The Interaction of Actual and Fundamental House Prices: A General Model with an Application to Sweden

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Abstract

The paper presents a general method for estimating a country's level of fundamental house prices and its interaction with actual house prices. We set up a unified empirical model which can be used to analyze the time-series behavior of the fundamental house price and to test various hypotheses regarding its relation to the actual house price. To illustrate how the method works, we apply it to data for Sweden. We find a tendency for actual house prices to converge on fundamental prices, albeit rather slowly.

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KEY WORDS: FUNDAMENTAL HOUSE PRICES, HOUSE PRICE DYNAMICS, HOUSING BUBBLES.

1 Introduction

Housing bubbles and their bursting have played a key role in many financial crises. Analytical tools that may help policy makers to spot a housing bubble before it grows dangerously big should therefore improve the basis for macroeconomic stabilization policy. This paper presents an empirical methodology for estimating whether a country's level of

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house prices is out of line with fundamentals. To illustrate its practical use, we apply the method to data for Sweden.

Our method for estimating fundamental house prices broadly follows the one developed by Hott and Monnin (2008) which is based on the dividend-discount model of asset prices pioneered by Campbell and Shiller (1988a,1988c). This model has been used recently and in different contexts to analyze housing markets by, i.a., Campbell et al. (2009), Costello et.al. (2011), Hiebert and Sydow (2011), Hott and Jokipii (2012), the European Commission (2012), Ambrose et al. (2013), Fairchild et.al. (2015) and Kishor and Morley (2015).

Our paper adds to the literature on housing bubbles in several directions. First, we extend the standard supply-demand model (Hott and Monnin, 2008 and others) by specifying a more general housing demand function allowing us to explicitly analyze the sensitivity of the estimated fundamental house price to the income and price elasticities. In previous work, it has been assumed that these elasticities are equal to unity. Second, we add to the empirical analysis of the dividend-discount model by setting up a unified empirical model of the interaction between actual and fundamental house prices. The model allows us to address the following questions which have not (to our knowledge) been answered in the previous literature in this area: Do actual house prices converge on fundamental house prices? If so, how long does it take before a gap between the actual and the fundamental price is closed? How can one identify shocks to fundamental and actual house prices? How sensitive are estimates of the fundamental house price to estimates of the long-run price and income elasticities of housing demand? How can one calculate a confidence band for the estimate of the fundamental house price and how broad is that band?

Most importantly from a policy perspective, we show that our empirical model generates ex ante real-time estimates of actual and fundamental house prices which are quite close to the subsequent ex post estimates, suggesting that the model can be used as an early warning indicator when a housing bubble is building up.

The paper is structured as follows: Section 2 presents the theoretical model of fundamental house prices. Section 3 describes the empirical strategy for modeling the forward-looking expectations determining fundamental house prices, and section 4 explains how the fundamental house price may be calculated, once the formation of expectations has been modeled. Section 5 presents our data sets and section 6 applies the general methodology laid out in sections 2 through 4 to estimate fundamental house prices in Sweden, comparing ex post and ex ante real-time estimates. In section 7 we set up a unified econometric model allowing an analysis of the interaction between actual and fundamental house prices, and section 8 illustrates how our model may be used to analyze the effects of policy-induced shocks to the housing market. Section 9 summarizes our main findings.

2 The theoretical model of the fundamental house price

The dividend-discount model has been applied to the analysis of housing markets and, in particular, to the estimation of the fundamental house price (Hott and Monnin, 2008; Campbell et al., 2009; Costello et.al., 2011; Hiebert and Sydow 2011; Hott and Jokipii, 2012; the European Commission, 2012; Ambrose et al., 2013; Fairchild et.al., 2015; and Kishor and Morley, 2015). The fundamental house price can be viewed as the equilibrium house price given that households have rational expectations about the fundamental factors (future income, interest rates and rents) affecting the future value of housing service. Under an assumption that there are no bubbles in the fundamental house price we can estimate the implied equilibrium price and analyze the deviations of actual housing prices from the fundamental prices. This comparison can shed light on the question whether the housing market is overpriced or underpriced. In this section, we define the concept of fundamental house prices and its determinants.

The imputed rent on a unit of owner-occupied housing is the amount the consumer is willing to pay for the housing service, i.e., the marginal rate of substitution between the housing service and all other goods. Standard consumer theory implies that, in a utility maximum, this marginal rate of substitution is

$$R_t^H = \left[i_t \left(1 - \tau_t^i\right) - \left\{ \frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c} \right\} + \tau + \delta + \eta \right] P_t - \left\{E_t\left[P_{t+1}\right] - P_t\right\} \qquad (1)$$

where R_t^H is the umputed rent on a unit of owner-occupied housing, i_t is the nominal mortgage interest rate, τ_t^i is the capital income tax rate, P_t^c is the consumer price level, τ is the effective property tax rate (constant), δ is the rate of depreciation of the real housing stock (constant), η is the user cost premium for risk and credit contraints (constant), P_t is the rela price of a unit of owner-occupied housing and $E_t\left[X_{t+i}\right]$ is the expectation held at time t regarding the value of variable X at time $t + i$. If the housing investment is fully debt-financed, the term $\left[i_t \left(1 - \tau_t^i\right) + \tau + \delta + \eta\right] P_t$ is the home-owner's nominal cash expenses, including expenses on repair and maintenance (δ), and the term $\left\{ \frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c} \right\} P_t + \left\{E_t\left[P_{t+1}\right] - P_t\right\}$ is the expected nominal capital gain, consisting of the expected gain $\left\{ \frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c} \right\} P_t$ arising from general inflation plus the expected real capital gain, $\left\{E_t\left[P_{t+1}\right] - P_t\right\}$. If the housing investment is equity-financed, the home-owner forgoes the after-tax interest income he could have earned by investing his wealth in the capital market, so $i_t \left(1 - \tau_t^i\right) P_t$ is still part of the (opportunity) cost of housing. The term ηP_t — which is unimportant for our present purpose — is a premium reflecting risk and possible credit constraints.[1] Rearranging (1), we get an expression for the house price in

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[1] The determinants of this term (which we treat as an exogenous constant in the present paper) are explained in detail in Sørensen (2012, Appendix).

period t:

$$P_t = \frac{R_t^H + E_t\left[P_{t+1}\right]}{1 + \gamma_t}, \qquad \gamma_t \equiv i_t\left(1 - \tau_t^i\right) - \left\{\frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c}\right\} + \tau_t + \delta + \eta. \qquad (2)$$

The variable γ_t is the user cost of owner-occupied housing, *excluding* the expected real capital gain. If agents are rational, they realize that the link between prices and imputed rents is given by (2). By forward iteration, one finds that (2) implies

$$P_t = E_t\left[\sum_{i=0}^{\infty} \frac{R_{t+i}^H}{\prod_{j=0}^{i}\left(1 + \gamma_{t+j}\right)}\right]. \qquad (3)$$

Equation (3) shows that the fundamental house price is the discounted value of expected future imputed rents, where the period-by-period discount rate is given by γ_{t+j}. The discounted sum on the right-hand side of (3) will be finite if the real imputed rent grows at an average rate lower than the average value of γ_{t+j}. The standard definition of the fundamental house price assumes that this condition is met. This is equivalent to ruling out bubbles in the fundamental house price on the housing market. In addition, one can say that there is under- or overpricing in the housing market if the actual house price deviates significantly from the fundamental price given by (3).

The expected future imputed rents in (3) are not directly observable. Inspired by Hott and Monnin (2008), we therefore consider a "Supply-and-Demand model" (S-D model) in which imputed rents are assumed to adjust so as to equilibrate the supply of and demand for housing services. Specifically, suppose the aggregate long-run demand for housing services (D) varies positively with aggregate real disposable income (Y) and negatively with imputed rents so that

$$D_t = BY_t^{\varepsilon_Y}\left(R_t^H\right)^{-\varepsilon_R}, \qquad (4)$$

where B is a constant, ε_Y is the long-run income elasticity of housing demand, and ε_R is a price elasticity measuring the numerical long-run elasticity of housing demand with respect to the imputed rent.[2] The aggregate supply of housing services is proportional to the aggregate housing stock (H), and the proportionality factor may be normalized at unity by appropriate choice of units. In a housing market equilibrium we thus have $H_t = D_t$. From (4) this implies

$$R_t^H = B^{1/\varepsilon_R} Y_t^{\varepsilon_Y/\varepsilon_R} H_t^{-1/\varepsilon_R}. \qquad (5)$$

In the next two sections we will show how the house price model consisting of (3) and (5) can be used to estimate the fundamental house price. For this purpose it will

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[2] This specification of housing demand is more general than the one used by Hott and Monnin (2008). They assume that $\varepsilon_Y = \varepsilon_R = 1$, as would be the case if consumers have Cobb-Douglas utility functions and the savings rate is constant.

be convenient to respecify the model in terms of the price-to-imputed-rent ratio, P_t/R_t^H. Defining

$$s_t \equiv \ln\left(P_t/R_t^H\right), \quad \Delta y_{t+j} \equiv \ln Y_{t+j} - \ln Y_{t+j-1}, \quad \Delta h_{t+j} \equiv \ln H_{t+j} - \ln H_{t+j-1},$$

and assuming rational expectations, we show in appendix A that the housing market model above implies the following approximate expression for the (log of the) price-to-imputed rent ratio,

$$s_t = c + \sum_{j=1}^{\infty} \phi^j E_t \left[\left(\varepsilon_Y/\varepsilon_R\right) \Delta y_{t+j} - \left(1/\varepsilon_R\right) \Delta h_{t+j} - \gamma_{t+j}\right] - \gamma_t, \tag{6}$$

$$\phi \equiv \frac{\exp\left(\overline{s} + \Delta \overline{r}^H\right)}{1 + \exp\left(\overline{s} + \Delta \overline{r}^H\right)}, \tag{7}$$

where c is an unimportant constant, $\overline{s}$ is the mean value of s_t, and $\Delta \overline{r}^H$ is the mean value of $\ln R_{t+1}^H - \ln R_t^H$. In the next section we turn to the issue of modeling expectations and estimating the fundamental house price.

3   Modeling expectations

To apply formula (6) for estimation purposes, we must model the way expectations are formed. Following Hott and Monnin (2008), Campbell et al. (2009), Ambrose et al. (2012) and others, we do so by assuming that agents act as if they based their forecasts on a VAR model. The VAR model should as a minimum include the variables which determine fundamental house prices according to our theoretical model, i.e., Δy_t, Δh_t and γ_t. For consistency, we also include the change in the actual house price, Δp_t^a, where p_t^a is the log of the actual real house price, since knowledge of the current house price is included in the agent's information set in equation (1) which was used to derive the fundamental house price. This is similar to the procedure followed by Campbell and Ammer (1993) and Engsted et al. (2012) who emphasize the need to include the actual current stock price in a VAR-model of the stock market.

