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THE ECONOMIC EFFECTS OF WELFARE ACCOUNTS: A SIMPLE MODEL

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A SIMPLE MODEL

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This technical appendix sets up a simple model of the labour market and uses the model to derive the economic effects of the system of individual welfare accounts described in Bovenberg, Hansen and Sørensen (2011).

The next section describes the features of the model economy in which the welfare accounts are introduced. Section 2 derives the effects on individual behaviour of the various fiscal instruments of the government, including those related to the welfare accounts. In section 3 we show that, in the absence of welfare accounts, the optimal tax-transfer policy involves distortions to labour supply as the government redistributes income towards the unemployed and towards low-income wage earners. Section 4 then shows that the government can engineer an ex-post Pareto improvement by introducing welfare accounts with a lifetime income guarantee for low-income earners, thus improving the equity-efficiency trade-off.

1. The model economy

We consider an economy inhabited by a group of high-income earners and a group of low-income earners. Due to a higher exogenous productivity level, the former group earns a higher wage rate than the latter.

All individuals live for two periods. At the start of the first period they must go through some initial subperiod of involuntary unemployment before they are able to find a job. To preserve incentives for work, the government keeps the rate of unemployment benefit below the after-tax wage rates of employed workers. People out of work would therefore like to borrow to smooth their consumption over time. However, since the unemployed have not had the opportunity to accumulate non-human capital that can be used as collateral, we assume that they are unable to borrow. Hence they are liquidity-constrained, consuming all of their benefit income. Once they find a job, workers earn a sufficient income to be able to start saving for retirement. By exerting greater job search effort, job seekers can reduce the duration of the initial unemployment spell. For simplicity, the working hours of employed workers are treated as fixed, so labour supply is only endogenous at the extensive margin via the endogenous job search intensity.

During the second period of life all individuals are retired, financing their consumption from previous savings and from a government retirement benefit.

Whether employed or not, all young individuals receive a flat government benefit throughout period 1. This benefit represents public transfers (or individual public services with a private good character) offered to people of working age regardless of their employment status. In addition, unemployed workers receive an unemployment benefit. Employed workers contribute a fraction of their earnings to a mandatory individual saving account (a "welfare account"). The welfare account is debited by a fraction of the unemployment benefit and the general benefit received during the first period. At the time of retirement, any surplus on the account is used to supplement retirement benefits. If the account balance is negative at the date of retirement, the account holder just receives the normal government retirement benefit which thus provides an income floor for retirees. In this way the system of welfare accounts offers a lifetime income guarantee. We assume that high-income earners are able to accumulate a surplus on their welfare accounts during working life whereas the low-income earners are not.

For simplicity, the tax-transfer system is constrained to be linear. When the welfare accounts are added, the system becomes piecewise linear, like real-world tax-transfer systems.

Following this general overview, we now describe the structure of the economy in detail.

2. Individual behaviour

2.1. High-income earners

2.1.1. The preferences and budget constraints of a high-income earner

We start by considering the behaviour of a high-income earner. To economize on notation, we will set the interest rate and the utility discount rate of all individuals equal to zero. We also simplify by assuming that a young person's employment rate - the fraction of period 1 during which she is employed - is equal to her initial job search effort e (these assumptions are quite innocent)[1]. The consumption rate of an unemployed worker is denoted by C_1^u, while C_1^e indicates the consumption rate of an employed worker and C_2 is the consumption rate during retirement. Normalizing the length of each time period to one, we may then write the lifetime utility of a high-income earner (indicated by superscript h) as

$$U^h = e^h v\left(C_1^{uh}\right) + \left(1 - e^h\right) v\left(C_1^{eh}\right) + v\left(C_2^h\right) - f\left(e^h\right), \tag{2.1}$$

$$v' > 0, \quad v'' < 0, \qquad f' > 0, \quad f'' > 0, \qquad \lim_{e^h \to 0} f' = 0, \quad \lim_{e^h \to 1} f' = \infty,$$

where $v\left(C^h\right)$ is the instantaneous utility of consumption and $f\left(e^h\right)$ is the disutility of search effort. The assumptions on the disutility function $f\left(e^h\right)$ stated in (2.1) ensure that an optimizing individual will always choose a search intensity e^h (and hence an employment rate) between zero and one.

The budget constraints of the high-income earner are

$$C_1^{uh} = B + y_1, \tag{2.2}$$

$$C_1^{eh} = W\left(1 - \tau - s\right) + y_1 - S^h, \tag{2.3}$$

$$C_2^h = e^h S^h + y_2 + A^h. \tag{2.4}$$

Equation (2.2) reflects that an unemployed worker is liquidity-constrained, consuming all of his income which consists of the unemployment benefit B plus the general benefit y_1 granted to all young individuals. The budget constraint for an employed worker is given by equation (2.3) where the variable W is the wage rate of the high-income earner, τ is the labour income tax rate, s is the rate of mandatory contribution to the individual welfare account (WA), and S^h is ordinary saving (excluding contributions to the WA). This equation reflects that an employed worker starts to save from the time he finds a job. Equation (2.4) shows the resources available to a retired high-income earner. They

[1] Instead of modelling the first-period employment rate as a deterministic function of search effort, we might assume that agents maximise expected lifetime utility in a setting where their *probability* of employment varies positively with their search effort. In such an economy all the qualitative results reported in section 4 of the present paper would still go through, as shown by Bovenberg and Sørensen (2004).

consist of his previous savings (recall that the interest rate has been normalized to zero) plus the ordinary retirement benefit (y_2) and the balance on the WA (A^h), which is paid out at the time of retirement. If α_B and α_y denote the fractions of the benefits B and y_1 that are debited to the WA during the first period, the account balance at retirement amounts to

$$A^h = sWe^h - \alpha_B \left(1 - e^h\right) B - \alpha_y y_1, \qquad 0 \leq \alpha_B \leq 1, \quad 0 \leq \alpha_y \leq 1, \quad A^h \geq 0. \quad (2.5)$$

The constraint $A^h \geq 0$ reflects the lifetime income insurance built into the WA system: if the balance on the WA is negative at the time of retirement, the account is set at zero, and the individual still receives his ordinary retirement benefit y_2. Using (2.3) through (2.5) and assuming that the constraint $A^h \geq 0$ is not violated, we obtain the consolidated budget constraint measuring the consumption possibilities of the high-income earner from the time he starts to work until his death:

$$e^h C_1^{eh} + C_2^h = We^h (1 - \tau) + y_1(e^h - \alpha_y) - \alpha_B(1 - e^h)B + y_2. \quad (2.6)$$

The WA contribution rate s has dropped out of (2.6) since contributions to the WA are effectively remitted to the consumer when the account balance is paid out. Equation (2.6) also shows that, for a consumer with a surplus on the WA, the account system reduces the effective rate of unemployment benefit by the fraction α_B over a lifetime horizon. Similarly, the effective general benefit to young individuals is reduced by the fraction α_y.

