepsilon_Y = \varepsilon_R = 1.0$.[10] Both figures include 68% confidence bands calculated using the bootstrap simulation outlined in Appendix C with 999 trials. The qualitative impulse responses are quite similar in both countries: A transitory shock, which we associate with a temporary shock to the actual house price, has a negative effect on the fundamental house price and a positive effect on the gap. A positive permanent shock to the fundamental house price leads to a strong positive initial effect and then the fundamental price gradually falls back to its new long-term level. The actual house price also increases somewhat but the effect is quantitatively smaller than the effect on the fundamental house price. As both prices adjust, the gap, which is initially negative, returns to its initial level of zero. These effects are consistent with the predictions of the theoretical model.

The central estimates of the impulse responses in figure 5 imply that in Denmark it takes about 3.8 years following a shock to the actual house price before half of the house-price gap is closed. The reason for the slow adjustment is that the impulse response is increasing over the first year and a half before it starts to decrease. In case of a shock to the Danish fundamental house price, the adjustment of the house price gap is somewhat faster, but the concept of the half-life of the gap is difficult to apply in this case since there is a significant overshooting of the gap before it returns to zero.

According to figure 6 the adjustment to a shock to the actual house price in Sweden is faster, with a half-life of 1.7 years for the house price gap. In case of a shock to the fundamental house price, the half-life of the Swedish gap is 1.9 years.

[10] The impulse responses shown in Figures 5 and 6 are robust to changes in the price and income elasticities, see Figures E.1 and E.2 in Appendix E.

Figure 5: Impulse responses of fundamental house prices, actual house prices and the gap to a one standard deviation shock to the actual house price (upper row) and to a one standard deviation shock to fundamental house prices (lower row), ($\varepsilon_Y = \varepsilon_R = 1$), Denmark.

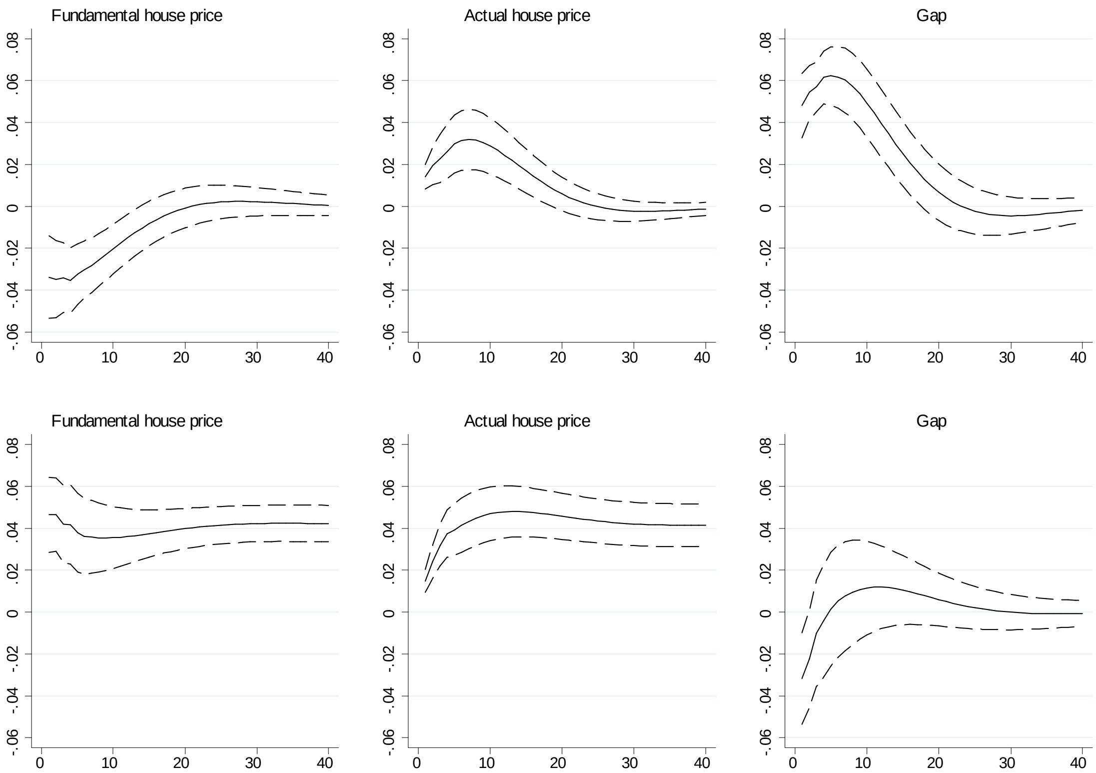
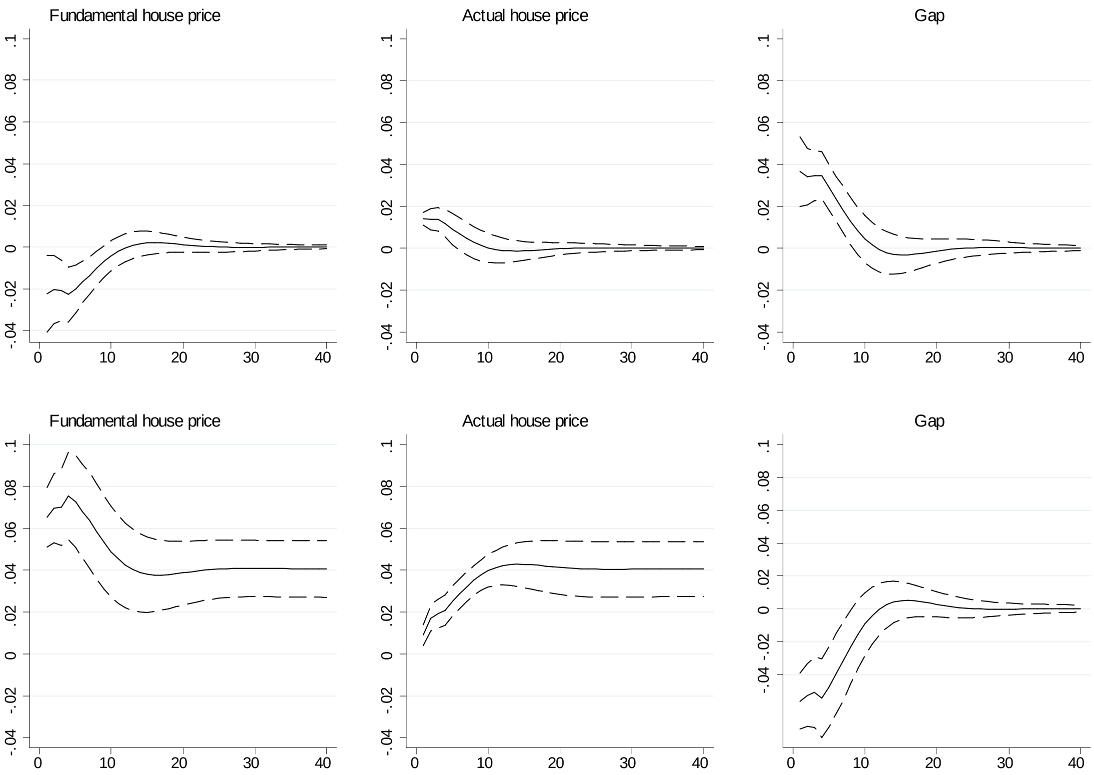


