

Keynesian-Monetarist Dynamics and the Corridor

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November 1993

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Abstract

The idea that for small disturbances the full employment equilibrium is stable while for large disturbances it is unstable was coined by Leijonhufvud in the notion of a “corridor”. We discuss the existence of a corridor in the standard Keynesian-Monetarist textbook macro-model. It turns out that though the full employment steady state of this model may be locally stable - which is the case when the well-known Cagan condition holds - the model is never globally stable. This is due to the inherent non-linearity in the demand for money function, arising from non-negativity of the nominal rate of interest. Thus, perhaps surprisingly, the Cagan condition is both necessary and sufficient for the existence of a corridor in the Keynesian-Monetarist model.

Key words: IS-LM dynamics. Global dynamics. Corridor. Deflation. Liquidity trap. Hopf bifurcation.

JEL Classification: E12, E31, E32, E44.

1 Introduction

This note is concerned with an unfamiliar implication of a familiar macrodynamic model. It is shown that the full employment equilibrium of the simple Keynesian-Monetarist medium run model has to be unstable in the large, whether it is locally

stable or not. By Keynesian-Monetarist medium run model we mean the IS-LM model dynamized by adding the expectations-augmented natural rate Phillips curve and adaptive expectations. The qualifier "simple" signifies that the consumption function is of the simple Keynesian type having current income as the only argument. It is well-known that, depending on the parameters, the steady state of this model may be locally stable or unstable (Tobin 1975, Scarth 1977, Taylor 1977, Yarrow 1977). The stabilizing force derives from price level adjustments (notably the Keynes effect), while the destabilizing force derives from the endogenously changing inflation expectations. Which is locally the stronger one essentially depends on the celebrated stability condition found by Cagan in his classical study of hyperinflation (Cagan 1956).

Tobin (1975) hinted at the possibility that the dynamics might give rise to stability for small disturbances, but instability for larger chocks. Tobin did not, however, analyze the problem in a rigorous way and left the definite conclusion for further investigation.

Using the geometry of the phase plane we show that Tobin's conjecture is confirmed when the inherent non-linearity in the demand for money function, arising from non-negativity of the nominal rate of interest, is taken into account. Large contractionary disturbances lead to a *dynamic* liquidity trap so that the system never returns to the steady state. Hence, in the simple Keynesian-Monetarist model deep slumps are selfsustaining. In case the Cagan condition holds, i.e., the steady state is *locally* stable, this implies the existence of a "corridor", a notion introduced by Leijonhufvud (1973). A corridor is the limited neighbourhood of stability around a steady state which is locally stable, but not globally stable. Conditions for the existence of a corridor in different models are discussed in Grossman (1974), Howitt (1978), Löfgren (1979), Siven (1981), Raymon (1981), Balasko & Royer (1985) and van de Klundert & Schaik (1990). The question of the existence of a corridor in the Keynesian-Monetarist model was briefly considered in Cugno & Montrucchio (1984). They approached the problem by means of the Hopf bifurcation theorem, analysing the dynamic scheme in a small neighbourhood of the bifurcation value of the control parameter. This provided no concise conditions as to the existence of a corridor. We show, however, the global result that the Cagan condition is both a necessary and a sufficient condition for the existence of a corridor.

Although the model we consider is very simplistic (and degenerates if expectations are rational instead of adaptive) it is one of the core models of intermediate

macroeconomic textbooks and of discussions of economic policy in the press and among policy makers (cf. Mankiw 1990, Tobin 1993). The fact that the model is necessarily unstable in the large therefore seems worthy of some attention.