2.1.2. The indirect utility function of a high-income earner

The high-income earner maximises his lifetime utility (2.1) subject to his budget constraints (2.2) and (2.6). Inserting (2.2) into the utility function for convenience, we write the Lagrangian for this problem as

$$L^h = \left(1 - e^h\right) v \left(B + y_1\right) + e^h v \left(C_1^{eh}\right) + v \left(C_2^h\right) - f \left(e^h\right)$$

$$+\lambda^h \left[We^h (1 - \tau) + y_1(e^h - \alpha_y) - \alpha_B(1 - e^h)B + y_2 - e^h C_1^{eh} - C_2^h\right], \quad (2.7)$$

where λ^h is the Lagrange multiplier associated with the budget constraint (2.6). Maximisation of (2.7) yields an indirect utility function of the form

$$V^h = V^h \left(B, y_1, y_2, \tau, \alpha_B, \alpha_y\right), \quad (2.8)$$

with the partial derivatives[2]

$$V_B^h \equiv \partial V^h / \partial B = \partial L^h / \partial B = \left(1 - e^h\right)\left(\lambda^u - \alpha_B \lambda^h\right), \qquad \lambda^u \equiv v'\left(B + y_1\right), \tag{2.9}$$

$$V_{y_1}^h \equiv \partial V^h / \partial y_1 = \partial L^h / \partial y_1 = \left(1 - e^h\right)\lambda^u + \lambda^h\left(e^h - \alpha_y\right), \tag{2.10}$$

$$V_{y_2}^h \equiv \partial V^h / \partial y_2 = \partial L^h / \partial y_2 = \lambda^h, \tag{2.11}$$

$$V_\tau^h \equiv \partial V^h / \partial \tau = \partial L^h / \partial \tau = -W e^h \lambda^h, \tag{2.12}$$

$$V_{\alpha_B}^h \equiv \partial V^h / \partial \alpha_B = \partial L^h / \partial \alpha_B = -\left(1 - e^h\right) B \lambda^h, \tag{2.13}$$

$$V_{\alpha_y}^h \equiv \partial V^h / \partial \alpha_y = \partial L^h / \partial \alpha_y = -y_1 \lambda^h. \tag{2.14}$$

From (2.11) we see that the Lagrange multiplier λ^h measures the high-income earner's marginal utility of exogenous income obtained during the stage of life when he is not liquidity-constrained, while (2.9) shows that λ^u measures the marginal utility of income for an unemployed (and hence liquidity-constrained) worker. The properties of the indirect utility function stated in (2.9) through (2.14) will be used repeatedly in sections 3 and 4.

2.1.3. The optimal behaviour of a high-income earner

We will now analyse the effects of the various fiscal instruments on the high-income earner's search effort (employment rate). For this purpose, it is convenient to insert the budget constraints (2.2) through (2.5) into the utility function (2.1) to get

$$\begin{aligned} U^h = {} & \left(1 - e^h\right) v(\overbrace{B + y_1}^{C_1^{uh}}) + e^h v(\overbrace{W\left(1 - \tau - s\right) + y_1 - S^h}^{C_1^{eh}}) \\ & + v(\overbrace{e^h S^h + sWe^h - y_1 \alpha_y - \alpha_B\left(1 - e^h\right) B + y_2}^{C_2^h}) - f\left(e^h\right). \end{aligned} \tag{2.15}$$

The first-order condition for optimal saving is

$$\partial U^h / \partial S^h = 0 \quad \Rightarrow \quad e^h\left[v'\left(C_1^{eh}\right) - v'\left(C_2^h\right)\right] = 0 \quad \Rightarrow \quad C_1^{eh} = C_2^h. \tag{2.16}$$

[2] Note that we are evaluating the partial derivatives of the Lagrangian L^h at the consumer's optimum. Hence we may ignore the marginal effects of the policy instruments on the behavioural variables C_1^{eh}, C_2^h and e^h since (by the Envelope Theorem) these effects will wash out when the first-order conditions for maximisation of L^h are met.

According to (2.16), the consumer chooses his level of saving so as to smooth consumption over his remaining lifetime from the time he finds a job. From (2.3) through (2.5), we find that (2.16) implies

$$W(1-\tau)-\left(1+e^{h}\right)\left(S^{h}+sW\right)+\alpha_{B}\left(1-e^{h}\right)B+y_{1}\left(1+\alpha_{y}\right)-y_{2}=0. \tag{2.17}$$

The first-order condition for optimal search effort is

$$\partial U^{h}/\partial e^{h}=0 \quad \Rightarrow \quad v\left(C_{1}^{eh}\right)-v\left(C_{1}^{uh}\right)+\left(S^{h}+sW+\alpha_{B}B\right)v'\left(C_{2}^{h}\right)=f'\left(e^{h}\right). \tag{2.18}$$

The left-hand side of the second equation in (2.18) is the marginal benefit from search effort. This benefit consists of the utility gain obtained in period 1 by moving from unemployment into employment - $v\left(C_{1}^{eh}\right)-v\left(C_{1}^{uh}\right)$ - plus the extra utility gained in period 2 due to the extra savings and the higher WA balance arising from the additional employment in period 1 - $\left(S^{h}+sW+\alpha_{B}B\right)v'\left(C_{2}^{h}\right)$. The right-hand side of the second equation in (2.18) is the marginal welfare cost of search effort which is simply the marginal disutility of search. (2.17) and (2.3) imply that

$$S^{h}+sW+\alpha_{B}B=\left(1/e^{h}\right)\left(C_{1}^{eh}-y_{2}+\alpha_{B}B+\alpha_{y}y_{1}\right). \tag{2.19}$$

The magnitude $S^{h}+sW+\alpha_{B}B$ measures the extra resources that will be available for consumption in period 2 when employment in period 1 increases by one unit. According to (2.19) a *sufficient* condition for this magnitude to be positive is that $y_2<C_1^{eh}$, i.e., the ordinary retirement benefit is lower than the consumption of an employed high-income earner. In the rest of this subsection we assume that this mild condition is met,[3] but it should be stressed that none of our results in sections 3 and 4 rely on this assumption.

Inserting the relevant budget constraints in (2.18), we can write the first-order condition for optimal search effort as

$$\begin{aligned} &v\left(W(1-\tau-s)+y_{1}-S^{h}\right)-v\left(B+y_{1}\right) \\ &+\left(S^{h}+sW+\alpha_{B}B\right)v'\left(e^{h}S^{h}+sWe^{h}-y_{1}\alpha_{y}-\alpha_{B}\left(1-e^{h}\right)B+y_{2}\right)-f'\left(e^{h}\right)=0. \end{aligned} \tag{2.20}$$

[3] Recall that y_2 is a *net* benefit which is not subject to the labour income tax and to the mandatory WA contribution.