When forming expectations about future user-costs, house prices, disposable income and the stock of housing, rational agents might consider more variables than those included in our theoretical partial equilibrium model of the market for owner-occupied housing. In particular, since owner-occupied and rental housing are close substitutes, it is natural to assume that house prices and the housing stock interact with rents in the rental housing sector. We therefore include the change in the log of the real rent on a unit of rental housing (Δr_t) in our VAR model, but for reasons of parsimony we do not include additional variables. In matrix form and abstracting from deterministic components, the VAR forecasting model may then be written as

$$\Phi\left(L\right) b_t = \Phi_0 + \varepsilon_t, \tag{8}$$

where the time series vector b_t is defined as

$$b_t \equiv \begin{bmatrix} \Delta p_t^a \\ \Delta r_t \\ \gamma_t \\ \Delta y_t \\ \Delta h_t \end{bmatrix}, \tag{9}$$

and where $\Phi(L) = I_5 - \sum_{j=1}^{n} A_j L^j$, L is the lag operator, I is a 5×5 identity matrix, ε_t is a 5×1 column vector of white noise processes, and n is the number of lags.

Defining the column vector z_t as

$$z_t \equiv \begin{bmatrix} b_t - \mu \\ b_{t-1} - \mu \\ \cdot \\ \cdot \\ b_{t-n+1} - \mu \end{bmatrix}, \qquad \mu = (I_5 - \Phi_1 - \ldots - \Phi_n)^{-1} \Phi_0, \tag{10}$$

we can rewrite the VAR(n) model in (8) in the following VAR(1) form

$$z_t = A z_{t-1} + \xi_t, \tag{11}$$

where

$$A \equiv \begin{bmatrix} \Phi_1 & \Phi_2 & \cdot & \Phi_{n-1} & \Phi_n \\ I_5 & 0 & \cdot & 0 & 0 \\ 0 & I_5 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & I_5 & 0 \end{bmatrix}, \qquad \xi_t \equiv \begin{bmatrix} \varepsilon_t \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}. \tag{12}$$

Our VAR(1)-model (11) will be covariance stationary if all eigenvalues of the companion A matrix in (12) are less than one in absolute value. We assume this condition to be met.[3]

Using the VAR(1) in (11) and (12), we can now calculate the expected future values of the variables in the VAR(n) model (8) from the relationship

$$E_t [z_{t+i}] = A^i z_t. \tag{13}$$

4   Calculating the fundamental house price

Next we note from the definition of b_t stated in (9) that — abstracting from an unimportant constant which will depend on c and the elements of the column vector μ — our

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[3]Note that this approach does not require that the model is stationary as it is always possible to reformulate a non-stationary VAR model in the VAR(1) form.

house price model in (6) can be written as

$$s_t = \sum_{j=1}^{\infty} \phi^j g_1 E_t \left[z_{t+j} \right] + g_2 z_t, \qquad (14)$$

where the $1 \times 5n$ vectors g_1 and g_2 are defined as

$$g_1 \equiv \left[\begin{array}{cccccccc} 0 & 0 & -1 & \varepsilon_Y/\varepsilon_R & -1/\varepsilon_R & 0 & \cdot & 0 \end{array} \right], \qquad g_2 \equiv \left[\begin{array}{cccccccc} 0 & 0 & -1 & 0 & 0 & 0 & \cdot & 0 \end{array} \right]. \qquad (15)$$

Inserting (13) into (14) and solving for s_t, we obtain

$$s_t = \sum_{j=1}^{\infty} \phi^j g_1 A^j z_t + g_2 z_t \quad \Rightarrow$$

$$s_t = \left[g_2 + \phi g_1 A \left(I - \phi A \right)^{-1} \right] z_t. \qquad (16)$$

Once we have estimated the coefficients in the VAR(n) model in (8), we can use (16) along with the definitions of A, g_1 and g_2 given in (12) and (15) to compute an estimate of the fundamental price-to-imputed-rent ratio. According to (5) and using the definition of s_t we can then back out the estimate of the fundamental house price from the relationship

$$\hat{p}_t = s_t + \left(\varepsilon_Y/\varepsilon_R \right) y_t - \left(1/\varepsilon_R \right) h_t + \beta_0, \qquad (17)$$

where β_0 captures the constant term in (5) plus the conversion factor needed to transform our data into comparable units. We calibrate β_0 so as to minimize the sum of the squared deviations of the (log of the) actual house prices from the estimated (log of the) fundamental house prices. This procedure yields

$$\beta_0 = \frac{1}{T} \sum_{t=1}^{T} \left[p_t^a - s_t - \left(\varepsilon_Y/\varepsilon_R \right) y_t + \left(1/\varepsilon_R \right) h_t \right] = \bar{p}^a - \bar{s} - \left(\varepsilon_Y/\varepsilon_R \right) \bar{y} + \left(1/\varepsilon_R \right) \bar{h}, \qquad (18)$$

which may be inserted in (17) to give

$$\hat{p}_t = \bar{p}^a + \left(s_t - \bar{s} \right) + \left(\varepsilon_Y/\varepsilon_R \right) \left(y_t - \bar{y} \right) - \left(1/\varepsilon_R \right) \left(h_t - \bar{h} \right). \qquad (19)$$

Note that the estimate of the fundamental house price is derived from observable variables, i.e., real disposable income and the real housing stock, and the fundamental price-to-imputed rent ratio which is computed from observables using (16) plus an estimate of ϕ (which is likewise derived from observables, cf. below). As shown by (19) our estimation procedure implies that, on average over the sample period, the level of fundamental house prices equals the actual house price level.

When applying equation (19), we use prior knowledge on the size of the long-run elasticities ε_Y and ε_R.[4] By varying these parameters, we can analyze the sensitivity of the estimated fundamental house price to the income and price elasticities of housing demand.

In order to estimate s_t from (16), we need an estimate of the parameter ϕ defined in (7). In the previous literature it has been common to simply postulate a plausible value of ϕ, but we want our estimate of ϕ to be model-consistent. Since $s_t \equiv p_t - r^H$, $r_t^H \equiv \ln R_t^H$, and since we have already assumed that the mean value of the fundamental house price equals the mean of the actual house price, $\bar{p}^a$, it is natural to assume that

$$\bar{s} = \bar{p}^a - \bar{r}^H, \tag{20}$$

where $\bar{r}^H$ is the mean value of the level of imputed rent. From (5) it follows that

$$\Delta \bar{r}^H = (\varepsilon_Y/\varepsilon_R)\,\Delta \bar{y} - (1/\varepsilon_R)\,\Delta \bar{h}. \tag{21}$$

The mean values $\bar{p}^a$, $\Delta \bar{y}$ and $\Delta \bar{h}$ are directly observable, but to obtain an estimate for ϕ from (7), (20) and (21) we also need an estimate for $\bar{r}^H$. Since we do not observe the initial value of the *level* of imputed rent, we cannot calculate $\bar{r}^H$ from our estimate of $\Delta \bar{r}^H$ in (21). However, since we do observe $\bar{p}^a$, and since we assume that $\bar{p}^a = \bar{p}$, we can use the relationship between R^H and P stated in (1) to infer what a plausible average level of R^H would be, given the observed average house price level. In particular, when measured relative to the average level of house prices, the average level of imputed rent should not deviate too much from an average long-term real interest rate. We have therefore calibrated $\bar{r}^H$ so as to imply that the ratio of the average imputed rent to the average house price level is equal to the average after-tax real mortgage interest rate.[5] Numerical experiments reveal that our estimate for ϕ is not very sensitive to reasonable variations in the assumed magnitude of the real interest rate.

5   Data

When estimating our five variable VAR model specified in (8) and (9) we use a data set for Sweden covering the period from 1986Q1 to 2015Q1. The data were collected from

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[4]We might also calibrate the elasticities ε_Y and ε_R so as to minimize the sum of the squared deviations between actual house prices and fundamental house prices. This would be equivalent to running an OLS regression of p_t^a on y_t and h_t. However, since all of these variables are likely to form part of a larger simultaneous system, the resulting OLS estimates of ε_Y and ε_R would probably be biased. Moreover, we prefer to be able to vary ε_Y and ε_R to check the sensitivity of our results to these parameters.
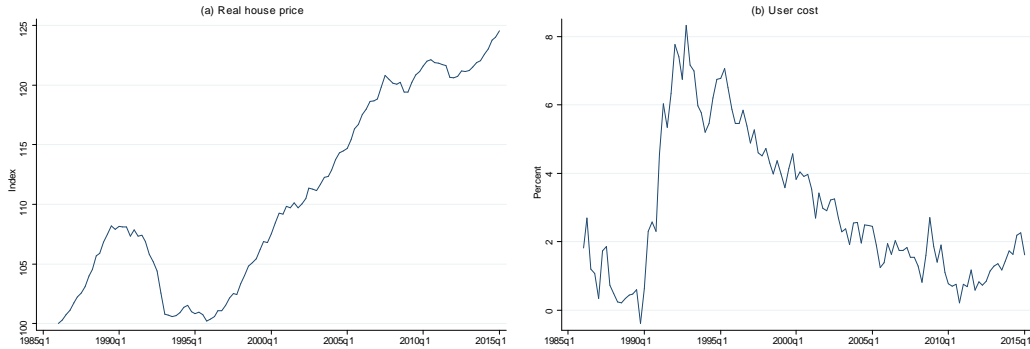
[5]In other words, we calibrate $\bar{r}^H$ so as to satisfy the equation

$$\bar{r}^H = \bar{p}^a + \log\left[i\left(1 - \tau^i\right) - \left\{\frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c}\right\}\right]$$

where $i\left(1 - \tau^i\right) - \left\{\frac{E_t[P_{t+1}^c] - P_t^c}{P_t^c}\right\}$ is the mean value of the after-tax real interest rate observed over the sample period.

Figure 1: Real house prices (natural logarithms, 1986Q1=100) and real user cost of owner-occupied housing in Sweden.

various sources listed in Appendix B.

In Figure 1(a) we illustrate how (the log of) real Swedish house prices have evolved over time. The graph suggests that Sweden experienced a housing bubble in the beginning of the 1990s. It also shows that house prices have increased rapidly since the mid-1990s with only a relatively mild interruption by the recent financial crisis.

The user cost of owner-occupied housing $\gamma_t \equiv i_t \left(1 - \tau_t^i\right) - \left\{ \frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c} \right\} + \tau_t + \delta + \eta$ plays a central role in our theoretical model. Figure 1(b) shows the evolution of user costs in Sweden, where we have normalized the constant term $\delta + \eta$ to zero. From the mid 1990s the user cost declined from a relatively high level, and it has recently returned to a level comparable to that prevailing before the banking crisis of the early 1990s.

The remaining three variables used in our estimations are shown in Figure B.1 in Appendix B.