2 Keynesian-Monetarist Dynamics

The model is the standard imperfectly flexible prices-version of the IS-LM model. The symbols are: P = price of output, M = nominal money supply (equal to monetary base), m = real money supply, M/P , y = real output, \dot{P}/P = actual rate of inflation, x = expected rate of inflation, i = nominal rate of interest, r = real rate of interest. A dot denotes a time derivative. F_x denotes the partial derivative with respect to the argument x of a function F with two or more arguments. The derivative of a function F with one argument is denoted F' or, sometimes, F_x . The model consists of the following six equations, equations (2.3) and (2.4) being merely identities.

$$y = C(y) + I(y, r), \quad 0 < C_y < C_y + I_y < 1, \quad I_r < 0, \quad (2.1)$$

$$m = L(y, i), \quad L_y > 0, \quad L_i < 0, \quad (2.2)$$

$$r = i - x, \quad (2.3)$$

$$m = M/P, \quad (2.4)$$

$$\dot{P}/P = \varphi(y) + x, \quad \varphi(y^*) = 0, \quad \varphi' > 0, \quad (2.5)$$

$$\dot{x} = b(\dot{P}/P - x). \quad (2.6)$$

The behavioural functions C , I , and L (i.e., consumption, investment and money demand), and the Phillips curve φ are continuously differentiable.

M , y^* and b are positive constants. y^* is the “natural” rate of output defined as that level of output which is consistent with a constant rate of inflation. We may also speak of y^* as “full employment output” meaning by this no more than the non-accelerating-inflation level of output. The conception behind (2.1) is that firms in every short period (in the limit a point in time) adjust output to the effective demand they meet at the given price level. The assumption is that this takes place not only below, but also above the natural level of output. It is understood that both the real wage, the capital stock, and the labour supply allow profitable production to take place also above the natural level, provided there is sufficient demand.¹

¹Thus the complexities of “shifting regimes” (see, e.g., Grandmont 1987 and van den Heuvel

From the point of view of the dynamic analysis there are six endogenous variables: y , m , x , i , r , and P . Given the initial values $x(0)$ and $P(0)$ the model generates time paths of these endogenous variables.

The short run

In the short run, i.e., for fixed t , there are a historically given money supply, a given output price and given inflation expectations. Therefore, when considering the LM equation, (2.2), and the IS equation,

$$y = C(y) + I(y, i - x), \quad (2.7)$$

derived from (2.1) and (2.3), m and x are given for fixed t . The “IS curve” is downward sloping and the “LM curve” upward sloping. The short run solution (y, i) is therefore unique, and we can write y and i as continuously differentiable functions of x and m :

$$y = f(x, m), \quad i = g(x, m), \quad (2.8)$$

to be characterized later. The question of *existence* of this solution (y, i) to (2.2) and (2.7) calls for a comment since a study of global dynamics should take account of a large range of variation of x and m . We assume the following boundary conditions:

- A1. (i) $C(0) > 0$, and
(ii) there exists β , $0 < \beta < 1$, such that $C_y + I_y \leq \beta$ everywhere.

A2. For all $y > 0$:

- (i) $\lim_{i \rightarrow 0} L(y, i) = \infty$, and
(ii) $\lim_{i \rightarrow \infty} L(y, i) = 0$.

Part (ii) of A1, i.e., that the marginal propensity to spend is always bounded away from one, strengthens the standard condition $0 < C_y + I_y < 1$ in an economically unimportant, but technically convenient way. As for A2 its rationale is: When the rate of interest approaches zero, everyone wants to hold his wealth in the form of cash and may indeed want to borrow and keep the proceeds in cash. This is because some interest reward is needed to compensate for the lower degree of liquidity which

1983) do not arise.

characterizes bonds and equities. On the other hand, at very high rates of interest nobody willingly holds money, and society tends to some other means of exchange. A1 is assumed throughout while A2 is assumed until further notice.²

In view of A2 and $L_i < 0$ we can in principle solve (2.2) with respect to i , for given $y > 0$, $m > 0$. This gives

$$i = h(y, m) > 0 \text{ with } h_y = -L_y/L_i > 0, h_m = 1/L_i < 0, \text{ and} \quad (2.9)$$

$$\text{for all } y > 0 \quad \lim_{m \rightarrow \infty} h(y, m) = 0, \quad \lim_{m \rightarrow 0} h(y, m) = \infty \quad (2.10)$$

by A.2. Inserting (2.9) in (2.7) it is straightforward to show:

Property 1. *Assume A1 and A2. Then given any x and m , where $m > 0$, the system (2.2) and (2.7) has a unique solution (y, i) , as indicated in (2.8), and y and i are positive.*