The effects of the policy instruments on employment may be found by taking total differentials of (2.17) and (2.20). Doing that, we obtain

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dS^h \\ de^h \end{bmatrix} = \begin{bmatrix} D_S^h \\ D_e^h \end{bmatrix}, \tag{2.21}$$

$$a_{11} \equiv -\left(1+e^h\right), \qquad a_{12} \equiv -\left(S^h + sW + \alpha_B B\right),$$

$$a_{21} \equiv \left(S^h + sW + \alpha_B B\right) e^h v'', \qquad a_{22} \equiv \left(S^h + sW + \alpha_B B\right)^2 v'' - f'',$$

$$\begin{aligned} D_S^h \;\; \equiv \;\; & dy_2 - (1+\alpha_y)\, dy_1 - \alpha_B \left(1-e^h\right) dB + W d\tau + W \left(1+e^h\right) ds \\ & -B\left(1-e^h\right) d\alpha_B - y_1 d\alpha_y \end{aligned}$$

$$\begin{aligned} D_e^h \;\; \equiv \;\; & -\left(S^h + sW + \alpha_B B\right) v'' dy_2 + \left[\lambda^u - \lambda^h + \alpha_y \left(S^h + sW + \alpha_B B\right) v''\right] dy_1 \\ & + \left[\lambda^u - \alpha_B \lambda^h + \alpha_B \left(1-e^h\right)\left(S^h + sW + \alpha_B B\right) v''\right] dB + W\lambda^h d\tau \\ & - \left(S^h + sW + \alpha_B B\right) W e^h v'' ds + \left(S^h + sW + \alpha_B B\right) y_1 v'' d\alpha_y \\ & + \left[B\left(1-e^h\right)\left(S^h + sW + \alpha_B B\right) v'' - B\lambda^h\right] d\alpha_B \end{aligned}$$

The second-order condition for a utility maximum requires that the determinant Δ^h of the Jacobian matrix in (2.21) be positive. This condition is met since

$$\Delta^h \equiv a_{11}a_{22} - a_{12}a_{21} = \left(1+e^h\right) f'' - \left(S^h + sW + \alpha_B B\right)^2 v'' > 0, \tag{2.22}$$

where the positive sign of Δ^h follows from the assumptions on preferences made in (2.1). Applying Cramer's Rule to the system (2.21) and using the subscript "c" to indicate compensated effects (substitution effects), we find the following employment effects of the various fiscal instruments:

$$\frac{\partial e^h}{\partial y_2} = \frac{\overbrace{\left(S^h + sW + \alpha_B B\right) v''}^{\text{pure income effect}}}{\Delta^h} < 0, \tag{2.23}$$

$$\frac{\partial e^h}{\partial y_1} = \overbrace{\left(\frac{\partial e^h}{\partial y_1}\right)_c}^{\text{substitution effect } < 0} + \overbrace{\left(e^h - \alpha_y\right)\frac{\partial e^h}{\partial y_2}}^{\text{income effect } \gtrless 0}, \qquad \left(\frac{\partial e^h}{\partial y_1}\right)_c \equiv \frac{\left(1+e^h\right)\left(\lambda^h - \lambda^u\right)}{\Delta^h}, \tag{2.24}$$

$$\frac{\partial e^h}{\partial B} = \overbrace{\left(\frac{\partial e^h}{\partial B}\right)_c}^{\text{substitution effect} < 0} \overbrace{- \alpha_B \left(1 - e^h\right) \frac{\partial e^h}{\partial y_2}}^{\text{income effect} \geq 0}, \qquad \left(\frac{\partial e^h}{\partial B}\right)_c \equiv \frac{\left(1 + e^h\right)\left(\alpha_B \lambda^h - \lambda^u\right)}{\Delta^h}, \tag{2.25}$$

$$\frac{\partial e^h}{\partial \tau} = \overbrace{\left(\frac{\partial e^h}{\partial \tau}\right)_c}^{\text{substitution effect} < 0} \overbrace{- W e^h \frac{\partial e^h}{\partial y_2}}^{\text{income effect} > 0}, \qquad \left(\frac{\partial e^h}{\partial \tau}\right)_c \equiv -\frac{W\left(1 + e^h\right)\lambda^h}{\Delta^h}, \tag{2.26}$$

$$\frac{\partial e^h}{\partial s} = 0, \tag{2.27}$$

$$\frac{\partial e^h}{\partial \alpha_B} = \overbrace{\left(\frac{\partial e^h}{\partial \alpha_B}\right)_c}^{\text{substitution effect} > 0} \overbrace{- B \left(1 - e^h\right) \frac{\partial e^h}{\partial y_2}}^{\text{income effect} > 0} > 0, \qquad \left(\frac{\partial e^h}{\partial \alpha_B}\right)_c \equiv \frac{B\left(1 + e^h\right)\lambda^h}{\Delta^h}, \tag{2.28}$$

$$\frac{\partial e^h}{\partial \alpha_y} = \overbrace{-y_1 \frac{\partial e^h}{\partial y_2}}^{\text{income effect}} > 0. \tag{2.29}$$

When determining the sign of the substitution effects in (2.24) and (2.25), we have used the assumption $\alpha_B \leq 1$ plus the fact that a liquidity-constrained unemployed worker features higher marginal utility than an employed worker ($\lambda^u > \lambda^h$), since the unemployed worker exhibits lower consumption. All of the above behavioural effects are intuitive. Note from (2.27) that the mandatory WA contribution rate s does not distort the high-income earner's labour supply since the contribution is remitted to the worker at the date of retirement. This is consistent with the observation in (2.6) that s does not affect the high-income earner's lifetime budget constraint.

2.2. Low-income earners

2.2.1. The preferences and budget constraints of a low-income earner

A low-income earner is assumed to have the same preferences as those of a high-income earner stated in (2.1). However, because of his low wage rate, we assume that a low-income earner is unable to accumulate a surplus on his WA. As a consequence of the lifetime income guarantee, his WA account balance is therefore set equal to zero at the date of retirement. Denoting the low-income earner's wage rate by w, we may then write the budget constraints of the low-income earner (indicated by superscript l) as

$$C_1^{ul} = B + y_1, \tag{2.30}$$

$$C_1^{el} = w(1 - \tau - s) + y_1 - S^l, \tag{2.31}$$

$$C_2^l = e^l S^1 + y_2. \tag{2.32}$$

From (2.31) and (2.32) it follows that the consolidated budget constraint measuring the low-income earner's total consumption possibilities from the time he starts to work until his death are:

$$e^l C_1^{el} + C_2^l = we^l(1 - \tau - s) + e^l y_1 + y_2. \tag{2.33}$$

Eqation (2.33) shows that the mandatory contribution to the WA (s) works exactly like an ordinary labour income tax for a low-income earner who is unable to accumulate a WA surplus.

2.2.2. The indirect utility function of a low-income earner

The low-income earner maximises a lifetime utility function identical to (2.1) subject to the constraints (2.30) and (2.33). The Lagrangian for this problem is

$$\begin{aligned} L^l = {} & \left(1 - e^l\right) v\left(B + y_1\right) + e^l v\left(C_1^{el}\right) + v\left(C_2^l\right) - f\left(e^l\right) \\ & + \lambda^l \left[we^l(1 - \tau - s) + e^l y_1 + y_2 - e^l C_1^{el} - C_2^l\right], \end{aligned} \tag{2.34}$$

from which we may derive the low-income earner's indirect utility function

$$V^l = V^l\left(B, y_1, y_2, \tau, s\right), \tag{2.35}$$

with the following properties:[4]

$$V_B^l \equiv \partial V^l / \partial B = \partial L^l / \partial B = \left(1 - e^l\right) \lambda^u, \tag{2.36}$$

$$V_{y_1}^l \equiv \partial V^l / \partial y_1 = \partial L^l / \partial y_1 = \left(1 - e^l\right) \lambda^u + e^l \lambda^l, \tag{2.37}$$

[4] Once again we evaluate the derivatives of the Lagrangian at the consumer's optimum, cf. footnote 2.

$$V_{y_2}^l \equiv \partial V^l / \partial y_2 = \partial L^l / \partial y_2 = \lambda^l, \tag{2.38}$$

$$V_\tau^l \equiv \partial V^l / \partial \tau = \partial L^l / \partial \tau = -we^l \lambda^l, \tag{2.39}$$

$$V_s^l \equiv \partial V^l / \partial s = \partial L^l / \partial s = -we^l \lambda^l = V_\tau^l. \tag{2.40}$$

Since τ and s enter the budget constraint (2.33) in an identical fashion, it is not surprising to find from (2.39) and (2.40) that these two policy instruments exert the same impact on the low-income earner's maximum attainable utility. The asymmetric effect of s on the welfare of high-income and low-income earners will play an important role in the analysis in section 4.