6 Estimating the fundamental house price

6.1 Estimating the VAR forecasting model

We will now apply the method laid out above to estimate fundamental house prices in Sweden. The first step is to estimate the VAR model described in section 3 which is used to forecast the variables determining expected future imputed rents. To determine the lag length in the VAR model we use the Schwarz Bayesian information criteria with a maximum of 12 lags allowed and then we test for autocorrelation, heteroscedasticity and normality in the residuals. If any of the tests suggests a rejection of the null hypothesis that there is no remaining autocorrelation or heteroscedasticity, we add one lag and repeat the tests. We stop at the lag length where we cannot reject the null. This procedure leads us to set the lag length equal to four quarters.

Panel A of Table 1 shows in the first five rows the p-values of tests of conditional autoregressive heteroskedasticity and normality in the residuals from each of the five equations in the VAR(n) model specified in (11) and (12). The last row shows the p-values of a multivariate LM test for autocorrelation (using six lags). According to the table we can never reject the null of no ARCH and normality in two equations.

Table 1: VAR model diagnostics.

Panel A			Panel B	
Equation	ARCH	Jarque-Bera	Max(modulus)	0.920
Δp_t^a	0.114	0.812	Hypothesis	Trace test
Δr_t	0.304	0.000	r=0	150.0***
γ_t	0.719	0.373	r=1	96.3***
Δy_t	0.542	0.825	r=2	60.1***
Δh_t	0.172	0.006	r=3	26.2**
LM-test(6)	0.700		r=4	7.5

Note: Panel A: Only p-values are shown in the table. ARCH refers to an LM-test for autoregressive conditional heteroskedasticity, Jarque-Bera is a test for normality and LM-test refers to a multivariate test for autocorrelation using 6 lags. Panel B: Max(modulus) is the maximum eigenvalue of the VAR model and Trace test refers to the standard Johansen trace test. All tests are based on VAR models with 4 lags with a constant and a linear trend. *** denotes significance at the 0.01 level, ** at the 0.05 level and * at the 0.10 level.

One underlying assumption in our theoretical model and the procedure by which we intend to estimate fundamental house prices is that the VAR model is stable. The first row of Panel B in Table 1 reports the maximum eigenvalue of the VAR model. To formally test for cointegration we apply the standard Johansen trace test. Panel B reports the trace test statistics which reject that there are up to 3 cointegration vectors.

This result does not change when we add or subtract lags. Testing for unit roots in the data reveals that there is a potential unit root in the user cost γ and housing investment Δh. Judging from the graphs in Figures 1 and B.1, the user cost and housing investment move along broken linear trends. The Swedish user cost increased sharply during the banking crisis in the early 1990s and reached its peak in 1993. From then on it fell to a level comparable to the level in the early 1990s. A similar pattern is also noted for housing investment in Sweden where investment fell substantially during the banking crisis and is now settled on a very low level. It may be that the cointegration tests pick up these secular changes and identify them as unit roots. In our empirical VAR model we therefore include a linear trend in order to capture these secular movements.

In principle, our method would work even if the VAR model is non-stationary. What we need to do in that case is to estimate the Vector Error Correction representation and

then compute the companion matrix in equation (12). Estimating the VEC representation under the assumption that there are three cointegration vectors in the model, we can then estimate the fundamental house price. Comparing this estimate to the one we obtain under the assumption of full rank, we find very similar results and that the quantitative conclusions drawn from both models are consistent. Therefore, since we found no strong evidence suggesting that the VAR model is unstable, we will continue under the assumption that the model is stationary.

Having estimated the VAR(n) model, we can transform it into the VAR(1) model (11) by using (12). We may then apply (13) to forecast the relevant variables in our theoretical house price model and plug the resulting forecasts into (16) to obtain an estimated time series for the ratio of the fundamental house price to the imputed rent. As a final step, we can use these estimates to derive a time series for the fundamental house price from (19).

6.2   Fundamental versus actual house prices

Figure 2 displays the actual Swedish house price along with the estimates of the fundamental house price implied by our model, computed from (19) plus (7), (20) and (21). As a baseline case the left graph assumes that $\varepsilon_Y = \varepsilon_R = 1$ while the right graph shows estimates for the case when $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$ as a robustness check. The dotted lines in the figures are 90 percent confidence bands around the fundamental house price computed using a non-parametric bootstrap with 999 trials, see Appendix C for details.

Figure 2 suggests that actual house prices are more sluggish than fundamental house prices. In particular, the fundamental house price level fluctuated more dramatically than the actual price level before and during the Swedish banking and currency crisis in the early 1990s. Irrespective of our assumptions about the elasticities, Figure 2 implies significant overpricing on the Swedish housing market since the beginning of 2010.[6] The amount of overpricing during the most recent period mimics the overpricing during the banking crisis, suggesting a potential threat to the Swedish economy.

Figure 2 indicates that deviations of actual house prices from their fundamental level may be quite persistent. Below we will provide measures of the speed of adjustment of actual house prices towards the fundamental level.

6.3   Real-time estimates of fundamental and actual house prices

The estimates of the fundamental house price presented above are based on the full sample estimates of the VAR model in (8). When compared to actual house prices, these *ex post* estimates of the fundamental house price reveal whether the housing market has been over- or under-valued historically, given the information available up until the end of the

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[6]This conclusion is robust to alternative assumptions about the elasticities. For brevity these graphs are not shown here but are available from the authors upon request.

Figure 2: Actual and fundamental house prices in Sweden ($\varepsilon_Y = \varepsilon_R = 1$ in the LHS graph and $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$ in the RHS graph).
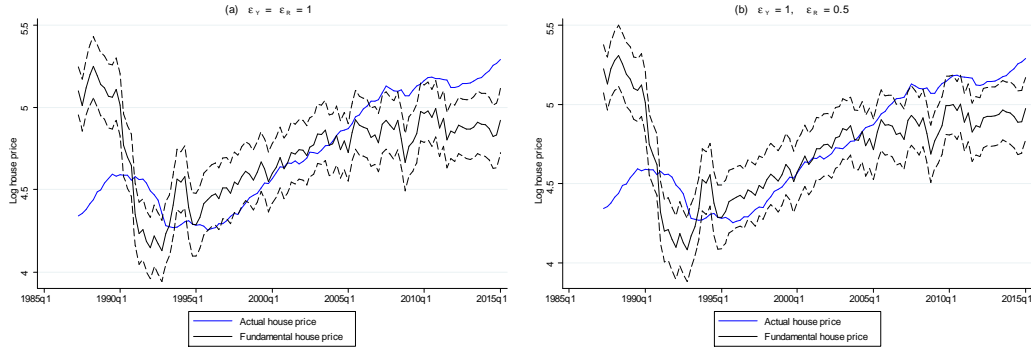

sample period. A large and growing gap between actual and fundamental house prices indicates periods when a bubble has been building up. However, from a policy perspective it is essential whether our model could also be used to spot the build-up of a bubble in a real-time ex ante setting.

To address this question, we will now estimate the fundamental house price recursively. We start by re-estimating the VAR model in (8) and (9) using the sample 1986Q1 until 1998Q4. Then, using the procedure outlined above, we back out an estimate of the fundamental house price in 1998Q4 based on the information available up until that time. We then add one observation to the sample and re-estimate the VAR model in (8) and (9) for the sample 1986Q1 until 1999Q1 and then we back out the fundamental house price in 1999Q1. We continue adding one observation at a time to construct successive estimates of the fundamental house price for these observations also. In this way we obtain real-time estimates of the fundamental house price from 1999Q1 until 2015Q1.

Figure 3(a) shows these estimates together with the actual house price. A couple of features stand out. First, the real-time estimates are very close to the full-sample estimates. Second, actual house prices continue to increase from 2005 even though the fundamental price is not increasing. A policymaker using our model to compute the fundamental house price in real time would therefore observe a continuing deviation of actual prices from the fundamental prices and would thus receive a signal of growing imbalances in the housing market. The close correlation between the real-time and the full-sample estimate of the fundamental house price suggests that our model could in fact serve as a useful tool for a policy maker trying to detect a housing bubble. Our model in (8) and (9) can also be used by public and private sector agents to compute forecasts of actual house prices. The real-time estimates as well as the full-sample estimate of the fundamental house price suggest that the increase in actual house prices since 2005 was not driven by an increase in the fundamental value. The question is whether our model can shed light on the sharp increases in the actual house price since 2005? To illustrate, we compute dynamic forecasts over the period 2005Q1 until 2015Q1. To generate the

forecasts we estimate our model in (8) and (9) for the sample 1986Q1 until 2004Q4. Based on these estimates we then compute dynamic forecasts for the remaining part of our sample. If our model is useful it should be able to track the actual price behavior closely. Figure 3(b) shows the dynamic forecast of the actual house price together with the actual house price. As is evident, the VAR model forecasts are very close to the actual price changes, reflecting that the model is also good at tracking the evolution of the other explanatory variables that interact with actual house prices. Overall, we find that our model seems to capture the main behavior of actual house prices. Households forecast, using the VAR model, suggests that the fundamental house price should remain on about the same level in the future, the forecasts of future actual house prices do imply ever rising future actual house prices. These observations are consistent with household expectations revealed in SEB's Swedish Housing Price Indicator.

Figure 3: Real time estimate of the fundamental house prices and dynamic forecasts of actual house prices 1999Q1-2015Q1.
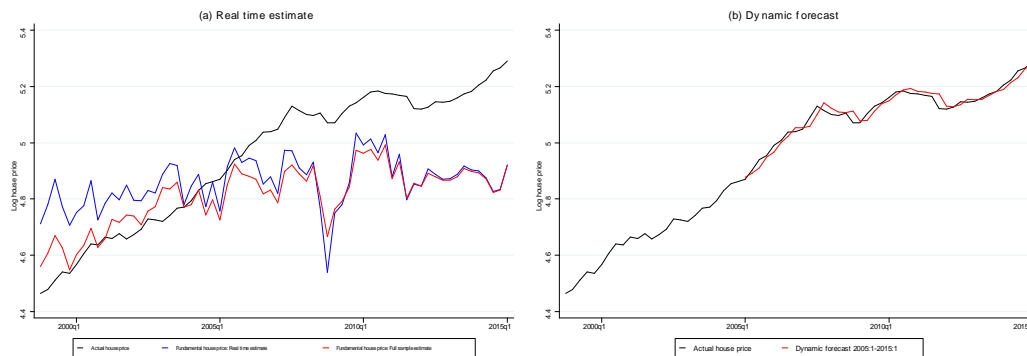
7 Analyzing the interaction of actual and fundamental house prices

7.1 Modeling the gap between actual and fundamental house prices

Having computed the fundamental house price, we can now analyze the behavior of the gap between the actual and the fundamental price. One interesting question is whether the actual price tends to converge towards the fundamental price? In the short and medium term actual house prices may deviate from the fundamental price level due to various frictions in the housing market and due to temporary house price bubbles, but in the long run one would expect actual prices to converge on fundamental prices. Empirically this would imply that the gap between actual and fundamental prices is mean-reverting. At the same time we would expect actual prices to react to changes in fundamental prices,

but not necessarily the opposite since the fundamental price does not depend directly on the actual price according to eq. (3).