For use later we observe that

$$\frac{\partial y}{\partial x} = f_x = \frac{-I_r}{1 - C_y - I_y + I_r L_y / L_i} > 0, \quad \frac{\partial y}{\partial m} = f_m = \frac{I_r / L_i}{1 - C_y - I_y + I_r L_y / L_i} > 0. \quad (2.11)$$

Dynamics

From (2.5), (2.6), and (2.8) we obtain

$$\dot{x} = b\varphi(f(x, m)). \quad (2.12)$$

Combining (2.4), (2.5), and (2.8) gives

$$\dot{m} = -[x + \varphi(f(x, m))]m \quad (2.13)$$

In view of Property 1 the dynamic system (2.12)-(2.13) is defined for all (x, m) , $m > 0$. The domain of definition of the system will be called U , that is $U = R \times R_{++}$.

A *steady state* is a time path along which x and m , and therefore y and i , are constant. The steady state values of x and m are called x^* and m^* , respectively. Now, $\dot{x} = 0$ implies, by (2.12) and (2.5), that output equals y^* , the natural rate of

²An example of a demand for money function obeying A2 is the case of constant interest elasticity: $L(y, i) = \psi(y)i^{-\varepsilon}$, $\psi' > 0$, $\varepsilon > 0$. A2(i) excludes the Cagan demand for money function $L(y, i) = \psi(y)e^{-ai}$, $a > 0$. We deal with that case separately below.

output. Then, by (2.13), $\dot{m} = 0$ implies $x^* = 0$. Furthermore, m^* is the positive solution in m to the equation $y^* = C(y^*) + I(y^*, h(y^*, m))$ which is derived by inserting (2.9), $y = y^*$, and $x = 0$ into (2.7). To ensure existence of such a solution we need sufficient variability of investment demand. We shall assume

$$A3. C(y^*) + I(y^*, \infty) < y^* < C(y^*) + I(y^*, 0).$$

I.e., at a low (high) real rate of interest, investment is (is not) sufficient to absorb full employment savings. Now follows:

Property 2. *Given A1, A2, and A3, the dynamic system (2.12)-(2.13) has a unique steady state (x^*, m^*) in the domain of definition, U , and $x^* = 0$. Like x^* , m^* is independent of b , the speed of adjustment of inflation expectations, and independent of φ' , the steepness of the short run Phillips curve.*

Investigating the Jacobian of (2.12)-(2.13), evaluated at the steady state, we find that the determinant is $b\varphi'f_m > 0$ and the trace is $\varphi'(bf_x - f_m m)$. Thus, the steady state $(0, m^*)$ is *locally asymptotically stable* – i.e., a sink – if $bf_x(0, m^*) < f_m(0, m^*)m^*$. By (2.11), this inequality is equivalent to

$$-b\frac{L_i^*}{m^*} < 1. \tag{2.14}$$

The steady state is *unstable*³ if this inequality is reversed. It follows that for b (the speed of adjustment of inflation expectations) or $-L_i^*/m^*$ (the semi elasticity of money demand with respect to the nominal rate of interest) sufficiently small, the steady state is a sink, and for b or $-L_i^*/m^*$ sufficiently large, the steady state is unstable.⁴

Thus, it cannot be decided a priori whether the steady state is stable or unstable for small displacements of the economy. This is due to the ambiguous role of price dynamics in relation to stability. While lower prices increase demand through the

³A steady state which is not locally stable is called *unstable*. Our definitions are as in Hirsch & Smale (1974).

⁴While φ' , i.e., the sensitivity of (unanticipated) inflation with respect to the activity level in the economy, influences neither the position of the steady state nor the question of asymptotic stability, it turns out that oscillations are less likely to occur, the larger is φ' .

so-called Keynes effect⁵, the expectation of falling prices decreases demand. Indeed, if the system is initially in the steady state, the immediate effect of a contractionary shock is to make $y < y^*$; then inflation goes down (becomes negative) and expected inflation follows suit, the more quickly so, the higher is b . As appears from (2.7) output can only return to y^* , if the real rate of interest, $i - x$, is moved downwards. While falling prices increase real money balances, thereby lowering the nominal rate of interest, i , and tending to pull $i - x$ downwards, the expectation of falling prices evidently works in exactly the opposite direction, tending to increase $i - x$.⁶ The first mentioned force is the stronger one when the sensitivity of the nominal rate of interest with respect to the money supply, $-m^*/L_i^*$, is high, i.e., when $-L^*/m^*$ is low.