2.2.3. The optimal behaviour of a low-income earner

To study the effects of fiscal policy on the low-income earner's employment rate, we start by inserting the budget constraints (2.30) through (2.32) into his lifetime utility function to get

$$\begin{aligned} U^l = \left(1 - e^l\right) v\left(B + y_1\right) + e^l v\left(w(1 - \tau - s) + y_1 - S^l\right) \\ + v\left(e^l S^1 + y_2\right) - f\left(e^l\right). \end{aligned} \tag{2.41}$$

From the first-order condition $\partial U^l / \partial S^l = 0$ it is easy to show from (2.41) that optimal savings behaviour implies $C_1^{el} = C_2^l$. From (2.31) and (2.32) this in turn implies

$$w(1 - \tau - s) + y_1 - \left(1 + e^l\right) S^l - y_2 = 0. \tag{2.42}$$

Note from (2.42) that

$$y_2 < w(1 - \tau - s) + y_1 \quad \Rightarrow \quad S^l > 0. \tag{2.43}$$

In other words, as long as the ordinary retirement benefit is lower than the net income of an employed low-income earner, his savings will be positive. We assume below that this weak condition is met, but again, none of the results reported in sections 3 and 4 depend on this assumption.

The optimal search intensity of the low-income earner is found from the first-order condition $\partial U^l / \partial e^l = 0$ which yields

$$v\left(w\left(1-\tau-s\right)+y_1-S^l\right)-v\left(B+y_1\right)$$

$$+S^l v'\left(e^l S^l+y_2\right)-f'\left(e^l\right)=0. \tag{2.44}$$

The structure of the optimum conditions (2.42) and (2.44) is identical to that of the analogous optimum conditions (2.17) and (2.20) for the high-income earner, with two exceptions. First, the policy instruments α_B and α_y do not appear in (2.44) since the negative WA of the low-income earner is set to zero at the time of retirement. Second, equations (2.42) and (2.44) confirm our earlier observation that the low-income earner's WA contribution s works exactly like an ordinary labour income tax. Hence we have $\partial e^l/\partial s = \partial e^l/\partial \tau$. To find the effects on e^l of the policy instruments y_1, y_2, B and τ, one can simply go back to the corresponding equations (2.23) through (2.26) for the high-income earner and set $\alpha_B = \alpha_y = s = 0$ to obtain

$$\frac{\partial e^l}{\partial y_2} = \frac{\overbrace{S^l v''}^{\text{pure income effect}}}{\Delta^l} < 0, \qquad \Delta^l \equiv \left(1+e^l\right)f'' - \left(S^l\right)^2 v'' > 0, \tag{2.45}$$

$$\frac{\partial e^l}{\partial y_1} = \overbrace{\left(\frac{\partial e^l}{\partial y_1}\right)_c}^{\text{substitution effect} < 0} + \overbrace{e^l \frac{\partial e^l}{\partial y_2}}^{\text{income effect} < 0} < 0, \qquad \left(\frac{\partial e^l}{\partial y_1}\right)_c \equiv \frac{\left(1+e^l\right)\left(\lambda^l - \lambda^u\right)}{\Delta^l}, \tag{2.46}$$

$$\frac{\partial e^h}{\partial B} = -\frac{\left(1+e^l\right)\lambda^u}{\Delta^l} < 0, \tag{2.47}$$

$$\frac{\partial e^l}{\partial \tau} = \frac{\partial e^l}{\partial s} = \overbrace{\left(\frac{\partial e^l}{\partial \tau}\right)_c}^{\text{substitution effect} < 0} \overbrace{- we^h \frac{\partial e^l}{\partial y_2}}^{\text{income effect} > 0}, \qquad \left(\frac{\partial e^l}{\partial \tau}\right)_c \equiv -\frac{w\left(1+e^l\right)\lambda^l}{\Delta^l}. \tag{2.48}$$

Again, all of these effects are intuitive.

The next two sections draw on our analysis of the employment effects of fiscal policy when studying the optimal policy and the potential for a Pareto-improvement through the introduction of welfare accounts.

3. The optimal tax-transfer policy in an economy without welfare accounts

This section shows that, in our model economy without welfare accounts (i.e., when $s = \alpha_B = \alpha_y = 0$), the optimal tax-transfer policy involves distortions to labour supply in the form a non-zero tax and benefit wedge between the potential wage income and the net benefit income of an unemployed worker. We will also show that the optimal tax and benefit wedge for high-income earners must be positive unless we have a "perverse" situation where the marginal social value of income for the unemployed is smaller than the marginal social value of income for employed workers. This analysis provides the background for the next section where we will demonstrate how the introduction of welfare accounts with a lifetime income guarantee can reduce the distortions to the labour supply of high-income earners without reducing anybody's welfare.

3.1. The social planning problem

We assume that the social planner is a utilitarian who seeks to maximise the sum total of all utilities (V) which is given by

$$V = \beta V^h + (1-\beta) V^l, \qquad 0 < \beta < 1. \tag{3.1}$$

In (3.1) we have normalized the total population to one, so β is the exogenous share of high-income earners in the total population while $1-\beta$ is the share of low-income earners. The social welfare function (3.1) must be maximised subject to the government budget constraint which is

$$\beta G^h + (1-\beta) G^l = 0, \tag{3.2}$$

where G^h and G^l are the generational accounts of a high-income earner and a low-income earner, respectively. A person's generational account is his net payments to the public sector over his lifetime,[5] so in the absence of WAs ($s = \alpha_B = \alpha_y = 0$) we have

$$G^h = \tau W e^h - B\left(1 - e^h\right) - y_1 - y_2, \tag{3.3}$$

[5] Strictly speaking, a person's generational account measures the *present value* of his net payments to the public sector, but recall that we have normalized the interest rate to zero.