All of these hypotheses can be analyzed in terms of a bivariate VAR model comprising the actual and the fundamental house price. Define $q_t \equiv \begin{bmatrix} p_t & p_t^a \end{bmatrix}'$, where we recall that p_t^a is the log of the actual real house price whereas p_t is the log of the fundamental real house price. Further, assume that q_t is generated by the following Vector Error Correction Model (VECM),

$$\Delta q_t = \widetilde{\mu} - \alpha \beta' q_{t-1} + \sum_{i=1}^{n-1} \Gamma_i \Delta q_{t-i} + \varepsilon_t, \tag{22}$$

where $\widetilde{\mu}$ is a 1×2 vector of constants (which may be zero) and ε_t is a 1×2 vector of shocks. The hypothesis that the gap between the actual and the fundamental house price is mean-reverting implies that these two variables must be cointegrated with the cointegration vector $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}$. Moreover, we expect that the actual house price reacts negatively to a positive deviation from the fundamental price, and vice versa. This implies that the adjustment coefficient α_2 in the α-vector in (22) should be negative and significantly different from zero. Our theoretical model also suggests that the fundamental house price should not respond to any gap between actual and fundamental prices, i.e., $\alpha_1 = 0$.

The VECM stated above also allows us to analyze the effects of a shock to the fundamental price or a shock to the actual price. Provided there exists one cointegration vector in the bivariate system, we can identify one common stochastic shock and one transitory shock in this system. The common stochastic shock (or stochastic trend) will have a permanent effect on both variables but no long-run effect on the gap, given that the gap is stationary. The transitory shock on the other hand can only have short-run effects on the variables and therefore also on the gap. In Appendix D we show how the procedure suggested by Bergman et al. (2011) may be applied to the VEC system (22) to identify permanent and transitory shocks to the housing market.

Having identified the two structural shocks, we can estimate the impulse response functions associated with each structural shock. These impulse responses can then be used to analyze the speed of adjustment in each variable, including the gap. In this manner our VECM allows us to measure the speed of convergence of actual house prices towards their fundamental level following a permanent change in the fundamental house price. Earlier studies of fundamental house prices such as Hott and Monnin (2008) and Hott and Jokipii (2012) do not examine these issues.[7]

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[7]Note that the fundamental house price in the bivariate VAR model (22) is computed from estimates of the VAR model in (8) and therefore includes measurement errors. In order to take this uncertainty into account when computing the impulse responses, we apply a bootstrap approach. As mentioned above, we use a non-parametric bootstrap to generate confidence bands around our estimate of the fundamental house price. For each trial we also set up the bivariate VAR model, identify the structural shocks and compute the implied impulse responses. It is then straightforward to construct confidence bands for the

7.2   Does the house price gap behave in accordance with theory?

We will now use our data set for Sweden to estimate the VECM in equation (22) and investigate whether house prices do in fact behave as predicted by theory. Based on model specification tests prior to the final estimation, we incorporate 4 lags. The cointegration vector includes a linear trend.

Our first step is to test for the number of cointegration vectors. Next we test whether the null that the gap between the actual and the fundamental house price is stationary can be rejected. Table 2 reports the Johansen trace tests for cointegration, the LR-test of the null that the gap is stationary, and estimates of the adjustment coefficients. Overall we find some support for our hypothesis that the model contains one cointegration vector. We find strong rejection of the null that the rank is 0 whereas the p-values for the null that the rank is 1 is above the 10 percent level for our benchmark case with $\varepsilon_Y = \varepsilon_R = 1$ but only above the 5 percent level for our alternative case with $\varepsilon_Y = 1$, $\varepsilon_R = 0.5$. Comparing the eigenvalues used in the Johansen test reported in the first two columns of Table 2 shows that the second eigenvalue is substantially smaller than the first eigenvalue, indicating that it is likely that the rank is one for both models.

Assuming there is one cointegration vector present, we then perform tests of the hypothesis that the gap between actual and fundamental house prices is stationary. The results are reported in the third column of Table 2. The p-values indicate that we cannot reject the null that the gap is stationary at conventional significance levels for any of the cases. Continuing under the assumption that the gap is indeed stationary, we estimate the adjustment coefficients. As mentioned above, we expect that the actual house price should respond negatively to a widening of the gap. Therefore we expect to find that $\alpha_2 < 0$. At the same time we expect that the fundamental house price should be independent of the gap, deviations of the actual house price from the fundamental price should have no effect on the fundamental house price. Therefore, we expect to find that $\alpha_1 = 0$. The estimates shown in Table 2 suggest that the actual house price is responding to the gap. The coefficient is negative, implying that when actual house prices exceed the fundamental price, actual house prices will fall, and vice versa. This holds for both sets of elasticity assumptions. The hypothesis that the fundamental house price should not adjust to the gap seems to be consistent with the Swedish data since the estimated value of α_1 is not significantly different from zero. Our conclusion then is that our empirical house price model for Sweden satisfies the underlying assumptions of our theoretical model.

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impulse responses.

Table 2: Johansen LR-trace tests, tests of the null that the gap between actual and fundamental house prices is stationary and estimates of the adjustment coefficients in VECM.

			LR-trace test				
	λ_1	λ_2	$r = 0$	$r = 1$	$\beta' = \begin{bmatrix} 1 & -1 \end{bmatrix}$	α_1	α_2
$\varepsilon_Y = 1, \varepsilon_R = 1$	0.216	0.098	33.69	10.34	0.24	0.076	-0.063
			[0.003]	[0.114]	[0.622]	[1.382]	[-4.794]
$\varepsilon_Y = 1, \varepsilon_R = 0.5$	0.199	0.106	32.37	11.32	0.68	0.075	-0.049
			[0.005]	[0.078]	[0.408]	[1.630]	[-4.183]

Note: The first two columns report the eigenvalues of the Johansen test. The next two columns report the Bartlett corrected Johansen trace tests. The fifth column reports LR test of the null that the gap between actual and fundamental house price is stationary. P-values are shown within parentheses below each test statistic. The last two columns report the estimated adjustment coefficients under the assumption that the gap is stationary. We use 4 lags in the VAR model and assume a restricted trend.

7.3 Impulse responses: How long does it take for the housing market to adjust to a shock?

The VECM in (22) can be used to estimate the impulse responses of the variables to structural shocks. The empirical results discussed in the previous subsection suggest that there is one cointegration vector in the system while the tests for stationarity of the house price gap suggest that this hypothesis cannot be rejected. As long as there exists one cointegration vector in the system, the variables are affected by one common trend shock and one transitory shock. If the gap is stationary, the effects of the common trend shock on each variable cancel out such that the long-run effect on the gap is zero. We will now assume that the gap is stationary and estimate the impulse responses using the method of Bergman et al. (2011) explained in detail in Appendix D. In addition, we will use the estimated impulse responses to calculate the implied half-life of a shock (the time it takes until half the initial effect of the shock disappears).

We associate the common trend shock with a shock to fundamental house prices which should have a permanent effect on the fundamental house price as well as on the actual house price. Our assumption that the gap is stationary imposes an identifying restriction on the impulse responses such that the difference between the actual and the fundamental house price approaches zero when the horizon goes to infinity. The transitory shock is associated with shocks to the actual house price. Based on our theoretical observations, we identify this shock by assuming it has no long-run effects on either variable in the system.

Figure 4 shows the effects of the two structural shocks in the gap between actual and fundamental house price under the assumptions that $\varepsilon_Y = \varepsilon_R = 1.0$ and that the

Figure 4: Impulse responses of fundamental house prices, actual house prices and the gap to a one standard deviation shock to the actual house price (LHS) and to a one standard deviation shock to fundamental house prices (RHS), ($\varepsilon_Y = \varepsilon_R = 1$), Sweden.
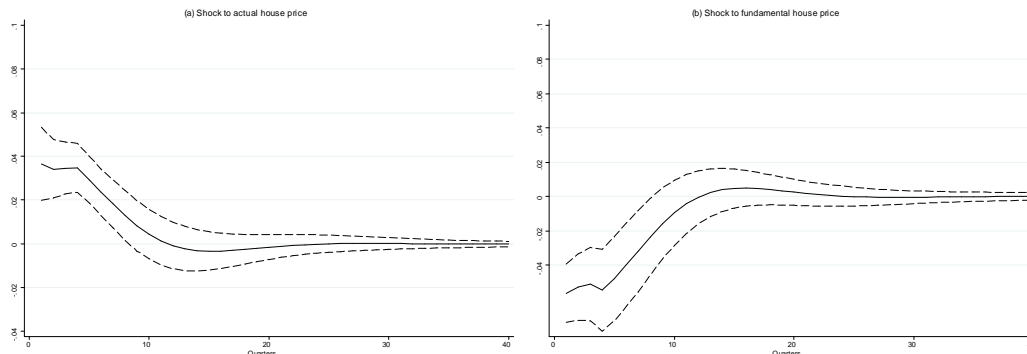
gap is stationary. The figure includes 68% confidence bands calculated using bootstrap simulations with 999 trials, see Appendix C for details.[8] We see that a positive transitory shock, which we associate with a temporary shock to the actual house price, has a positive effect on the gap. The gap between actual and fundamental prices widen as a result of the positive shock.[9] A positive permanent shock to the fundamental house price leads to a strong negative initial effect on the gap. As actual prices adjust, they increase over time, the gap closes and in the long-run, as a result of our stationarity assumption, the gap closes completely. These general effects of shocks on actual and fundamental house prices are consistent with predictions from our theoretical model discussed above. We also note in the graphs that the adjustement is slow, it takes about 2 years before the effect cancels to be significantly different from zero.

The central estimates of the impulse responses in figure 4 imply that in Sweden it takes about 1.7 years following a shock to the actual house price before half of the house-price gap is closed. In case of a shock to the fundamental house price, the half-life of the Swedish gap is 1.9 years.

7.4 Is the relationship between fundamental and actual house price stable over time?

An important question is whether the relationship between the actual and the fundamental house price is stable over time? Anundsen (2015) finds that actual house prices in the U.S. disconnected from their previous determinants since the beginning of 2000. Based on a

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[8]The impulse responses shown in Figure 4 are robust to changes in the price and income elasticities, see Figure E.1 in Appendix E.

[9]Note that we are not imposing any other restriction on the impulse responses other than the temporary and permanent distinction arising from our assumption of a stationary gap. Both shocks are allowed to affect both variables freely.

user cost expression similar to our equation (1) plus an assumption on arbitrage between the markets for rental and owner-occupied housing, he derives cointegration relationships between the actual house price and its determinants. Then using both single-equation and multivariate cointegration tests he shows that there was a stable relationship between the actual house price and its determinants until the beginning of 2000. After 2000 he finds no equilibrium relationship and therefore concludes that actual house prices started to disconnect from fundamentals.