Figure 6: Impulse responses of fundamental house prices, actual house prices and the gap to a one standard deviation shock to the actual house price (upper row) and to a one standard deviation shock to fundamental house prices (lower row), ($\varepsilon_Y = \varepsilon_R = 1$), Sweden.



8 Simulating policy-induced shocks to the housing market

Section 6.2 showed how our model may be used to evaluate whether actual house prices are out of line with fundamentals. Our analysis suggested that the Swedish housing market is currently overvalued. In such a situation policy makers may wish to prick the housing bubble before it grows to dangerous proportions. A policy intervention in the housing market will typically work via a change in the user cost of housing which we have specified as $\gamma_t \equiv i_t (1 - \tau_t^i) - \pi_t + \tau_t + \delta + \eta$. For example, a limitation on or abolition of the deductibility of mortgage interest payments will affect the user cost via its impact on the effective capital income tax rate τ_t^i; a change in the property tax regime will affect the effective property tax rate τ_t, and restrictions on loan-to-value ratios will increase the parameter η which captures the effects of credit constraints (see Sørensen, 2012, Appendix).

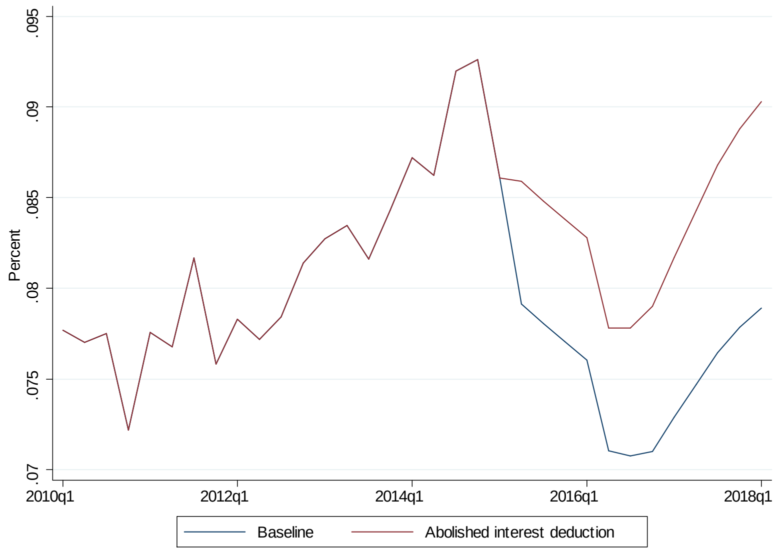
In this section we illustrate how the VAR model in equations (20) and (21) may be used to simulate the effects on actual and fundamental house prices of a policy-induced shock to the user cost. We consider a policy reform which causes a permanent level change in the user cost of owner-occupied housing. We assume that the policy change is unanticipated before it is implemented but is fully built into the forecasts of all future user costs from the time of implementation. With such a reform causing a permanent structural break in the time series for the user cost, the VAR model (20) and (21) estimated on historical data cannot be used to forecast the future level of user cost. However, we assume that the model still gives a correct description of the links between the user cost and the *other* variables in the VAR model. To simulate the effects of a permanent policy-induced shock to the user cost, we therefore feed an exogenous future time path for the user cost γ_t into the VAR model and then use the model to calculate the future values of the other variables determining the evolution of the housing market (i.e., Δp_t^a, Δr_t, Δy_t, and Δh_t).