$$G^l = \tau w e^l - B\left(1 - e^l\right) - y_1 - y_2. \tag{3.4}$$

Using (3.1) through (3.4) and remembering that individual indirect utilities are functions of the government's fiscal instruments, we can write the Lagrangian for the social planning problem as

$$\begin{aligned} L^{SP} &= \beta V^h\left(y_1, y_2, B, \tau\right) + (1-\beta) V^l\left(y_1, y_2, B, \tau\right) \\ &+\mu\left\{\tau\left[\beta W e^h + (1-\beta) w e^l\right] - B\left[\beta\left(1-e^h\right) + (1-\beta)\left(1-e^l\right)\right] - y_1 - y_2\right\}, \end{aligned} \tag{3.5}$$

where μ is the Lagrange multiplier associated with the government budget constraint, measuring the marginal social value of an additional unit of net revenue. From (3.5) and the properties of the indirect utility functions derived in section 2, we obtain the following first-order conditions for the optimal choice of the four policy instruments:

$$\begin{aligned} y_1: \quad & \beta\left[\left(1-e^h\right)\frac{\lambda^u}{\mu} + e^h\frac{\lambda^h}{\mu}\right] + (1-\beta)\left[\left(1-e^l\right)\frac{\lambda^u}{\mu} + e^l\frac{\lambda^l}{\mu}\right] \\ & = 1 - \beta\left(\tau W + B\right)\frac{\partial e^h}{\partial y_1} - (1-\beta)\left(\tau w + B\right)\frac{\partial e^l}{\partial y_1}, \end{aligned} \tag{3.6}$$

$$y_2: \quad \beta\frac{\lambda^h}{\mu} + (1-\beta)\frac{\lambda^l}{\mu} = 1 - \beta\left(\tau W + B\right)\frac{\partial e^h}{\partial y_2} - (1-\beta)\left(\tau w + B\right)\frac{\partial e^l}{\partial y_2}, \tag{3.7}$$

$$\begin{aligned} B: \quad & \beta\left(1-e^h\right)\frac{\lambda^u}{\mu} + (1-\beta)\left(1-e^l\right)\frac{\lambda^u}{\mu} = \beta\left(1-e^h\right) + (1-\beta)\left(1-e^l\right) \\ & -\beta\left(\tau W + B\right)\frac{\partial e^h}{\partial B} - (1-\beta)\left(\tau w + B\right)\frac{\partial e^l}{\partial B}, \end{aligned} \tag{3.8}$$

$$\begin{aligned} \tau: \quad & -\beta W e^h\frac{\lambda^h}{\mu} - (1-\beta) w e^l\frac{\lambda^l}{\mu} = -\beta W e^h - (1-\beta) w e^l \\ & -\beta\left(\tau W + B\right)\frac{\partial e^h}{\partial \tau} - (1-\beta)\left(\tau w + B\right)\frac{\partial e^l}{\partial \tau}. \end{aligned} \tag{3.9}$$

The left-hand sides of (3.6) through (3.9) indicate the marginal effects on private welfare (measured in monetary terms) of the various policy instruments, while the right-hand sides measure the marginal impact on net public revenue of the policy instrument considered. In a social optimum each benefit rate is raised (and the tax rate lowered) to the point where the resulting marginal gain in private welfare is just offset by the marginal budgetary cost.

3.2. Optimality of a non-zero tax and benefit wedge

We will now use the above first-order conditions to show that, despite the availability of the lump-sum policy instruments y_1 and y_2, a zero tax and benefit wedge ($\tau = B = 0$) cannot be an optimal policy. In other words, the optimal policy involves distortions to labour supply. To see this, suppose τ and B were in fact both equal to zero. The social optimum conditions (3.6) and (3.7) would then reduce to

$$\beta\left[\left(1-e^h\right)\frac{\lambda^u}{\mu}+e^h\frac{\lambda^h}{\mu}\right]+(1-\beta)\left[\left(1-e^l\right)\frac{\lambda^u}{\mu}+e^l\frac{\lambda^l}{\mu}\right]=1, \tag{3.10}$$

$$\beta\frac{\lambda^h}{\mu}+(1-\beta)\frac{\lambda^l}{\mu}=1. \tag{3.11}$$

Equating the left-hand sides of (3.10) and (3.11) and rearranging, we get

$$\beta\left(1-e^h\right)\left(\frac{\lambda^u}{\mu}-\frac{\lambda^h}{\mu}\right)+(1-\beta)\left(1-e^l\right)\left(\frac{\lambda^u}{\mu}-\frac{\lambda^l}{\mu}\right)=0. \tag{3.12}$$

Since the government is free to choose a non-zero value of B, optimality of a zero unemployment benefit (in a situation where $\tau = 0$) would require that the first-order condition (3.8) be satisfied for $B = \tau = 0$. In that case we would have

$$\left(\frac{\lambda^u}{\mu}-1\right)\left[\beta\left(1-e^h\right)+(1-\beta)\left(1-e^l\right)\right]=0 \quad\Rightarrow\quad \frac{\lambda^u}{\mu}=1. \tag{3.13}$$

Equations (3.11) and (3.13) imply that

$$\beta\frac{\lambda^h}{\mu}+(1-\beta)\frac{\lambda^l}{\mu}=\frac{\lambda^u}{\mu}, \tag{3.14}$$

which may be inserted into (3.12) to give

$$\left(\frac{\lambda^l}{\mu}-\frac{\lambda^h}{\mu}\right)\left(e^l-e^h\right)=0. \tag{3.15}$$

For condition (3.15) to be met, we must either have $\lambda^l = \lambda^h$ (which in turn requires $C_1^{el} = C_1^{eh}$) or $e^l = e^h$. However, within a linear tax system the government does not have sufficient policy instruments to equalize the consumption levels of individuals with different wage rates. Further, given the combination of different wage rates and identical tax and benefit rates plus identical preferences, it is inevitable from the individual optimality conditions (2.18) and (2.44) that the two income groups will choose different employment rates ($e^l \neq e^h$). Hence (3.15) cannot be satisfied, so $B = \tau = 0$ cannot be an optimal policy.

3.3. Optimality of a positive tax and benefit wedge for high-income earners

Our next task is to show that the optimal tax and benefit wedge for the group of high-income earners must be positive in the normal case where the marginal social value of income for the unemployed is higher than the marginal social value of income for employed workers. We start by defining the marginal social value of income for unemployed workers as

$$\theta^{uh} \equiv \frac{\lambda^u}{\mu} + (\tau W + B)\frac{\partial e^h}{\partial B}, \qquad \theta^{ul} \equiv \frac{\lambda^u}{\mu} + (\tau w + B)\frac{\partial e^l}{\partial B}. \tag{3.16}$$

According to these definitions, the marginal social value of an additional pound of unemployment benefit equals the social planner's valuation of additional disposable income for an unemployed person, λ^u/μ, plus the impact on net public revenue of the change in employment caused by the rise in the benefit rate. By analogy with (3.16), we may also define the marginal social value of (second-period) income for employed workers as

$$\theta^{h} \equiv \frac{\lambda^h}{\mu} + (\tau W + B)\frac{\partial e^h}{\partial y_2}, \qquad \theta^{l} \equiv \frac{\lambda^l}{\mu} + (\tau w + B)\frac{\partial e^l}{\partial y_2}. \tag{3.17}$$

With these definitions the social optimum condition (3.7) may be restated as

$$\theta^e \equiv \beta\theta^h + (1-\beta)\,\theta^l = 1, \tag{3.18}$$

where θ^e denotes the marginal social value of income for the average employed worker. Using (3.16) and (3.18), we may rewrite the social optimum condition (3.8) as

$$\begin{aligned} &\beta\left(1-e^h\right)\left(\theta^{uh}-\theta^e\right) + (1-\beta)\left(1-e^l\right)\left(\theta^{ul}-\theta^e\right) \\ &= -\beta\,(\tau W + B)\,e^h\frac{\partial e^h}{\partial B} - (1-\beta)\,(\tau w + B)\,e^l\frac{\partial e^l}{\partial B}. \end{aligned} \tag{3.19}$$