In terms of our bivariate model, a disconnect would imply either that there is no cointegration relationship between the actual and the estimated fundamental house price or that the actual house price does not respond to deviations of the actual price from the fundamental price. We will apply our bivariate VAR model above and test both these hypotheses. First, we estimate the bivariate model using the sample 1986Q1 until 1998Q4 testing for cointegration using the Johansen trace test and estimating the adjustment parameter α_2. Then we successively add one observation at a time and compute the trace test statistic and α_2 each time we have included an extra observation until we reach the end of the sample in 2015Q1. This provides us with a recursive estimate of the trace test statistics and of the adjustment parameter α_2. If there is a disconnect we should find that actual and fundamental house prices are not cointegrated, or that the parameter α_2 measuring the response of actual prices to deviations of actual prices from the fundamental price is insignificantly different from zero.

Consider first the recursive estimates of the trace test statistics shown in Figure 5 where we add further observations of data as we move along the horizontal time axis. Note that we have divided each test statistic with the 5% critical value implying that if the ratio exceeds the unit line, we reject the null hypothesis that the rank is 0 (black line) or that the rank is one (blue line). To generate these test statistics we assume that the lag length in the underlying bivariate VAR model is equal to 4 and we assume a restricted trend; the same model specification we use for the full sample estimates presented in Table 2 above. As can be seen in the graph, the relationship between actual and fundamental house prices in Sweden seems to have been strengthened over time, in contrast to Anundsen's finding for the U.S. Using a sample ending in 1998Q4, we find that there is no cointegration relationship between the actual and the fundamental house price in Sweden, the p-value of the null hypothesis that there is no cointegration relationship suggests a rejection at the 46% level.[10] Gradually adding one observation at a time changes the result substantially: there is now solid empirical evidence suggesting one but not two cointegration vectors. According to these results, Swedish house prices have become connected to fundamental house prices since the 1990s, not disconnected. This may be due to the Swedish banking crisis during the early 1990s. Consistent with what

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[10] The trace tests are insensitive to assumptions about the elasticities; we always find empirical evidence suggesting that we cannot reject the null that there is one cointegration vector using the sample 1986Q1-1998Q4.

Anundsen (2015) finds, house prices may disconnect from fundamental values during the bursting of a housing bubble or a financial crisis. After the banking and currency crisis in Sweden in the beginning of the 1990's, the government implemented budget consolidation programs and other measures to resolve the government debt problem. It is perhaps to be expected that the linkages between house prices and fundamentals break down during a financial crisis and its subsequent resolution.[11]

Figure 5: Recursive trace tests, ratio of Trace test statistic and the 5% critical value.
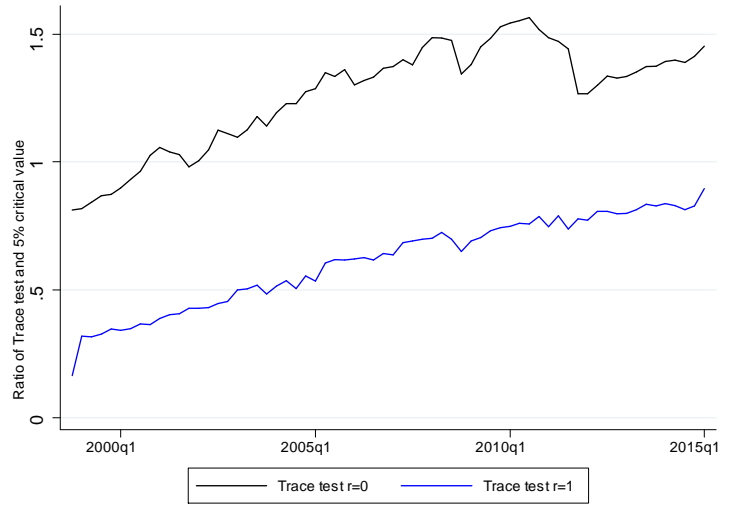
Next we investigate whether the adjustment parameters α_1 and α_2 have changed significantly since the 1990s. According to our theoretical model we expect α_1 to be insignificant whereas α_2 should be negative and significantly differetn from zero.Figure 6(a) shows the estimate of α_1, the response of fundamental houseprices to deviations of the actual price from the fundamental price. As is evident from this graph, the estimate is insignificant over the full estimation period.[12] Figure 6(b) showing the response of actual house prices to deviations of the actual price from the fundamental price, the adjustment parameter α_2. To generate these recursive estimates we assume that there is one cointegration vector present in the system, which is consistent with our findings in section 7.2 that the house price gap appears to be stationary around a linear trend. However, we allow the estimate of the trend to be sample-dependent. As above, we assume that the income and price elasticities are both equal to one. According to our recursive estimates, α_2 is always statistically different from zero, indicating that actual house prices have not disconnected from fundamentals since the 1990s. However, the graph in Figure 6(b) suggests that the response of actual house prices has become weaker during the last five years. From

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[11]Lack of data prevents us from testing whether there was a linkage between actual and fundamental house prices prior to the crisis.

[12]Our results concerning α_1 and α_2 are insensitive to assumptions about the elasticities. These results are not shown here for brevity, but are available upon request from the authors.

Figure 6: Recursive estimates of α_1 and α_2 under the assumption that the gap between actual and fundamental house prices is stationary.

1999Q1 until 2010Q4, α_2 was in the range of -0.08 and -0.1, but since then the size of the response has changed to -0.063 in 2015Q1, suggesting a slower speed of adjustment. Thus actual house prices have responded to a lesser degree to the gap between actual and fundamental prices during the last five years of our sample compared to the previous period. Comparing the results in Figures 5 and 6 we note that our results suggest that the empirical evidence supporting the assumption that there is one cointegration vector in the system strengthens over time whereas the speed of adjustment of actual house prices to a deviation from the fundamental house price falls over time. The linkage between actual and fundamental house prices have become stronger after 2000 whereas the influence of deviations in the gap is weaker after 2010.

8 Simulating policy-induced shocks to the housing market

Section 6.2 showed how our model may be used to evaluate whether actual house prices are out of line with fundamentals. Our analysis suggested that the Swedish housing market has been overvalued for some time. In such a situation policy makers may wish to prick the housing bubble before it grows to dangerous proportions. A policy intervention in the housing market will typically work via a change in the user cost of housing which we have specified as $\gamma_t \equiv i_t \left(1 - \tau_t^i\right) - \left\{ \frac{E_t\left[P_{t+1}^c\right] - P_t^c}{P_t^c} \right\} + \tau_t + \delta + \eta$. For example, a limitation on or abolition of the deductibility of mortgage interest payments will affect the user cost via its impact on the effective capital income tax rate τ_t^i; a change in the property tax regime will affect the effective property tax rate τ_t, and restrictions on loan-to-value ratios will increase the parameter η which captures the effects of credit constraints (see Sørensen, 2012, Appendix).

In this section we illustrate how the VAR model in equations (11) and (12) may be used

to simulate the effects on actual and fundamental house prices of a policy-induced shock to the user cost. We consider a policy reform which causes a permanent level change in the user cost of owner-occupied housing. We assume that the policy change is unanticipated before it is implemented but is fully built into the forecasts of all future user costs from the time of implementation. With such a reform causing a permanent structural break in the time series for the user cost, the VAR model (11) and (12) estimated on historical data cannot be used to forecast the future level of user cost. However, we assume that the model still gives a correct description of the links between the user cost and the *other* variables in the VAR model. To simulate the effects of a permanent policy-induced shock to the user cost, we therefore feed an exogenous future time path for the user cost γ_t into the VAR model and then use the model to calculate the future values of the other variables determining the evolution of the housing market (i.e., Δp_t^a, Δr_t, Δy_t, and Δh_t).

We apply this procedure to our data set for Sweden and assume that, starting from the second quarter of 2015, the user cost in each period increases by an amount $i_t \tau_t^i$, as would be the case if the deductibility of mortgage interest payments were abolished.[13] To fix the exogenous time path for the user cost from 2015Q2 onwards, we assume that the interest rate variable i_t follows the interest rate forecast included in the Monetary Policy report published by the Swedish central bank (the Riksbank) in February 2015. This forecast covered the three-year period 2015Q2 to 2018Q2, so we cut off our simulation at the end of the latter quarter. The interest rate forecasted by the Riksbank is the monetary policy rate (the repo rate) whereas our variable i_t is the mortgage rate offered by the dominant Swedish mortgage lender (Spintab). Over the period from June 1994 until February 2015 the average spread between the repo rate and the mortgage rate was about 2.7 percentage points. We assume that the same (constant) spread will prevail during the time span 2015Q2 through 2018Q2. In addition to future mortgage rates we also need an estimate of inflation expectations to calculate future user costs. We use the inflation forecasts in the Monetary Policy report from February 2015 since the Riksbank's predictions of future repo rates are dependent on the bank's expected future inflation rates. Finally, we assume that the effective property tax rate τ_t remains at the value prevailing in 2015Q2 and that the user cost component $\delta + \eta$ remains constant. Figure 7 shows the implied user cost paths in the case with and without abolition of interest deductibility. We see that the user cost is expected to fall in both cases, but abolishing interest deductibility would significantly reduce the magnitude of the fall.

Having fixed a future time path for the user cost in this way, we can feed it into our VAR model to produce a conditional forecast of actual and fundamental house prices, given the assumed permanent policy shock to the user cost. We can then compare these simulations to a baseline case where interest deductibility is maintained, but where the

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[13]Strictly speaking, if the taxation of positive net capital income is maintained, the opportunity cost of capital for individuals with positive financial net wealth would not be affected. We ignore this complication here since we are merely interested in simulating a shock to the user cost of a certain magnitude.

Figure 7: User cost under different scenarios.
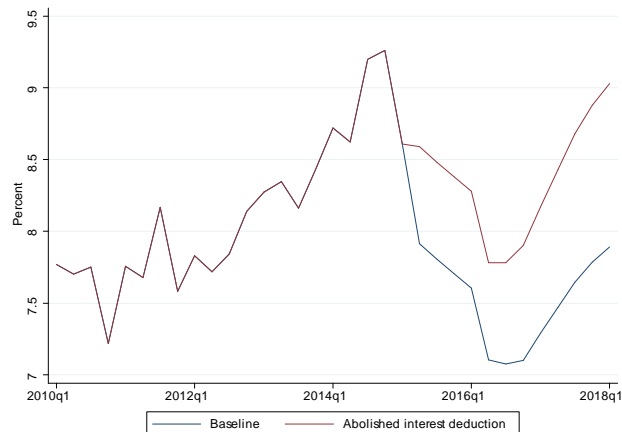
interest rate and the expected inflation rate (and hence the user cost) are still given by the forecasts of the Riksbank. All our simulations use our benchmark elasticity values $\varepsilon_Y = \varepsilon_R = 1$. To check whether our assumed exogenous future time path for the user cost could seriously bias our predictions regarding future house prices, we also compare our baseline scenario with exogenous user costs and no policy reform to another baseline scenario where the future user cost is calculated endogenously by means of our VAR model.