We apply this procedure to our data set for Sweden and assume that, starting from the second quarter of 2015, the user cost in each period increases by an amount $i_t \tau_t^i$, as would be the case if the deductibility of mortgage interest payments were abolished.[11] To fix the exogenous time path for the user cost from 2015:2 onwards, we assume that the interest rate variable i_t follows the interest rate forecast included in the Monetary Policy report published by the Swedish central bank (the Riksbank) in February 2015. This forecast covered the three-year period 2015:2 to 2018:2, so we cut off our simulation at the end of the latter quarter. The interest rate forecasted by the Riksbank is the

[11] Strictly speaking, if the taxation of positive net capital income is maintained, the opportunity cost of capital for individuals with positive financial net wealth would not be affected. We ignore this complication here since we are merely interested in simulating a shock to the user cost of a certain magnitude.

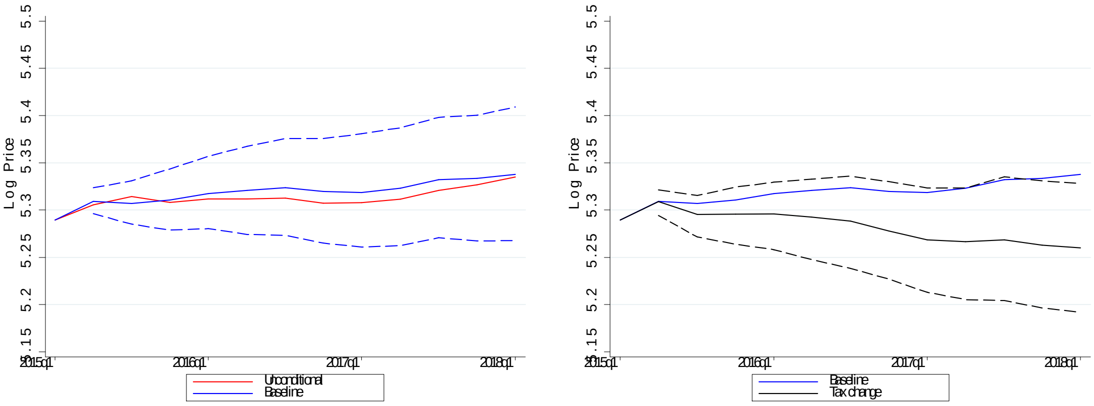
monetary policy rate (the repo rate) whereas our variable i_t is the SPINTAB mortgage rate. Over the period from June 1994 until February 2015 the average spread between the repo rate and the mortgage rate was about 2.7 percentage points. We assume that the same (constant) spread will prevail during the time span 2015:2 through 2018:2. In addition to future mortgage rates we also need an estimate of inflation expectations to calculate future user costs. We use the inflation forecasts in the Monetary Policy report from February 2015 since the Riksbank's predictions of future repo rates are dependent on the bank's expected future inflation rates. Finally, we assume that the effective property tax rate τ_t remains at the value prevailing in 2015:2 and that the user cost component $\delta + \eta$ remains constant. Figure 7 shows the implied user cost paths in the case with and without abolition of interest deductibility. We see that the user cost is expected to fall in both cases, but abolishing interest deductibility would significantly reduce the magnitude of the fall.

Figure 7: User cost under different scenarios.



Having fixed a future time path for the user cost in this way, we can feed it into our VAR model to produce a conditional forecast of actual and fundamental house prices, given the assumed permanent policy shock to the user cost. We can then compare these simulations to a baseline case where interest deductibility is maintained, but where the interest rate and the expected inflation rate (and hence the user cost) are still given by the forecasts of the Riksbank. All our simulations use the Supply-and-Demand version of our model with $\varepsilon_Y = \varepsilon_R = 1$. To check whether our assumed exogenous future time path for the user cost could seriously bias our predictions regarding future house prices, we also compare our baseline scenario with exogenous user costs and no policy reform to another baseline scenario where the future user cost is calculated endogenously by means

Figure 8: Effects of tax changes on actual house prices in Sweden ($\varepsilon_Y = \varepsilon_R = 1$).

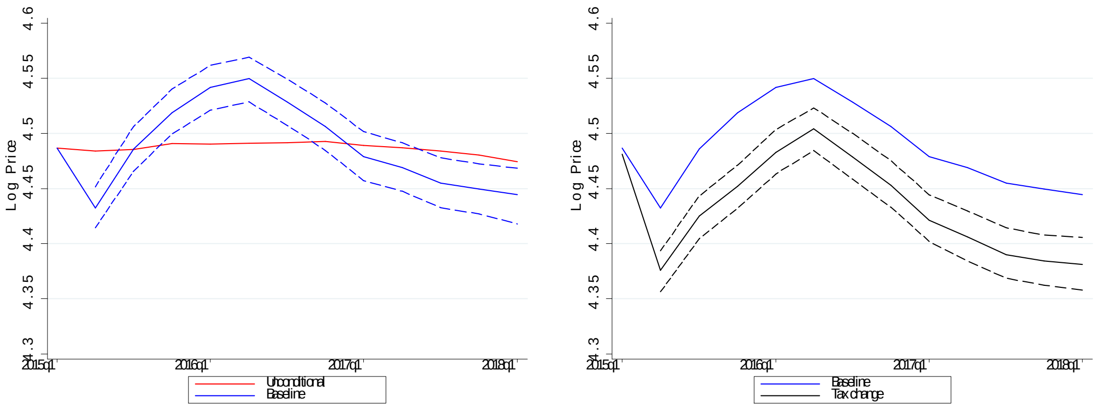


of our VAR model.