We know from (2.25) and (2.47) that $\partial e^h/\partial B$ and $\partial e^l/\partial B$ are both negative, given our assumption that $\alpha_B = 0$. Suppose now that the tax and benefit wedges $\tau W + B$ and $\tau w + B$ are also both negative. From (3.19) it then follows that the marginal social value of income would have to be lower for at least one of the two groups of unemployed workers than for the average employed worker. Given that both groups of unempoyed workers receive the same flat benefits, the normal case would be that $\theta^{uh} > \theta^e$ as well as $\theta^{ul} > \theta^e$, since otherwise the government would wish to redistribute income away from unemployed individuals towards the employed. Unless such a "perverse" situation exists, equation (3.19) thus implies that at least one of the tax and benefit wedges $\tau W + B$ and $\tau w + B$ must be positive under the optimal fiscal policy.

If $\tau w + B > 0$ is optimal, we also have $\tau W + B > 0$ in the social optimum, since $W > w$. If $\tau w + B < 0$ is an optimal policy, it is still possible to have $\tau W + B > 0$, and as we have seen this will be necessary for optimality in the normal case where the government wishes to redistribute income towards the unemployed. Hence we may reasonably assume that the group of high-income earners will always face a positive tax and benefit wedge under the optimal tax-transfer policy.

4. Introducing welfare accounts: The potential for a Pareto improvement

The labour income tax and the benefits B and y_1 (and y_2) serve partly to redistribute lifetime income from high-income earners to low-income earners and partly to redistribute income over the life cycle of each individual. To some extent these benefits therefore represent resources that are returned to the taxpayer himself over the life course. The introduction of welfare accounts allows the government to reduce the distortions from the part of an individual's tax payments and benefit receipts that only redistributes income over his own life cycle. Hence a system of welfare accounts with a lifetime income guarantee has the potential to generate a Pareto improvement, as we will show in this section.

4.1. Unemployment accounts

We start by focusing on the effects of introducing individual welfare accounts for unemployment benefits. When a fraction α_B of these benefits is debited to his WA and a fraction s of his wage income is paid into his account, the account balance of a high-income earner at the time of retirement will be

$$A^h = sWe^h - \alpha_B B \left(1 - e^h\right), \tag{4.1}$$

so the generational account of a high-income earner becomes

$$\begin{aligned} G^h &= (\tau + s) W e^h - B\left(1 - e^h\right) - y_1 - y_2 - A^h \\ &= \tau W e^h - (1 - \alpha_B)\left(1 - e^h\right) B - y_1 - y_2. \end{aligned} \tag{4.2}$$

The low-income earner also contributes a fraction s of his wage income to his WA, but since his wage rate is lower, we assume that he is unable to accumulate a surplus on his unemployment account. The account balance is therefore set to zero at the time of retirement, implying the following generational account for the low-income earner:

$$G^l = (\tau + s) w e^l - B\left(1 - e^l\right) - y_1 - y_2. \tag{4.3}$$

Suppose now that, when the mandatory contribution to the WAs is introduced, the ordinary labour income tax rate is cut by a similar amount so that

$$d\tau = -ds. \tag{4.4}$$

The WA reform will then leave the low-income earner's total tax wedge $\tau + s$ unaffected, having no impact on his labour supply and his generational account. The low-income earner's utility level will also stay the same since it follows from (2.40) and (4.4) that

$$dV^l = V_s^l ds + V_\tau^l d\tau = V_s^l (ds + d\tau) = 0. \tag{4.5}$$

Suppose further that the fraction α_B of unemployment benefits that is debited to the WA is calibrated in such a way that the WA reform leaves the welfare of high-income earners unaffected. Recalling that the mandatory WA contribution has no impact on the high-income earner's welfare, and using (2.12) and (2.13), we find[6]

$$\begin{gathered} dV^h = V_\tau^h d\tau + V_{\alpha_B}^h d\alpha_B = -W e^h \lambda^h d\tau - B\left(1 - e^h\right) \lambda^h d\alpha_B = 0 \quad \Rightarrow \\ d\alpha_B = -\frac{W e^h}{B\left(1 - e^h\right)} d\tau. \end{gathered} \tag{4.6}$$

[6]Using (4.1), (4.4) and (4.6) plus the fact that $s = \alpha_B = 0$ in the initial situation, one can show that the high-income earner would end up with a zero WA account balance if his labour supply did not respond to the reform. But since the compensated labour supply response to a lower marginal tax rate and a lower effective rate of unemployment benefit will be positive, the WA account balance for the high-income earner will in fact be positive, as we have assumed.

From (2.26), (2.28), (4.2) and (4.4) and the fact that the initial value of α_B is zero, we find the effect of the WA reform on the generational account of the high-income earner to be

$$\begin{aligned} dG^h &= \overbrace{We^h d\tau + B\left(1-e^h\right)d\alpha_B}^{\text{mechanical effect}} + \overbrace{(\tau W + B)\left(\frac{\partial e^h}{\partial \tau}d\tau + \frac{\partial e^h}{\partial \alpha_B}d\alpha_B\right)}^{\text{behavioural effect}} \\ &= (\tau W + B)\left[\frac{We^h}{B\left(1-e^h\right)}\left(\frac{\partial e^h}{\partial \alpha_B}\right)_c - \left(\frac{\partial e^h}{\partial \tau}\right)_c\right] ds > 0. \qquad (4.7) \end{aligned}$$

We see that the mechanical revenue effects and the income effects on labour supply wash out (as a result of (4.6)), leaving only the compensated labour supply responses which are unambiguously positive (see (2.26) and (2.28)). Hence the generational account of the high-income earner improves, in part because of the positive labour supply response to a lower effective rate of unemployment benefit, and partly due to the positive effect on labour supply of the fall in the marginal labour income tax rate.

Since the WA reform was designed to leave the lifetime utilities of both income groups as well as the generational account of low-income earners unaffected, the greater net revenue contributed by high-income earners enables the government to generate a Pareto improvement. For example, without violating its budget constraint, the government could raise the welfare of both income groups by raising one of the universal benefits y_1 or y_2 or by lowering the labour income tax rate further.