Our simulation results for actual house prices are shown in figure 8 which includes 68 percent confidence bands derived from bootstrap simulations with 1000 trials. The left panel in figure 8 shows that the forecasts for actual house prices in the baseline case without policy reform do not differ very much depending on whether future user costs are treated as exogenous (the scenario denoted as "Baseline" which is calculated from the Riksbank forecasts of future rates of interest and expected inflation) or whether they are calculated endogenously by our VAR model (the scenario denoted as "Unconditional"). This suggests that the baseline scenario marked by the blue lines in figure 8 is in fact plausible. Given this baseline scenario, we then add an abolition of mortgage interest deductibility. The predicted outcome is shown in the right panel of Figure 8 where the black line is the conditional forecast based on abolition of interest deductibility (known by households from the time of implementation in the second quarter of 2015). Our estimations suggest that the abolition of interest deductibility would reduce actual real house prices by about 7.8 percent in 2018Q1, a significant effect.

Figure 9 shows how fundamental house prices are estimated to develop in the baseline scenario and the policy reform scenario. The left panel in the figure displays the expected evolution of the fundamental house price in the baseline scenario (based on the Riksbank forecasts of interest and inflation) compared to the unconditional forecast (using the VAR model forecast of the user cost marked by the red line). In the baseline scenario the fundamental house price falls initially, then increases and then finally falls significantly in

Figure 8: Effects of an abolished mortgage interest deductability on actual house prices in Sweden ($\varepsilon_Y = \varepsilon_R = 1$).
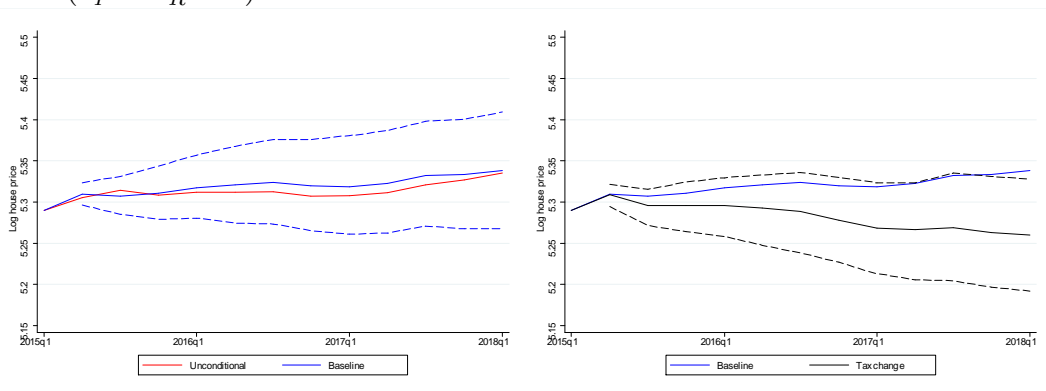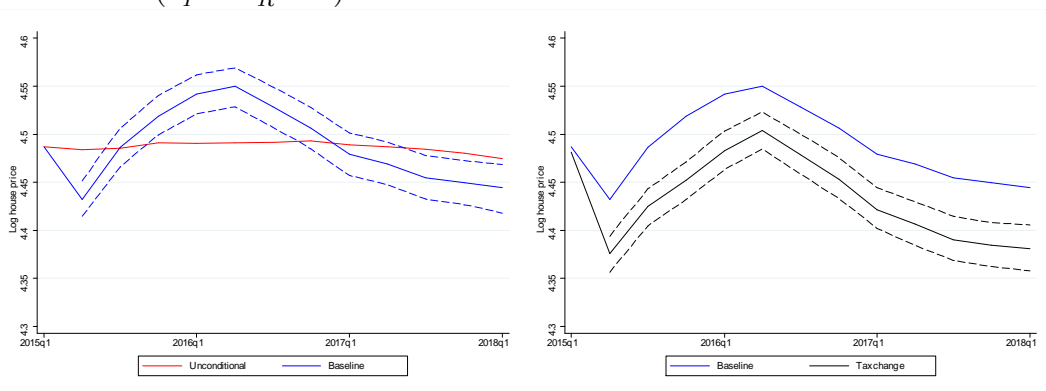

Figure 9: Effects of an abolished mortgage interest deductability on fundamental house prices in Sweden ($\varepsilon_Y = \varepsilon_R = 1$).

2017. The overshooting effect is due to the fact that the user cost falls in the short-term and then increases as can be seen in Figure 7 above. The right panel traces the evolution of the fundamental house price after the policy reform compared to the baseline scenario. There is a significant and relatively strong effect of the abolition of interest deductibility on the fundamental house price. In the first quarter of 2018 the fundamental house price would be 6 percent lower. In both scenarios the fundamental house price is expected to increase over the first year and then decline; both the short-term increase and the long-term decrease are statistically significant. The house price gap is decreasing over the forecast period, i.e., the fall in actual house prices exceeds the fall in fundamental house prices when comparing the baseline with the policy reform. In 2018Q1, the gap between actual and fundamental house prices has decreased by 5.5 percentage points. From the measurements of the half-life in the previous section we know that the gap will be closed eventually but that it takes approximately 4 years for actual house prices to approach the fundamental price level.

Our results in Figures 8 and 9 are relatively unaffected by the choice of income and

– 23 –

price elasticities. In Appendix D we show the results of our policy experiments for the case when $\varepsilon_Y = 1$ and $\varepsilon_R = 0.5$, see Figures E.2 and E.3. Comparing the graphs in the Appendix to the ones above we find only minor unsubstantial differences, the main conclusions are unaffected. Abolition of interest deductibility has a large and significant impact on actual as well as fundamental prices and the effects at the three year horizon are in accordance with the ones we obtain in Figures 8 and 9.

The methodology used above can also be used to shed light on the effectiveness of monetary policy as a tool for stabilizing the housing market. According to the baseline scenario described above, an *ex ante* analysis conducted in the beginning af 2015 would have suggested that Swedish house prices would continue to increase whereas the fundamental house price decreases somewhat implying a widening of the gap by early 2018. We may now ask the following question: By how much would the interest rate path announced in the Riksbank Monetary Policy report in February 2015 have had to be elevated if the Riksbank had wanted to cut the expected future actual house price significantly by early 2018?

Starting with the base case analyzed above, we increase the repo rate in increments by 0.05 compared to the base case, holding taxes, expected inflation and the markup of mortgage rates over the repo rate constant and equal to the baseline case but allowing the mortgage rate to follow the interest rate path published by the Riksbank in February 2015.

Figure 10 compares the unconditional and conditional forecasts of both actual and fundamental house prices produced by our model under these assumptions and the assumption that $\varepsilon_Y = \varepsilon_R = 1$. As can be seen from the graph, see Figure 10(a) actual house prices are expected to fall significantly starting in the first quarter of our forecasts (dotted lines are the 68% confidence bands). After three years, actual house prices are expected to have fallen by around 18 percentage points. Comparing to the unconditional baseline case the difference is striking. The unconditional forecast suggests that actual house prices would have increased somewhat in the first quarter of 2018.

The effects on the fundamental house price is shown in Figure 10(b). Here we also find that there will be a strong effect, fundamental house prices are expected to fall by 15 percentage points implying a fall in the gap between actual and fundamental house prices by 3 percentage points. From previous sections we know that this gap will be closed but that the speed of adjustment is slow, the half-life was measured to lie close to 2 years when decomposing the fluctuations in the gap into its temporary and permanent components.

To put these results into perspective we have conducted a number of additional experiments. First of all, we find that the results reported in Figure 10 are fairly robust to changes in the price and income elasticities. A lower price elasticity, holding the income elasticity constant, tends to reduce the fall in fundamental house prices leaving the fall in actual house prices unaffected. This implies that the gap will close faster. If $\varepsilon_Y = \varepsilon_R = 0.5$, the fall in fundamental house prices are further reduced such that the

Figure 10: Unconditional and conditional forecasts of actual house prices following an increase in user cost of 2.5 percentage points in each period compared to the base scenario ($\varepsilon_Y = \varepsilon_R = 1$).
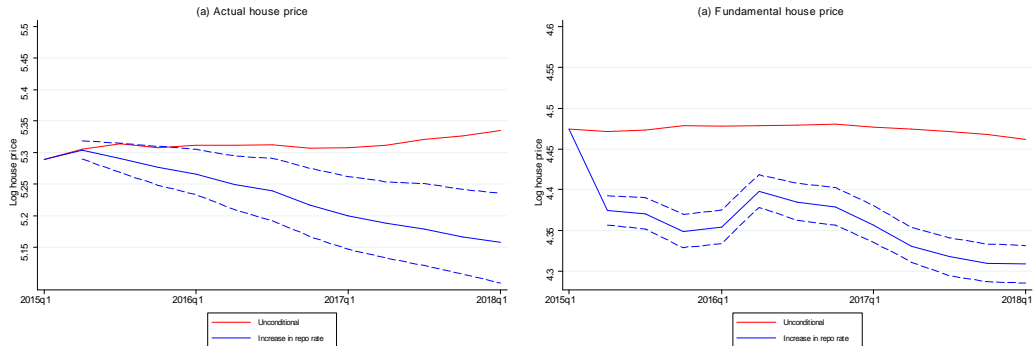
gap is also reduced. We may also ask whether a combination of higher repo rate and tax changes produce similar changes in actual and fundamental house prices as in Figure 10. Figure 8 suggests that the abolishment of the mortgage interest deductibility will imply a fall in actual house prices by 7.7 percentage points. Combining this tax change with increases in the repo rate we find that if the repo rate is increased by 2.2 percentage points in the first quarter of 2015 and the mortgage interest rate of 4.82% is held constant over the forecast period combined with the abolishment of the mortgage interest then actual house prices will fall by approximately 17 percentage points in the first quarter of 2018. Since it is likely that the repo rate will increase in the future, probably by more than 2.2 percentage points, the price falls would be even larger.

Our overall conclusion is that the interest rate is a powerful measure to use in order to reduce house prices, but we also found that in order to heavily influence actual house prices, it is necessary to increase the repo rate (and therefore also the mortgage rate) substantially. Therefore, we conclude, that in terms of our model, monetary policy is a powerful measure to be used to influence house prices but in order to obtain strong and significant effects, a very large increase is necessary. This questions the view that the interest rate weapon should be used actively to combat price increases on the housing market. Instead, our model suggest that a combination of changes in taxes and moderate increases in the repo rate is a useful measure.

9   Concluding remarks

Starting from a standard theoretical model of the housing market, this paper has described a methodology for estimating fundamental house prices and analyzing their interaction with actual house prices. The main contribution of the paper was to develop a unified bivariate vector error correction model allowing a rigorous test of various hypotheses

regarding the relationship between actual and fundamental house prices as well as an identification of temporary and permanent shocks to the housing market. Our methodology enabled us to analyze the sensitivity of our results to alternative assumptions regarding the income and price elasticities of housing demand.