Our simulation results for actual house prices are shown in figure 8 which includes 68 percent confidence bands derived from bootstrap simulations with 1000 trials. The left panel in figure 8 shows that the forecasts for actual house prices in the baseline case without policy reform do not differ very much depending on whether future user costs are treated as exogenous (the scenario denoted as “Baseline” which is calculated from the Riksbank forecasts of future rates of interest and expected inflation) or whether they are calculated endogenously by our VAR model (the scenario denoted as “Unconditional”). This suggests that the baseline scenario marked by the blue lines in figure 8 is in fact plausible, there is no significant difference between the baseline scenario and the unconditional forecasts using the VAR model. Given this baseline scenario, we then add an abolition of mortgage interest deductibility. The predicted outcome is shown in the right panel of Figure 8 where the black line is the conditional forecast based on abolition of interest deductibility (known by households from the time of implementation in the second quarter of 2015). Our estimations suggest that the abolition of interest deductibility would reduce actual real house prices by about 7.8 percent in 2018:1, a significant effect.

Figure 9 shows how fundamental house prices are estimated to develop in the baseline scenario and the policy reform scenario. The left panel in the figure displays the expected evolution of the fundamental house price in the baseline scenario (based on the Riksbank forecasts of interest and inflation) compared to the unconditional forecast (using the VAR model forecast of the user cost marked by the red line). In the baseline scenario the fundamental house price falls initially, then increases and then finally falls significantly in 2017. The overshooting effect is due to the fact that the user cost falls in the short-term and then increases as can be seen in Figure 7 above. The right panel traces the evolution of the fundamental house price after the policy reform compared to the baseline scenario. There is a significant and relatively strong effect of the abolition of interest deductibility on the fundamental house price. In the first quarter of 2018, the fundamental house

Figure 9: Effects of tax changes on fundamental house prices in Sweden ($\varepsilon_Y = \varepsilon_R = 1$).



price would be 6 percent lower in 2018. In both scenarios the fundamental house price is expected to increase over the first year and then decline; both the short-term increase and the long-term decrease are statistically significant. The house price gap is decreasing over the forecast period, i.e., the fall in actual house prices exceeds the fall in fundamental house prices when comparing the baseline with the policy reform. From the measurements of the half-life in the previous section we know that the gap will be closed eventually but that it takes approximately 4 years for actual house prices to approach the fundamental price level.

Our results in Figures 8 and 9 are relatively unaffected by the choice of income and price elasticities. In Appendix E we show the results of our policy experiments for the case when $\varepsilon_Y = 1$ and $\varepsilon_R = 0.5$, see Figures E.3 and E.4. Comparing the graphs in the Appendix to the ones above we find only minor unsubstantial differences, the main conclusions are unaffected. Abolition of interest deductibility has a large and significant impact on actual as well as fundamental prices and the effects at the three year horizon are in accordance to the ones we obtain in Figures 8 and 9.

9 Concluding remarks

Starting from a standard theoretical model of the housing market, this paper has described a methodology for estimating fundamental house prices and analyzing their interaction with actual house prices. The main contribution of the paper was to develop a unified bivariate vector error correction model allowing a rigorous test of various hypotheses regarding the relationship between actual and fundamental house prices as well as an identification of temporary and permanent shocks to the housing market. Our methodology enabled us to analyze the sensitivity of our results to alternative assumptions regarding the income and price elasticities of housing demand.

To illustrate how the method works, we have applied it to data sets for Denmark and Sweden. Our results indicate that the theoretical concept of the fundamental house price is indeed useful for empirical purposes in the sense that the estimated fundamental house price tends to work as an anchor for the actual house price. Specifically, we found that the data tend to support the hypothesis that the gap between the actual and the fundamental house price is mean-reverting with a zero mean. However, our analysis also suggests that actual house prices adjust rather slowly to fundamentals and may occasionally display bubble-like behavior.

Since the fundamental house price is a forward-looking variable, we would expect it to be useful for predicting future actual house prices. We showed how our empirical model may be used to make conditional forecasts of the effects on actual and fundamental house prices of policy interventions aimed at pricking bubbles in the housing market. By constructing confidence bands around our estimates, we were able to illustrate the considerable uncertainty relating to any estimate of the fundamental house price. Nevertheless, we found that, with high probability, the Swedish housing market is currently out of line with fundamentals.

References

Ambrose, B. W., P. Eichholtz and T. Lindenthal, (2013), "House Prices and Fundamentals: 355 Years of Evidence," *Journal of Money, Credit and Banking*, 45, 477–491.

Bergman, U. M., (1996), "International Evidence on the Sources of Macroeconomic Fluctuations," *European Economic Review*, 40, 1237–1258.

Bergman, U. M., Y.-W. Cheung and K. S. Lai, (2011), "The Common–Trend and Transitory Dynamics in Real Exchange Rate Fluctuations," *Applied Economics*, 43, 1–18.

Campbell, J. Y. and J. Ammer, (1993), "What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns," *Journal of Finance*, 48, 3–37.