4.2. Efficient liquidity insurance through unemployment accounts

The potential for a Pareto improvement means that the unemployment accounts improve the government's equity-efficiency trade-off. In particular, unemployment accounts allows the government to offer liquidity insurance to the unemployed at a lower efficiency cost. To illustrate this, consider first the situation without WAs and suppose that the government wishes to improve liquidity insurance for the unemployed by raising the labour income tax rate in order to finance a rise in the rate of unemployment benefit. Suppose further that this is done in a way that does not affect the lifetime utility of a low-income earner. From (2.36) and (2.39) this requires

$$dV^l = V_\tau^l d\tau + V_B^l dB = -we^l\lambda^l d\tau + \left(1-e^l\right)\lambda^u dB = 0 \quad \Rightarrow$$

$$d\tau = \left(\frac{1-e^l}{we^l}\right)\left(\frac{\lambda^u}{\lambda^l}\right)dB. \tag{4.8}$$

According to (2.9) and (2.12) the impact of the reform (4.8) on the lifetime utility of a high-income earner will be

$$dV^h_{\text{No WA}} = V^h_\tau d\tau + V^h_B dB = -We^h\lambda^h d\tau + \left(1-e^h\right)\lambda^u dB \quad \Rightarrow$$

$$dV^h_{\text{No WA}} = \left[\left(1-e^h\right)\lambda^u - We^h\lambda^h\left(\frac{1-e^l}{we^l}\right)\left(\frac{\lambda^u}{\lambda^l}\right)\right]dB, \tag{4.9}$$

where the subscript "No WA" indicates that we are considering a situation without welfare accounts. Using (3.3), we find the following effect of the reform (4.8) on a high-income earner's generational account:

$$dG^h_{\text{No WA}} = We^h d\tau - \left(1-e^h\right)dB + (\tau W + B)\left(\frac{\partial e^h}{\partial \tau}d\tau + \frac{\partial e^h}{\partial B}dB\right) \quad \Rightarrow$$

$$\begin{aligned} dG^h_{\text{No WA}} \;&=\; \left[(\tau W + B)\frac{\partial e^h}{\partial B} - \left(1-e^h\right)\right]dB \\ &\quad + \left[We^h + (\tau W + B)\frac{\partial e^h}{\partial \tau}\right]\left(\frac{1-e^l}{we^l}\right)\left(\frac{\lambda^u}{\lambda^l}\right)dB. \end{aligned} \tag{4.10}$$

Now suppose instead that the rise in unemployment benefits is implemented in an economy with unemployment accounts and suppose further that, instead of raising the labour income tax rate, the government finances the higher unemployment benefits through an increase in the mandatory WA contribution rate ds of exactly the same magnitude as the tax increase given by (4.8). Since τ and s are equivalent taxes from the viewpoint of a low-income earner without a WA surplus, the welfare, behaviour, and generational account of all members of this group will be exactly the same as under the reform (4.8).

For high-income earners we assume that α_B is calibrated to ensure that their lifetime utility changes by the same amount as before, i.e., by the amount stated in (4.9). Using (2.9) and (2.12) and the subscript "WA" to indicate a situation with welfare accounts, we thus choose a value of $d\alpha_B$ such that

$$dV^h_{\text{WA}} = V^h_B dB + V^h_{\alpha_B} d\alpha_B = \left(1-e^h\right)\lambda^u dB - B\left(1-e^h\right)\lambda^h d\alpha_B = dV^h_{\text{No WA}} \quad \Rightarrow$$

$$d\alpha_B = \left(\frac{W}{B}\right)\left(\frac{e^h}{1-e^h}\right)\left(\frac{1-e^l}{we^l}\right)\left(\frac{\lambda^u}{\lambda^l}\right)dB. \tag{4.11}$$

In an economy with unemployment accounts, a high-income earner's generational account is given by (4.2). Using this along with (4.11) and assuming that $\alpha_B = 0$ initially, we find that the improved liquidity insurance via unemployment accounts will affect a high-income earner's generational account in the following way:

$$dG^h_{\text{WA}} = B\left(1-e^h\right)d\alpha_B - \left(1-e^h\right)dB + (\tau W + B)\left(\frac{\partial e^h}{\partial \alpha_B}d\alpha_B + \frac{\partial e^h}{\partial B}dB\right) \quad \Rightarrow$$

$$dG^h_{\text{WA}} = \left[(\tau W + B)\frac{\partial e^h}{\partial B} - \left(1-e^h\right)\right]dB$$

$$+\left[B\left(1-e^h\right) + (\tau W + B)\frac{\partial e^h}{\partial \alpha_B}\right]\left(\frac{W}{B}\right)\left(\frac{e^h}{1-e^h}\right)\left(\frac{1-e^l}{we^l}\right)\left(\frac{\lambda^u}{\lambda^l}\right)dB. \qquad (4.12)$$

Subtracting (4.10) from (4.12) and inserting the expressions for $\partial e^h/\partial \alpha_B$ and $\partial e^h/\partial \tau$ stated in (2.26) and (2.28), we end up with

$$dG^h_{\text{WA}} - dG^h_{\text{No WA}} = (\tau W + B)\left[\frac{We^h}{B\left(1-e^h\right)}\overbrace{\left(\frac{\partial e^h}{\partial \alpha_B}\right)_c}^{>0} - \overbrace{\left(\frac{\partial e^h}{\partial \tau}\right)_c}^{<0}\right]\left(\frac{1-e^l}{we^l}\right)\left(\frac{\lambda^u}{\lambda^l}\right)dB > 0. \qquad (4.13)$$

The result in (4.13) shows that a rise in unemployment benefits *with a given impact on private welfare* can be provided at a lower net revenue cost via a system of unemployment accounts than via the traditional tax-transfer system. The explanation is that, under the WA system, the rise in unemployment benefits causes a smaller drop in labour supply. There are two reasons for this. First, for individuals with a surplus on their WA the rise in the mandatory WA contribution rate does not work like a tax and hence does not discourage work. This effect is reflected by the presence of the compensated labour supply response $\left(\partial e^h/\partial \tau\right)_c$ in the square bracket in (4.13). Second, for people with a WA surplus the rise in the rate of unemployment benefit is less discouraging to job search because they must finance a fraction α_B of the benefit themselves via a lower surplus on their unemployment account. This is captured by the presence of the compensated labour supply reaction $\left(\partial e^h/\partial \alpha_B\right)_c$ in (4.13). Note that even if high-income earners must finance (part of) their higher benefit themselves, a rise in the rate of unemployment benefit could still be beneficial by shifting resources to the stage in a worker's life cycle where he is liquidity-constrained (and hence has a higher marginal utility of consumption than when his unemployment account balance is paid out).

4.3. Welfare accounts for other benefits

The combination of a proportional labour income tax (τ) and a flat benefit y_1 to all individuals of working age does imply some redistribution of lifetime income from high-income to low-income earners. However, part of the labour income tax paid by high-income earners is transferred back to these individuals themselves via the benefit y_1. Yet even this non-redistributive part of the labour income tax is distortionary, since there is no direct link between taxes paid and benefits received. The present subsection shows that this distortion may be avoided by including the benefit y_1 in the system of welfare accounts. Abstracting from unemployment accounts, the balance on a high-income earner's WA at retirement would then be

$$A^h = sWe^h - \alpha_y y_1, \tag{4.14}$$

and his generational account would be

$$\begin{aligned} G^h &= (\tau + s) We^h - B\left(1 - e^h\right) - y_1 - y_2 - A^h \\ &= \tau We^h - B\left(1 - e^h\right) - (1 - \alpha_y) y_1 - y_2. \end{aligned} \tag{4.15}$$