To illustrate how the method works, we have applied it to a data set for Sweden. Our results indicate that the theoretical concept of the fundamental house price is indeed useful for empirical purposes in the sense that the estimated fundamental house price tends to work as an anchor for the actual house price. Specifically, we found that the data tend to support the hypothesis that the gap between the actual and the fundamental house price is mean-reverting with a zero mean. However, our analysis also suggests that actual house prices adjust rather slowly to fundamentals and may occasionally display bubble-like behavior.

Since the fundamental house price is a forward-looking variable, we would expect it to be useful for predicting future actual house prices. We showed how our empirical model may be used to detect a growing house price gap in real time and to make conditional forecasts of the effects on actual and fundamental house prices of policy interventions aimed at pricking bubble in the housing market. By constructing confidence bands around our estimates, we were able to illustrate the considerable uncertainty relating to any estimate of the fundamental house price. We also showed that the relationship between actual and fundamental house prices has not been entirely stable over time.

Nevertheless we found that, with high probability, the Swedish housing market has been out of line with fundamentals around the middle of the present decade. This conclusion is at odds with that of a Riksbank research paper by Dermani et al. (2016) which argues that the recent behavior of Swedish house prices can be explained by a regression equation that includes "fundamental" factors like disposable income, user costs, housing supply, population growth, and net financial wealth. Notice, however, that our VAR model presented in section 6 is also able to track the evolution of actual house prices fairly accurately by incorporating "fundamental" variables such as disposable income, user costs, housing supply, and rents in the rental housing sector. But as our analysis has demonstrated, being able to explain actual house prices statistically on the basis of these variables does not mean that the actual house price corresponds to the fundamental house price implied by economic theory. By contrast, the two prices usually differ, but the fundamental price works as an attractor for the actual price, so policy makers should be concerned when the actual price seems to be far above the fundamental level.

By assuming a constant risk premium in the user cost of housing, our analysis has implicitly abstracted from changes in credit standards. The papers by Duca et al. (2011) and Anundsen (2015) have shown that a relaxation of credit constraints played a significant role in explaining the house price boom in the U.S. prior to the financial crisis. An interesting topic for future research could be to investigate whether the estimated fluctuations of house prices around their fundamental level become smaller once changes in

credit standards are allowed for.

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Appendix A: Linearizing the model of the fundamental house price

This appendix derives equations (6) and (7) in section 2. Using the definition $S_t \equiv P_t / R_t^H$, and introducing the simplifying notation $P_{t+1}^e \equiv E_t[P_{t+1}]$, $R_{t+1}^{He} \equiv E[R_{t+1}^H]$ and $S_{t+1}^e \equiv P_{t+1}^e / R_{t+1}^{He}$, we can rewrite equation (2) as[14]

$$(1 + \gamma_t) P_t = R_t^H + P_{t+1}^e \quad \Rightarrow$$

$$(1 + \gamma_t) S_t = 1 + S_{t+1}^e \frac{R_{t+1}^{He}}{R_t^H} \quad \Rightarrow$$

$$s_t = \ln \left(1 + \exp \left(s_{t+1}^e + \Delta r_{t+1}^{He}\right)\right) - \ln \left(1 + \gamma_t\right),$$

$$\text{(A.1)}$$

$$s_t \equiv \ln S_t, \quad s_{t+1}^e \equiv \ln S_{t+1}^e, \quad \Delta r_{t+1}^{He} \equiv \ln R_{t+1}^{He} - \ln R_t^H.$$

Let

$$\bar{m} \equiv \bar{s}^e + \Delta \bar{r}^{He} \tag{A.2}$$

denote the mean value of the term $s_{t+1}^e + \Delta r_{t+1}^{He}$ in (A.1) over the sample period considered. Taking a first-order Taylor approximation of (A.1) around $s_{t+1}^e + \Delta r_{t+1}^{He} = \bar{m}$ and $\gamma_t = 0$, we get

$$s_t \approx \ln \left(1 + \exp \left(\bar{m}\right)\right) + \left(\frac{\exp \left(\bar{m}\right)}{1 + \exp \left(\bar{m}\right)}\right) \left(s_{t+1}^e + \Delta r_{t+1}^{He} - \bar{m}\right) - \gamma_t. \tag{A.3}$$

Defining

$$\phi \equiv \frac{\exp \left(\bar{m}\right)}{1 + \exp \left(\bar{m}\right)}, \tag{A.4}$$

we can restate (A.3) as[15]

$$s_t = \kappa + \phi \left(s_{t+1}^e + \Delta r_{t+1}^{He}\right) - \gamma_t, \qquad \kappa \equiv -\phi \ln \phi - (1 - \phi) \ln (1 - \phi). \tag{A.5}$$

Assuming that agents are forward-looking, and defining $\Delta r_{t+j}^{He} \equiv E_t \left[\Delta r_{t+j}^H\right]$, we find by forward iteration of (A.5) that

$$s_t = c + \sum_{j=1}^{\infty} \phi^j E_t \left[\Delta r_{t+j}^H - \gamma_{t+j}\right] - \gamma_t, \qquad c \equiv \frac{\kappa}{1 - \phi}. \tag{A.6}$$

In our S-D model of the housing market it follows from (5) that

$$\Delta r_{t+j}^H = (\varepsilon_Y / \varepsilon_R) \Delta y_{t+j} - (1/\varepsilon_R) \Delta h_{t+j}, \tag{A.7}$$

where $\Delta y_{t+j} \equiv \ln Y_{t+j} - \ln Y_{t+j-1}$ and $\Delta h_{t+j} \equiv \ln H_{t+j} - \ln H_{t+j-1}$. Substituting (A.7) into (A.6), we obtain equation (6). Equation (7) follows from (A.2) and (A.4) and the assumption of rational expectations.

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[14]This normalization procedure follows the one suggested by Hott and Monnin (2008).

[15]In deriving (A.5), we use the facts that $\ln (1 + \exp (\bar{m})) = -\ln (1 - \phi)$ and $\ln (\phi) = \bar{m} - \ln (1 + \exp (\bar{m})) = \bar{m} + \ln (1 - \phi)$.

Appendix B: Data

The Swedish nominal mortgage interest rate i_t is measured by the nominal 5-year mortgage lending rate for SPINTAB (first day of quarter) and is downloaded from Ecowin. The expected inflation rate is household inflation expectations taken from surveys conducted by the Swedish National Institute of Economic Research. The capital income tax rate is taken from Englund (2011) and the property tax is from Hansson (2013). The real price of a unit of owner–occupied housing, p_t^a, is the nominal price of one- or two-dwelling buildings deflated by the Consumer Price Index (CPI); the real rent on a unit of owner-occupied housing, r_t, is the CPI component of rents for housing, COICOP 04.S downloaded from Statistics Sweden, and Δy_t is the real net household disposable income downloaded from OECD. In order to calculate the fundamental price-to-imputed-rent ratio in the Supply-and-Demand model summarized in (19), we also need an estimate of the stock of housing. This measure is computed by cumulating the time series for gross fixed housing capital formation (taken from the Swedish national accounts) on the assumption that the depreciation rate is constant over the sample.

Figure B.1: Real disposable income, real rent and real housing investment in Sweden
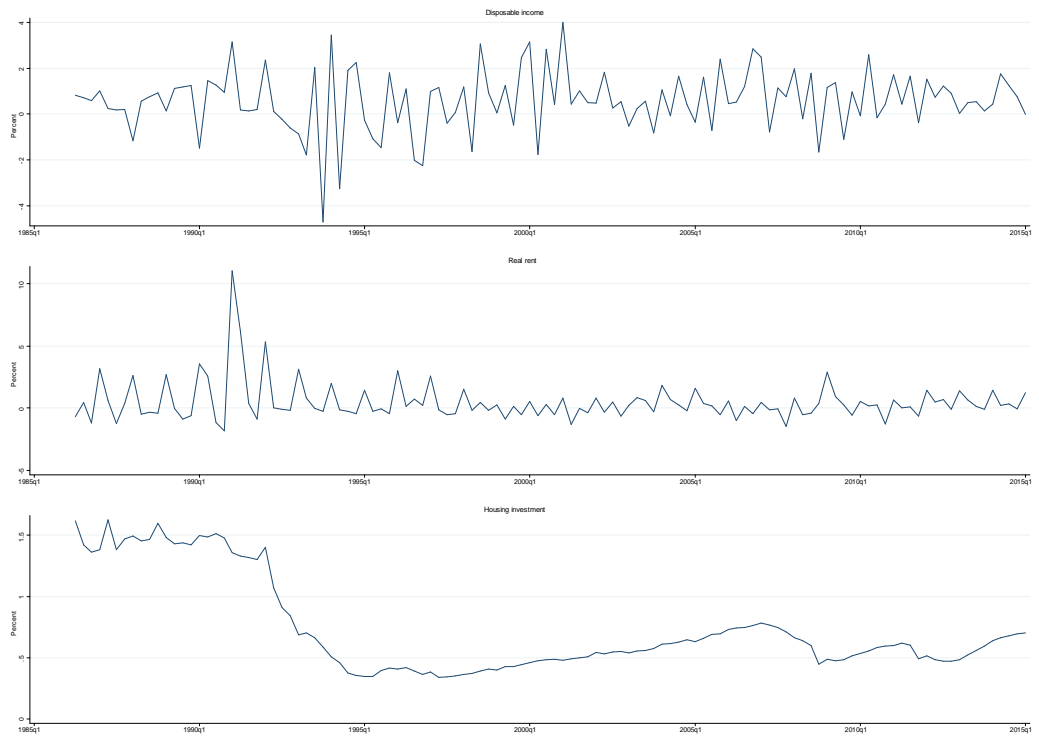
Appendix C: A Non-Parametric Bootstrap Technique for Estimating Confidence Bands for the Estimated Fundamental House Price and Impulse Responses in the Bivariate VAR model.

This Appendix explains the non-parametric bootstrap technique used in section 6.2 to generate confidence bands for the estimated fundamental house price and applied in section 7.3 to calculate confidence bands for the impulse response functions generated from estimates of the bivariate VECM in (22).

Consider the VAR-model in equation (8), i.e.,

$$b_t = \Phi_0 + \Phi_1 b_{t-1} + \ldots + \Phi_n b_{t-p} + \varepsilon_t,$$

where t is the time period, Φ_0 is the deterministic component and Φ_j, $j = 1, \ldots, p$, are the parameters to be estimated, b_t is the time series vector defined in equation (9), and ε_t is an $i.i.d.$ error term with zero mean. In the main text we show that this model can be rewritten as a VAR(1) model, i.e.,

$$z_t = A z_{t-1} + \xi_t,$$

where A and ξ_t are defined in (12). Using estimates from this VAR(1) model, we then apply the method outlined in section 4 to compute the fundamental house price.