Campbell, J. Y. and R. J. Shiller, (1988a), "The Dividend–Price Ratio and Expectations of Future Dividends and Discount Factors," *The Review of Financial Studies*, 1, 195–228.

Campbell, J. Y. and R. J. Shiller, (1988b), "Interpreting Cointegrated Models," *Journal of Economic Dynamics and Control*, 12, 505–522.

Campbell, J. Y. and R. J. Shiller, (1988c), "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance*, 43, 661–676.

Campbell, S. D., M. A. Davis, J. Gallin and R. F. Martin, (2009), "What Moves Housing Markets: A Variance Decomposition of the Rent–Price Ratio," *Journal of Urban Economics*, 66, 90–102.

Cavaliere, G., A. Rahbek and A. M. R. Taylor, (2012), "Bootstrap Determination of the Co-Integration Rank in Vector Autoregressive Models," *Econometrica*, 80, 1721–1740.

Costello, G., P. Fraser and N. Groenewold, (2011), "House Prices, Non–Fundamental Components and Interstate Spillovers: The Australian Experience," *Journal of Banking & Finance*, 35, 653–669.

Englund, P., (2011), "Swedish House Prices in an International Perspective," in, *The Riksbank's inquiry into the risks in the Swedish housing market*. Sveriges Riksbank.

Engsted, T., T. Q. Pedersen and C. Tanggaard, (2012), "Pitfalls in VAR Based Return Decompositions: A Clarification," *Journal of Banking & Finance*, 36, 1255–1265.

European Commission, (2012), "Quarterly Report on the Euro Area," Economic and Financial Affairs, Vol. 11, No. 4, December 2012, 31–38.

Fairchild, J., J. Ma and S. Wu, (2015), "Understanding Housing Market Volatility," *Journal of Money, Credit and Banking*, 47, 1309–1337.

Hiebert, P. and M. Sydow, (2011), "What Drives Returns to Euro Area Housing? Evidence from a Dynamic Dividend–Discount Model," *Journal of Urban Economics*, 70, 88–98.

Hott, C. and T. Jokipii, (2012), "Housing Bubbles and Interest Rates," Swiss national Bank Working Paper 2012-7.

Hott, C. and P. Monnin, (2008), "Fundamental Real Estate Prices: An Empirical Estimation with International Data," *Journal of Real Estate Finance and Economics*, 36, 427–450.

King, R. G., C. I. Plosser, J. H. Stock and M. W. Watson, (1991), "Stochastic Trends and Economic Fluctuations," *American Economic Review*, 81, 819–840.

Kishor, N. K. and J. Morley, (2015), "What Factors Drive the Price-Rent Ratio for the Housing Market? A Modified Present-Value Analysis," *Journal of Economic Dynamics and Control*, 58, 235–249.

Mellander, E., A. Vredin and A. Warne, (1992), "Stochastic Trends and Economic Fluctuations in a Small Open Economy," *Journal of Applied Econometrics*, 7, 369–394.

Sørensen, P. B., (2012), "The Swedish Housing Market: Trends and Risks," Report to the Swedish Fiscal Policy Council.

Stock, J. H., (1987), "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors," *Econometrica*, 55, 1035–1056.

Stock, J. H. and M. W. Watson, (1988), "Testing for Common Trends," *Journal of the American Statistical Association*, 83, 1097–1107.

Appendix A: Data sources for Sweden

The Swedish nominal mortgage interest rate i_t is measured by the nominal 5-year mortgage lending rate for SPINTAB (first day of quarter) and is downloaded from Ecowin. The expected inflation rate is household inflation expectations taken from surveys conducted by the Swedish National Institute of Economic Research. The capital income tax rate is taken from Englund (2011) and the property tax is from Hansson (2013). The real price of a unit of owner–occupied housing, p_t^a, is the nominal price of one- or two-dwelling buildings deflated by the Consumer Price Index (CPI); the real rent on a unit of owner-occupied housing, r_t, is the CPI component of rents for housing, COICOP 04.S downloaded from Statistics Sweden, and Δy_t is the real net household disposable income downloaded from OECD. In order to calculate the fundamental price-to-imputed-rent ratio in the Supply-and-Demand model summarized in (32), we also need an estimate of the stock of housing. This measure is computed by cumulating the time series for gross fixed housing capital formation (taken from the Swedish national accounts) on the assumption that the depreciation rate is constant over the sample.

Appendix B: Data

Figure B.1: Real disposable income, real rent and real housing investments in Denmark (upper panel) and Sweden (lower panel).

Appendix C: A Non-Parametric Bootstrap Technique for Estimating Confidence Bands for the Estimated Fundamental House Price and Impulse Responses in the Bivariate VAR model.

This Appendix explains the non-parametric bootstrap technique used in section 6.2 to generate confidence bands for the estimated fundamental house price and used in section 7.3 to generate confidence bands for the impulse response functions generated from estimates of the bivariate VECM in equation (37).

Consider the VAR-model in equation (17), i.e.,

$$b_t = \Phi_0 + \Phi_1 b_{t-1} + \ldots + \Phi_n b_{t-n} + \varepsilon_t$$

where t is the time period, Φ_0 is the deterministic component and Φ_j, $j = 1, \ldots, n$ are the parameters to be estimated, b_t is the time series vector defined in equation (18), and ε_t is an *i.i.d.* error term with zero mean. In the main text we show that this model can be rewritten as a VAR(1) model, i.e.,

$$z_t = A z_{t-1} + \xi_t$$

where A and ξ_t are defined in (21). Using estimates from this VAR(1) model, we then apply the method outlined in section 4 to compute the fundamental house price.