This explains the stability condition (2.14). The condition is called the *Cagan condition* because it is formally identical to the stability condition found by Cagan (1956) in his classical analysis of the purely monetary dynamics in situations of hyperinflation. As to the Keynesian-Monetarist model the condition (2.14) was, in essentially the same form, discovered by Tobin (1975). The condition is mentioned in the second edition of Dornbusch & Fischer (1981, p. 444), but - strangely enough - not explicitly in later editions, and in Scarth (1988, p. 60).⁷

3 The Corridor

Leaving the merely local stability analysis we turn to global dynamics. Tobin (1975, p. 201) gave some hints that the stabilizing force, the Keynes effect, tends to be relatively weaker the further below equilibrium output the system is. We shall prove this conjecture, i.e., that the simple Keynesian-Monetarist model is necessarily unstable for large contractionary disturbances. Deep slumps are not self-correcting. This implies that *if* (2.14) holds, i.e., the steady state is *locally* stable, then there exists what Leijonhufvud (1973) calls a “corridor”. By a corridor is meant the limited neighbourhood of asymptotic stability around a steady state which is locally

⁵The term *Keynes effect* is really a misnomer since this effect, measured by f_m as given in (2.11), is stronger the *less* Keynesian bias the assumptions have, i.e., the higher is L_r/L_i .

⁶This point was already stressed by Keynes (Keynes 1936, p. 263).

⁷For some extensions, see Groth (1988).

From another perspective the relation between price flexibility and stability is discussed in De Long and Summers (1986) and King (1988). They deal with stability in the sense of lack of statistical variance of output rather than as convergence. See also ch. 4 in Sheffrin (1989).

asymptotically stable but not globally asymptotically stable.

To be more precise we introduce the following definitions. Remember that the domain of definition, $R \times R_{++}$, of our dynamic system (2.12), (2.13) is called U . The steady state (x^*, m^*) of the system (2.12), (2.13) is called *globally asymptotically stable* if every solution $(x(t), m(t))$ with $(x(0), m(0))$ in U converges to (x^*, m^*) for $t \rightarrow +\infty$. A neighbourhood N of (x^*, m^*) in U is called a *neighbourhood of asymptotic stability* if any solution starting in N converges to (x^*, m^*) for $t \rightarrow +\infty$. Given a steady state which is locally asymptotically stable but not globally asymptotically stable, the union of all its neighbourhoods of asymptotic stability is called a *corridor*.⁸

By using the strict notion of asymptotic stability as a criterion we diverge from Howitt who found this notion inappropriate. He argued that “for the concept of asymptotic stability corridor-effects seem unlikely to occur” (Howitt 1978, p. 268). We do not agree since in reality non-linearities of some kind are always present and may cause a system which is locally asymptotically stable to be unstable in the large.

In the present case it is the non-linearity in the demand for real balances that is important. This non-linearity follows from the fact that zero is an absolute floor to the nominal rate of interest (as expressed in A2). Whatever the value of the interest elasticity, increases in real money supply become less and less effective in reducing the nominal rate of interest. To see the implication of this we draw the phase portrait of the system (2.12), (2.13), cf. Figure 4.1. The slopes of the $\dot{x} = 0$ and $\dot{m} = 0$ loci are given by

$$\frac{dm}{dx} \Big|_{\dot{x} = 0} = -\frac{f_x}{f_m}, \quad \frac{dm}{dx} \Big|_{\dot{m} = 0} = -\frac{f_x}{f_m} - \frac{1}{\varphi' f_m}.$$

At the point of intersection the slope of the $\dot{m} = 0$ locus is therefore smaller than the slope of the $\dot{x} = 0$ locus. Observe that above the $\dot{x} = 0$ locus we have $y > y^*$ and below the $\dot{x} = 0$ locus $y < y^*$. Moving North-East in the diagram is associated with rising output.