The low-income earners who do not manage to accumulate a WA surplus would still have a generational account given by (4.3). Now suppose again that when the WA system is introduced, the labour income tax rate is cut by the same amount as the mandatory WA contribution rate is increased so that (4.4) still holds. The welfare and the generational account of a low-income earner will then still be unaffected by the reform. Assume further that α_y is calibrated such that the lifetime utility of a high-income earner is likewise unchanged. From (2.12), (2.14) and (4.4) this implies

$$\begin{aligned} dV^h = V^h_\tau d\tau + V^h_{\alpha_y} d\alpha_y = -We^h \lambda^h d\tau - y_1 \lambda^h d\alpha_y = 0 \quad &\Rightarrow \\ d\alpha_y = \frac{We^h}{y_1} ds. & \end{aligned} \tag{4.16}$$

The impact of the WA reform on the generational account of a high-income earner may be derived from (4.15), using (2.26), (2.29), (4.4) and (4.16):

$$\begin{aligned} dG^h &= We^h d\tau + y_1 d\alpha_y + (\tau W + B)\left[\left(\frac{\partial e^h}{\partial \tau}\right) d\tau + \left(\frac{\partial e^h}{\partial \alpha_y}\right) d\alpha_y\right] \\ &= -(\tau W + B)\left(\frac{\partial e^h}{\partial \tau}\right)_c ds > 0. \end{aligned} \tag{4.17}$$

We see from (4.17) that the WA reform improves the public budget while keeping all private utilities constant. Hence, the reform allows a Pareto improvement since it avoids the distortions to labour supply arising when ordinary taxes are levied on high-income earners to finance benefits that are transferred back to themselves.

4.4. Efficient redistribution of lifetime income through welfare accounts

The analysis in the previous subsection suggests that redistribution of lifetime incomes can be achieved in a more efficient manner by including the flat universal benefit y_1 in a system of welfare accounts that are coupled with a lifetime income gurantee. We will now show that this is indeed the case. Our proof follows the general procedure laid out in subsection 4.2. Consider first a government that wishes to achieve more redistribution within the framework of a conventional tax-transfer system by raising the labour income tax rate τ to finance a rise in the benefit y_1. Like before, suppose the increases in τ and y_1 are calibrated to keep the welfare of low-income earners constant. Exploiting (2.37) and (2.39), we find that this requires

$$dV^l = V_\tau^l d\tau + V_{y_1}^l dy_1 = -we^l\lambda^l d\tau + \left[\left(1-e^l\right)\lambda^u + e^l\lambda^l\right] dy_1 = 0 \quad \Rightarrow$$

$$d\tau = \frac{\left(1-e^l\right)\lambda^u + e^l\lambda^l}{we^l\lambda^l} dy_1. \tag{4.18}$$

The impact of the reform (4.18) on the welfare of high-income earners is found from (2.10) and (2.12):

$$dV^h_{\text{No WA}} = V_\tau^h d\tau + V_{y_1}^h dy_1 = -We^h\lambda^h d\tau + \left[\left(1-e^h\right)\lambda^u + e^h\lambda^h\right] dy_1 \quad \Rightarrow$$

$$dV^h_{\text{No WA}} = \left\{\left(1-e^h\right)\lambda^u + e^h\lambda^h - \left(\frac{We^h}{we^l}\right)\left(\frac{\lambda^h}{\lambda^l}\right)\left[\left(1-e^l\right)\lambda^u + e^l\lambda^l\right]\right\} dy_1. \tag{4.19}$$

From (3.3) we may also derive the effect of the reform (4.18) on a high-income earner's generational account:

$$dG^h_{\text{No WA}} = We^h d\tau - dy_1 + (\tau W + B)\left(\frac{\partial e^h}{\partial \tau} d\tau + \frac{\partial e^h}{\partial y_1} dy_1\right) \quad \Rightarrow$$

$$\begin{aligned} dG^h_{\text{No WA}} \quad &= \quad \left[(\tau W + B)\frac{\partial e^h}{\partial y_1} - 1\right] dy_1 \\ &\quad + \left[We^h + (\tau W + B)\frac{\partial e^h}{\partial \tau}\right]\left(\frac{\left(1-e^l\right)\lambda^u + e^l\lambda^l}{we^l\lambda^l}\right) dy_1. \end{aligned} \tag{4.20}$$

Let us now compare the effects of the above reform to a setting with welfare accounts where the rise in y_1 is financed by a rise in the WA contribution rate s and where a fraction α_y of the higher benefit is debited to the WA. Under the assumption that the rise in s is identical to the rise in τ in the situation without WAs, there will be no difference in the impact of the two reforms on the welfare and the generational account of a low-income earner. We may also calibrate $d\alpha_y$ to ensure that the impact on a high-income earner's welfare is the same under the two reforms. From (2.10), (2.14) and (4.19) we find that, starting from $\alpha_y = 0$, this will require

$$dV^h_{\text{WA}} = V^h_{y_1} dy_1 + V^h_{\alpha_y} d\alpha_y = \left[\left(1 - e^h\right) \lambda^u + e^h \lambda^h\right] dy_1 - y_1 \lambda^h d\alpha_y = dV^h_{\text{No WA}} \quad \Rightarrow$$

$$d\alpha_y = \left(\frac{1}{y_1}\right) \left(\frac{We^h}{we^l}\right) \left(\frac{\left(1 - e^l\right) \lambda^u + e^l \lambda^l}{\lambda^l}\right) dy_1. \tag{4.21}$$

Using (4.15) and (4.21), we obtain the change in a high-income earner's generational account under the policy reform with welfare accounts:

$$dG^h_{\text{WA}} = y_1 d\alpha_y - dy_1 + (\tau W + B) \left(\frac{\partial e^h}{\partial y_1} dy_1 + \frac{\partial e^h}{\partial \alpha_y} d\alpha_y\right) \quad \Rightarrow$$

$$dG^h_{\text{WA}} = \left[(\tau W + B) \frac{\partial e^h}{\partial y_1} - 1\right] dy_1$$

$$+ \left[1 + \left(\frac{\tau W + B}{y_1}\right) \frac{\partial e^h}{\partial \alpha_y}\right] \left(\frac{We^h}{we^l}\right) \left(\frac{\left(1 - e^l\right) \lambda^u + e^l \lambda^l}{\lambda^l}\right) dy_1. \tag{4.22}$$

Subtracting (4.20) from (4.22) and inserting the expressions for $\partial e^h / \partial \tau$ and $\partial e^h / \partial \alpha_y$ given by (2.26) and (2.29), we now find that

$$dG^h_{\text{WA}} - dG^h_{\text{No WA}} = -(\tau W + B) \left(\frac{\partial e^h}{\partial \tau}\right)_c \left(\frac{\left(1 - e^l\right) \lambda^u + e^l \lambda^l}{we^l \lambda^l}\right) dy_1 > 0. \tag{4.23}$$

This result demonstrates that a given amount of redistribution (through a rise in y_1) with a given impact on private sector welfare can be achieved at a lower net revenue cost in an economy with welfare accounts. Thus it is more efficient to redistribute lifetime incomes through a system of welfare accounts coupled with a tax-financed lifetime income guarantee for low-income earners.

REFERENCES

Bovenberg, A.L. and P.B. Sørensen (2004). Improving the equity-efficiency trade-off: Mandatory savings accounts for social insurance. *International Tax and Public Finance* 11, 507-529.

Bovenberg, A.L. M.I. Hansen and P.B. Sørensen (2011). Efficient redistribution of lifetime income through welfare accounts. Paper under review for *Fiscal Studies.*