To generate confidence bands for the estimated fundamental house price we use the following non-parametric bootstrap.

i. Estimate the VAR(1) model to obtain the estimation residuals $\hat{\xi}_t$ and use the estimated parameters in A, denoted $\hat{A}$, to compute the fundamental house price $\hat{p}_t$. Compute centered residuals $\hat{\xi}_1 - \bar{\hat{\xi}}, \ldots, \hat{\xi}_T - \bar{\hat{\xi}}$ where $\bar{\hat{\xi}}$ is the sample average of the estimated residuals and T is the sample size.

ii. Generate bootstrap residuals $\tilde{\xi}_1, \ldots, \tilde{\xi}_T$ by randomly drawing with replacements from the centered residuals.

iii. Compute bootstrap time series recursively using the VAR(1) model

$$\tilde{z}_t = \hat{A}\tilde{z}_{t-1} + \tilde{\xi}_t$$

where $(\tilde{z}_{-n+1}, \ldots, \tilde{z}_0) = (z_{-n+1}, \ldots, z_0)$.

iv. Reestimate the parameters in A based on the bootstrap time series $\tilde{z}_t$. Calculate the modulus of the largest root of $\tilde{A}$. If the largest modulus is less than 1, proceed to the next step, otherwise return to step (ii) as suggested by Cavaliere, Rahbek and Taylor (2012).

v. Based on the estimated parameters $\tilde{A}$, compute the bootstrap estimate of the fundamental house price $\tilde{p}_t$. Use the bootstrap estimate of the fundamental house price and the actual house price and apply the procedure outlined in Appendix D to identify the two structural shocks and estimate the impulse response function.

vi. Repeat steps (ii) to (v) N times.

Having obtained N estimates of the fundamental house price we finally construct the 90 percent confidence bands using the standard percentile method.

Appendix D: Identifying structural shocks to the housing market

This appendix applies the procedure suggested by Bergman et al. (2011) to illustrate how the VEC system (37) may be used to identify permanent and transitory shocks to the housing market. Let us assume that that the gap between the actual and the fundamental house price is mean-reverting so that the cointegration vector $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$. The system innovations ε_t can then be decomposed into a common permanent component and a transitory component. The permanent innovation has a permanent effect on both p_t^a and p_t, but no long-run effect on the gap $\beta' q_t$. The transitory innovation only has short-term effects on the variables in q_t as well as on the gap $\beta' q_t$.

Since Δq_t is stationary, the Wold decomposition theorem implies that the VECM in (37) can be given the following Vector Moving Average (VMA) representation,

$$\Delta q_t = \delta + C(L)\varepsilon_t, \tag{D.1}$$

where L is the lag operator, $C(L) = I + \Sigma_{k=1}^{\infty} C_k L^k$, and I is the 2×2 identity matrix with rank $[C(1)] = 1$, $\beta' C(1) = 0$, $\delta = C(1)\widetilde{\mu}$, and $\beta' \delta = 0$. Equation (D.1) can alternatively be expressed as the common-trends model (Stock and Watson, 1988),

$$\Delta q_t = \delta + C(1)\varepsilon_t + (1 - L)C^*(L)\varepsilon_t, \tag{D.2}$$

by writing $C(L)$ as $C(1) + (1 - L)C^*(L)$ where $C^*(L) = \sum_{i=0}^{\infty} C_i^* L^i$ and $C_i^* = -\sum_{j=i+1}^{\infty} C_j$ for $i \geq 0$ (see Stock, 1987). Hence, assuming that $\varepsilon_0 = 0$, we find by forward iteration of (D.2) that

$$q_t = q_0 + \delta t + C(1) \sum_{i=1}^{t} \varepsilon_i + C^*(L)\varepsilon_t. \tag{D.3}$$

Assume that the *structural* common-trend (CT) model is:

$$q_t = \mu_0 + \Phi \eta_t + Q^*(L) w_t, \tag{D.4}$$

where $\eta_t = \rho + \eta_{t-1} + \varphi_t$; $Q^*(L)$ is a stationary lag polynomial, and $w_t = \begin{bmatrix} \varphi_t & \psi_t \end{bmatrix}'$, with φ_t being the common stochastic shock and ψ_t being the transitory shock. The common stochastic trend, η_t, determines the trending behavior of the variables in q_t through the loading matrix, Φ. The transitory dynamics of the system are governed by $Q^*(L)w_t$. In addition, since cointegration implies that $\beta' \Phi = 0$, the dynamics of the gap between the actual and the fundamental house price is given by $\beta' \mu_0 + \beta' Q^*(L) w_t$.

Note that the structural common trends model in (D.4) is linked to a structural VMA representation since we can rewrite $Q(1) + (1 - L)Q^*(L)$ as $Q(L)$. Let the transformation matrix F link the estimated residuals in ε_t to the structural shocks in w_t, $w_t = F\varepsilon_t$. Then we have the following link between the structural VMA model and the unrestricted VMA model

$$\Delta q_t = \delta + C(L)\varepsilon_t = \delta + C(L)F^{-1}F\varepsilon_t = \delta + Q(L)w_t. \tag{D.5}$$

The matrix $F = \begin{bmatrix} F_\varphi & F_\psi \end{bmatrix}'$ identifies the individual shocks to the common stochastic trend and the transitory shocks of the system.