Now, whatever the specific shape of the demand for money function, as long as A2 (i) holds, the $\dot{x} = 0$ locus in the phase diagram tends to become vertical as x declines towards some critical value. Indeed, along the $\dot{x} = 0$ locus, $y = y^*$ and

⁸A corridor is thus a *basin* (Hirsch & Smale 1974, p. 190) which does not contain the whole of $U \setminus \{(x^*, m^*)\}$. The intuitive meaning of the term “corridor” is perhaps clearest if we think of the economy in (x, m, t) -space rather than in (x, m) -space.

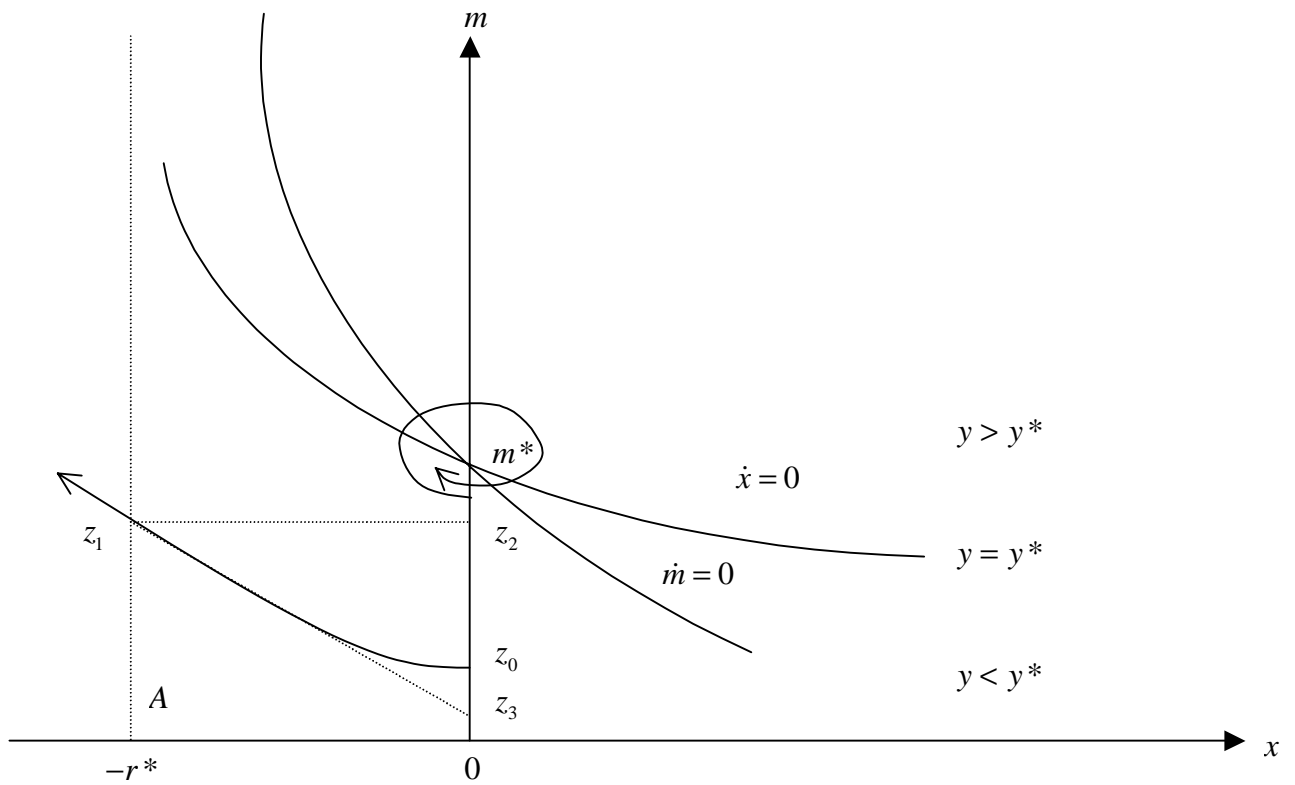


Figure 1:

therefore, by (2.1), the real rate of interest, r , has a constant value, the steady state value $r^* = h(y^*, m^*)$. Thus, along the $\dot{x} = 0$ locus we have $i = r^* + x = h(y^*, m)$, by (2.2) and (2.9). Hence, as x tends to $-r^*$ along the $\dot{x} = 0$ locus, $h(y^*, m)$ tends to zero and m tends to $+\infty$, by (2.10), as shown in Figure 1. This is a manifestation of the non-negativity of the nominal rate of interest, and instability in the large follows immediately. If the initial position $(x(0), m(0))$ of the system is on the vertical line $x = -r^*$, then the solution $(x(t), m(t))$ moves North-West and never returns to the steady state.

However, one might be more interested in the movement of the system, when the initial position is at some point on the vertical line $x = 0$ below the steady state point. This corresponds to a situation where the system has been in the steady state for all $t < 0$, but at $t = 0$ it is disturbed by a contractionary shock, say a downward shift of the investment schedule $I(y, r)$ caused by a fall in long run optimism. The effect of this is that the new steady state point $(0, m^*)$ has a higher m^* than the old, because r^* in the new steady state has to be lower than in the old, due to the

shift of $I(y, r)$. Thus, immediately after the shock the economy is at a point, say $z_0 = (0, m_0)$, below the new steady state point $(0, m^*)$, as indicated in Figure 1, now interpreted as showing the situation after the shock. Hence, at $t = 0$, $y < y^*$. Will this recession be self-correcting? The following proposition tells us that if the initial disturbance is great enough, this will not be the case and deflation will be permanent.

Proposition 1 (No recovery) *Assume A1, A2, and A3. Let a recession as described above take place. Then there exists δ , $0 < \delta < 1$, such that for $m_0 \leq \delta m^*$ the solution $(x(t), m(t))$ with $(x(0), m(0)) = (0, m_0)$ never leaves the deflationary region⁹ and y does not tend to y^* for $t \rightarrow \infty$.*

The proof rests on tracing a solution curve crossing the vertical line $x = -r^*$, say at z_1 , backwards in time. Using that the line segment $z_1 z_2$, cf. Figure 1, is compact and that solution curves in the relevant region are less steep the lower is m for fixed x , it can be shown that the solution curve does indeed cross the vertical halfline $\{(x, m) \in U | x = 0, 0 < m < m^*\}$. See appendix A.

Corollary 1 *If and only if the Cagan condition (2.14) holds, then a corridor exists.*

Thus, according to this model the deflationary process set into motion by a deep slump does not create a return to full employment. The explanation is that the lower the nominal rate of interest has become during the slump, the more difficult it is to decrease it further. And as actual and expected inflation continue to fall, it becomes more and more likely that the real rate of interest will increase instead of decrease.¹⁰ In this manner one might say that it is a *dynamic liquidity trap* which causes deep slumps to be self-sustaining¹¹. Observe that no postulate of a

⁹The deflationary region is the region $A = \{(x, m) \in U | \dot{x} < 0, \dot{m} > 0\}$, cf. Figure 1.

¹⁰Interestingly, in the boom there is no similar tendency for the stabilizing Keynes effect to be weaker the further away from equilibrium the system is. On the opposite, $-L_i/L$ tends to be smaller, or at least not larger, the higher is the rate of interest. Indeed, in the case of constant interest elasticity ε , $-L_i/L = -\varepsilon/i$, which is low when i is high.

¹¹Confining his analysis to the problem of local stability, Johnson (1977) identified the phenomenon of a dynamic liquidity trap with the case of local instability ($-bL_i^*/m^* > 1$). The above shows, however, that even in case of local stability – and whatever the value of the interest elasticity of the demand for money – the system falls into a dynamic liquidity trap if subjected to a large contractionary disturbance.

“conventional”, static liquidity trap - infinite interest elasticity of money demand - is involved. Likewise, no denial of the existence of and the possible local stability of a full employment equilibrium is involved. Nevertheless, if a recession is large enough, a dynamic liquidity trap is set in motion, preventing the recovery.