To generate confidence bands for the estimated fundamental house price we use the following non-parametric bootstrap:

i. Estimate the VAR(1) model to obtain the estimation residuals $\hat{\xi}_t$ and use the estimated parameters in A, denoted $\hat{A}$, to compute the fundamental house price $\hat{p}_t$. Compute centered residuals $\hat{\xi}_1 - \bar{\hat{\xi}}, \ldots, \hat{\xi}_T - \bar{\hat{\xi}}$, where $\bar{\hat{\xi}}$ is the sample average of the estimated residuals and T is the sample size.

ii. Generate bootstrap residuals $\tilde{\xi}_1, \ldots, \tilde{\xi}_T$ by randomly drawing with replacements from the centered residuals.

iii. Compute bootstrap time series recursively using the VAR(1) model

$$\tilde{z}_t = \hat{A}\tilde{z}_{t-1} + \tilde{\xi}_t$$

where $(\tilde{z}_{-p+1}, \ldots, \tilde{z}_0) = (z_{-p+1}, \ldots, z_0)$.

iv. Reestimate the parameters in A based on the bootstrap time series $\tilde{z}_t$. Calculate the modulus of the largest root of $\tilde{A}$. If the largest modulus is less than 1, proceed to the next step, otherwise return to step (ii) as suggested by Cavaliere, Rahbek and Taylor (2012).

v. Based on the estimated parameters $\tilde{A}$, compute the bootstrap estimate of the fundamental house price $\tilde{p}_t$. Use the bootstrap estimate of the fundamental house price and the actual house price and apply the procedure outlined in Appendix D to identify the two structural shocks and estimate the impulse response function.

vi. Repeat steps (ii) to (v) N times.

Having obtained N estimates of the fundamental house price we finally construct the 90 percent confidence bands using the standard percentile method.

Appendix D: Identifying structural shocks to the housing market

This appendix applies the procedure suggested by Bergman et al. (2011) to illustrate how the VEC system (22) may be used to identify permanent and transitory shocks to the housing market. Let us assume that that the gap between the actual and the fundamental house price is mean-reverting, implying a cointegration vector $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$. The system innovations ε_t can then be decomposed into a common permanent component and a transitory component. The permanent innovation has a permanent effect on both p_t^a and p_t, but no long-run effect on the gap $\beta' q_t$. The transitory innovation only has short-term effects on the variables in q_t as well as on the gap $\beta' q_t$.

Since Δq_t is stationary, the Wold decomposition theorem implies that the VECM in (22) can be given the following Vector Moving Average (VMA) representation,

$$\Delta q_t = \delta + C(L)\varepsilon_t, \tag{D.1}$$

where L is the lag operator, $C(L) = I + \Sigma_{k=1}^{\infty} C_k L^k$, and I is the 2×2 identity matrix with rank $[C(1)] = 1$, $\beta' C(1) = 0$, $\delta = C(1)\widetilde{\mu}$, and $\beta' \delta = 0$. Equation (D.1) can alternatively be expressed as the common-trends model (Stock and Watson, 1988),

$$\Delta q_t = \delta + C(1)\varepsilon_t + (1 - L)C^*(L)\varepsilon_t, \tag{D.2}$$

by writing $C(L)$ as $C(1) + (1-L)C^*(L)$ where $C^*(L) = \sum_{i=0}^{\infty} C_i^* L^i$ and $C_i^* = -\sum_{j=i+1}^{\infty} C_j$ for $i \geq 0$ (see Stock, 1987). Hence, assuming that $\varepsilon_0 = 0$, we find by forward iteration of (D.2) that

$$q_t = q_0 + \delta t + C(1) \sum_{i=1}^{t} \varepsilon_i + C^*(L)\varepsilon_t. \tag{D.3}$$

Assume that the *structural* common-trend (CT) model is:

$$q_t = \mu_0 + \Phi \eta_t + Q^*(L)w_t, \tag{D.4}$$

where $\eta_t = \rho + \eta_{t-1} + \varphi_t$; $Q^*(L)$ is a stationary lag polynomial, and $w_t = \begin{bmatrix} \varphi_t & \psi_t \end{bmatrix}'$, with φ_t being the common stochastic shock and ψ_t being the transitory shock. The common stochastic trend, η_t, determines the trending behavior of the variables in q_t through the loading matrix, Φ. The transitory dynamics of the system are governed by $Q^*(L)w_t$. In addition, since cointegration implies that $\beta' \Phi = 0$, the dynamics of the gap between the actual and the fundamental house price is given by $\beta' \mu_0 + \beta' Q^*(L)w_t$.

Note that the structural common trends model in (D.4) is linked to a structural VMA representation since we can rewrite $Q(1) + (1 - L)Q^*(L)$ as $Q(L)$. Let the transformation matrix F link the estimated residuals in ε_t to the structural shocks in w_t, $w_t = F\varepsilon_t$. Then we have the following link between the structural VMA model and the unrestricted VMA model:

$$\Delta q_t = \delta + C(L)\varepsilon_t = \delta + C(L)F^{-1}F\varepsilon_t = \delta + Q(L)w_t. \tag{D.5}$$

The matrix $F = \begin{bmatrix} F_\varphi & F_\psi \end{bmatrix}'$ identifies the individual shocks to the common stochastic trend and the transitory shocks of the system.

To determine the matrix $F = \begin{bmatrix} F_\varphi & F_\psi \end{bmatrix}'$ in the system (D.5), we first derive $C(L)$ and then find the CT representation of the VMA model. The basic analysis is based on King, Plosser, Stock and Watson (1991), Mellander, Vredin and Warne (1992), and Bergman (1996). Following Campbell and Shiller (1988), define

$$M = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}. \tag{D.6}$$

Also, let $\Gamma(L) = I - \sum_{j=1}^{p-1} \Gamma_j$. Premultiplying both sides of the VEC model in (22) yields

$$M\Gamma(L)\Delta q_t = M\widetilde{\mu} - M\Pi q_{t-p} + M\varepsilon_t, \tag{D.7}$$

where Π is equal to the matrix $\alpha\beta'$ in the VECM (22). Now define a stationary variable $s_t = D_\perp(L)MX_t$, where $D_\perp$ is a diagonal matrix with its diagonal elements given by $D_\perp(L)_{11} = 1 - L$ and $D_\perp(L)_{22} = 1$. We can then write $B(L)s_t = M\widetilde{\mu} + M\varepsilon_t$ where $B(L) = M[\Gamma(L)M'D(L) + \alpha^*L]$ and $\alpha^* = \begin{bmatrix} 0 & \alpha \end{bmatrix}$. Moreover, the adjustment coefficients are given by $\alpha_1 = B(1)_{12}$ and $\alpha_2 = B(1)_{12} - B(1)_{22}$.

Comparing the structural CT model in (D.4) with equation (D.3) gives $\Phi\varphi_t = C(1)\varepsilon_t$. Since $\beta'\Phi = 0$ and $\beta = \begin{bmatrix} 1 & -1 \end{bmatrix}'$, we then obtain

$$\varphi_t = \det(B(1))^{-1} \begin{bmatrix} -\alpha_2 & \alpha_1 \end{bmatrix} \varepsilon_t = F_\varphi \varepsilon_t. \tag{D.8}$$

Following the procedure described by Mellander, Vredin and Warne (1992), we also find that

$$\psi_t = \left(\alpha'\Omega^{-1}\alpha\right)^{-1/2} \alpha'\Omega^{-1}\varepsilon_t = F_\psi \varepsilon_t, \tag{D.9}$$

where Ω is the variance-covariance matrix associated with the vector of error terms, ε_t. The transitory innovation, ψ_t, generates temporary effects on the actual and the fundamental house price and hence no permanent effects on the gap between these two variables. The common trend innovation, φ_t, generates long-lasting effects on both variables, but these effects will cancel out over the long run, leaving no permanent effect on the gap between the actual and the fundamental house price, given that this gap is stationary.[16] The impulse responses of the actual and the fundamental house price are given by $C(L)F^{-1}$, and those of the gap between the actual and the fundamental house price are given by $\beta'C(L)F^{-1}$.

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[16]If the gap is not stationary while there still exists a cointegration vector, then CT innovations have long-run effects on the gap as well as on the two variables in q_t.

Figure E.1: Impulse responses of fundamental house prices, actual house prices and the gap to a one standard deviation shock to the actual house price (LHS) and to a one standard deviation shock to fundamental house prices (RHS), ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$), Sweden.
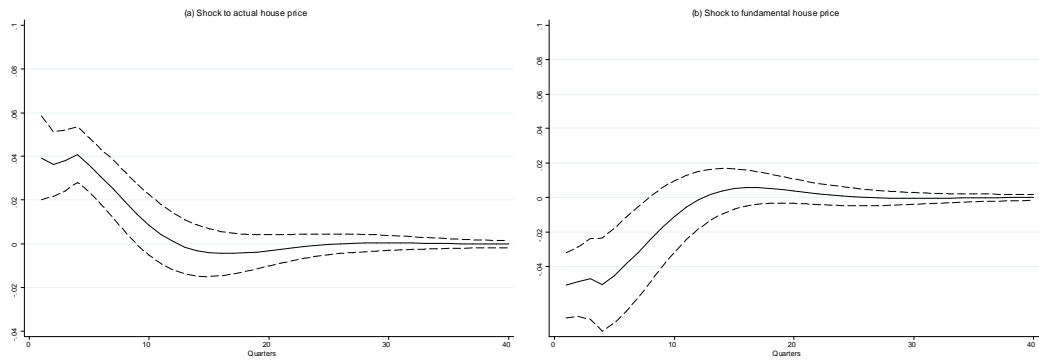
Appendix E: Robustness Analysis

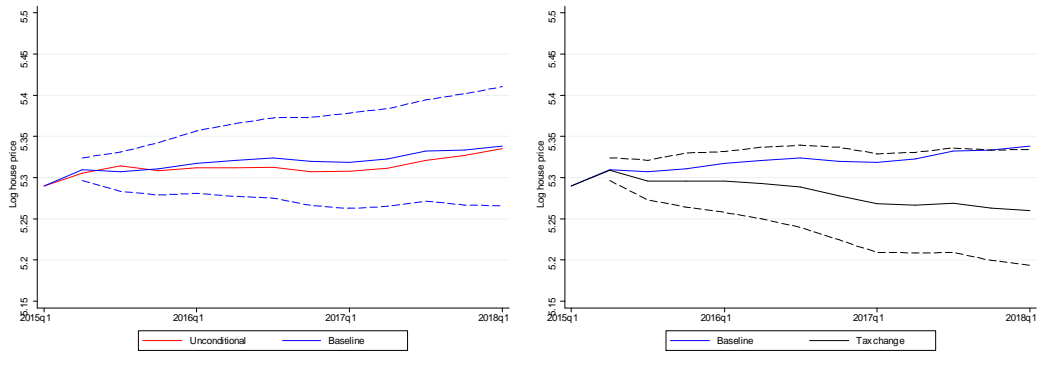Figure E.2: Effects of tax changes on actual house prices in Sweden ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$).

Figure E.3: Effects of tax changes on fundamental house prices in Sweden ($\varepsilon_Y = 1$, $\varepsilon_R = 0.5$).