To determine the matrix $F = \begin{bmatrix} F_\varphi & F_\psi \end{bmatrix}'$ in the system (D.5), we first derive $C(L)$ and then find the CT representation of the VMA model. The basic analysis is based on King, Plosser, Stock and Watson (1991), Mellander, Vredin and Warne (1992), and Bergman (1996). Following Campbell and Shiller (1988b), define

$$M = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}. \tag{D.6}$$

Also, let $\Gamma(L) = I - \sum_{j=1}^{p-1} \Gamma_j$. Premultiplying both sides of the VEC model in (37) yields

$$M\Gamma(L)\Delta q_t = M\widetilde{\mu} - M\Pi q_{t-p} + M\varepsilon_t, \tag{D.7}$$

where Π is equal to the matrix $\alpha\beta'$ in the VECM (37). Now define a stationary variable $s_t = D_\perp(L)MX_t$, where $D_\perp$ is a diagonal matrix with its diagonal elements given by $D_\perp(L)_{11} = 1-L$ and $D_\perp(L)_{22} = 1$. We can then write $B(L)s_t = M\widetilde{\mu} + M\varepsilon_t$ where $B(L) = M\left[\Gamma(L)M'D(L) + \alpha^* L\right]$ and $\alpha^* = \begin{bmatrix} 0 & \alpha \end{bmatrix}$. Moreover, the adjustment coefficients are given by $\alpha_1 = B(1)_{12}$ and $\alpha_2 = B(1)_{12} - B(1)_{22}$.

Comparing the structural CT model in (D.4) with equation (D.3) gives $\Phi\varphi_t = C(1)\varepsilon_t$. Since $\beta'\Phi = 0$ and $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$, we then obtain

$$\varphi_t = \det\left(B(1)\right)^{-1} \begin{bmatrix} -\alpha_2 & \alpha_1 \end{bmatrix} \varepsilon_t = F_\varphi \varepsilon_t. \tag{D.8}$$

Following the procedure described by Mellander, Vredin and Warne (1992), we also find that

$$\psi_t = \left(\alpha'\Omega^{-1}\alpha\right)^{-1/2} \alpha'\Omega^{-1}\varepsilon_t = F_\psi \varepsilon_t, \tag{D.9}$$

where Ω is the variance-covariance matrix associated with the vector of error terms, ε_t. The transitory innovation, ψ_t, generates temporary effects on the actual and the fundamental house price and hence no permanent effects on the gap between these two variables. The common trend innovation, φ_t, generates long-lasting effects on both variables, but these effects will cancel out over the long run, leaving no permanent effect on the gap between the actual and the fundamental house price, given that this gap is stationary.[12] The impulse responses of the actual and the fundamental house price are given by $C(L)F^{-1}$, and those of the gap between the actual and the fundamental house price are given by $\beta'C(L)F^{-1}$.

[12] If the gap is not stationary while there still exists a cointegration vector, then CT innovations have long-run effects on the gap as well as on the two variables in q_t.

Appendix E: Robustness Analysis

Figure E.1: Impulse responses of fundamental house prices, actual house prices and the gap to a one standard deviation shock to the actual house price (upper row) and to a one standard deviation shock to fundamental house prices (lower row), ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$), Denmark.

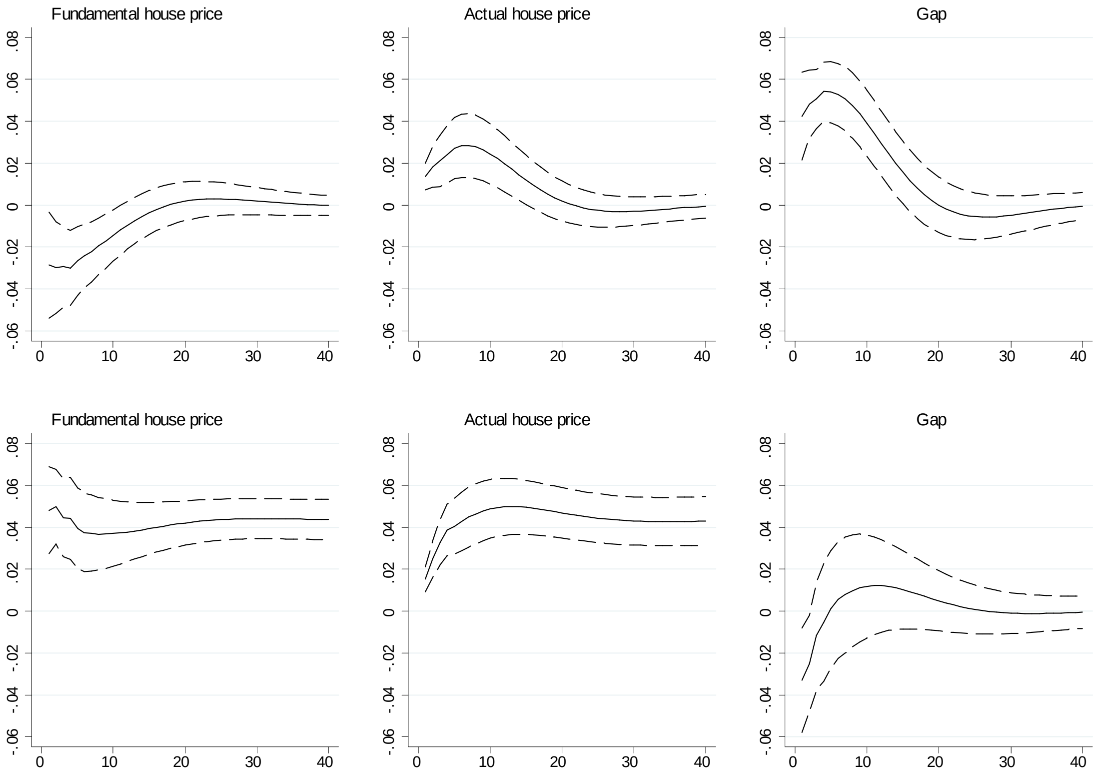


Figure E.2: Impulse responses of fundamental house prices, actual house prices and the gap to a one standard deviation shock to the actual house price (upper row) and to a one standard deviation shock to fundamental house prices (lower row), ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$), Sweden.

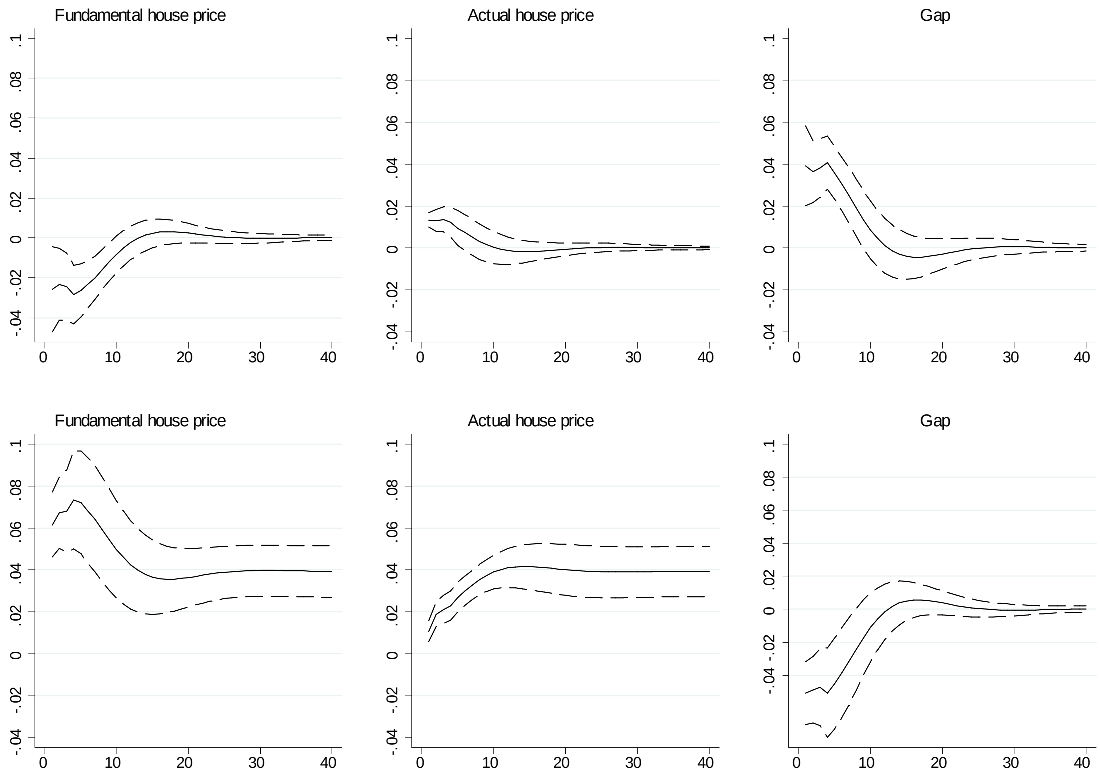


Figure E.3: Effects of tax changes on actual house prices in Sweden ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$).

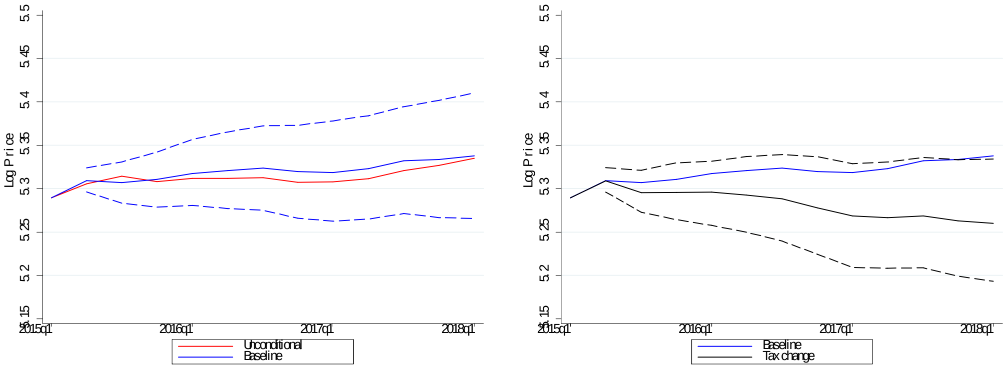


Figure E.4: Effects of tax changes on fundamental house prices in Sweden ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$).