In essentially the same model Cugno & Montrucchio (1984) analyzed, by means of the Hopf bifurcation theorem, the related problem of the existence of periodic orbits surrounding the steady state. In view of the difficulty of establishing whether the “subcritical” case (the periodic orbit is attracting and occurs to the right of the bifurcation value of the control parameter, here b) or the “supercritical” case (the periodic orbit is repelling and occurs to the left of the bifurcation value) is present, this provided no concise condition as to the existence of a corridor. However, the Hopf bifurcation theorem is useful to give an idea of the boundary of the corridor when it exists. The theorem implies that for all b on the one side of the bifurcation value $\bar{b} = f_m^* m^* / f_x^* = -m^* / L_i^*$ and close enough to it, there exists a single periodic orbit surrounding the steady state with amplitude approximately proportional to $|b - \bar{b}|^{1/2}$ (use theorem A.20 in Azariadis 1993 p. 156). When, in addition, (2.14) holds, we know a corridor exists and we may conjecture that its boundary can be identified with this periodic orbit. Then, since $\bar{b} = i^* / \varepsilon^*$, where ε^* is the absolute value of the interest elasticity of money demand, given the equilibrium rate of interest i^* , the greater is ε^* the smaller is the “radius” of the corridor. It should be added, that this is only a local claim pertaining to b sufficiently near \bar{b} ($b < \bar{b}$). I have not been able to *prove*- by means of, e.g., the Poincaré-Bendixson theorem - existence of a periodic orbit for b less than \bar{b} and thereby that the corridor is bounded above.

4 Extension to the Cagan Demand for Money Function

The demand for money function preferred by Cagan (1956) and others is

$$L(y, i) = \psi(y)e^{-ai}, \quad a > 0, \quad y > 0, \quad i \leq 0, \quad (4.1)$$

where $\psi(y)$ is some positively valued function with $\psi' > 0$ (representing the transaction motive). Since in this case $\lim_{i \rightarrow 0} L(y, i) = \psi(y) < \infty$, (4.1) does not obey A2(i). Money demand remains finite for i approaching zero.

While rendering the mathematics a little more complicated, allowing for this case does not change the conclusion. This is because introducing (4.1) does not

change the basic fact that a negative nominal rate of interest is not compatible with equilibrium in the assets markets. If the rate of interest was negative, nobody would want to hold bonds, since money would be a better alternative, earning zero interest while providing more liquidity. The Cagan function should therefore be interpreted in the following way

$$\tilde{L}(y, i) = \begin{cases} \psi(y)e^{-ai} & \text{if } i > 0, \\ [\psi(y), \infty) & \text{if } i = 0, \end{cases} \quad (2.15a)$$

i.e., the demand function becomes set valued at $i = 0$. With assumption A2 modified accordingly the argument for the existence of a corridor if and only if $ba < 1$ still goes through, the only difference being that the $\dot{x} = 0$ locus in Figure 1 now has a kink at $x = -r^*$, where it becomes vertical. See appendix B.

Interestingly, if the demand for money function (4.1) was not restricted to $i \leq 0$, the system would indeed be globally asymptotically stable, if the Cagan condition, $ba < 1$, holds. This can be shown by using Olech's Theorem (Sydsæter 1981).

5 Conclusion

We have established that in the simple Keynesian-Monetarist model, prevalent in intermediate macroeconomic textbooks, the Cagan condition only entails *local* stability and is in fact, given the non-negativity of the nominal rate of interest, both a *necessary* and a *sufficient* condition for the existence of a corridor. This is an example of the limitations of local stability analysis as emphasized by the modern theory of nonlinear dynamical systems (see, e.g., Azariadis 1993). Of course, the scenario of complete collapse of the economy and continuing deflation is not plausible. Apart from its all-round simplistic nature the model abstracts from monetary growth, the real balance effect and the inflation tax. These matters tend to counteract the dynamic liquidity trap so that the economy tends sooner or later to be lifted from the floor. The model then becomes more like a business cycle model.

6 Appendix A

Proof of Proposition 1. Letting $n = \log m$, the system (2.12), (2.13) is transformed into the equivalent system

$$\begin{aligned}\dot{x} &= b(\varphi(f(x, e^n))), \\ \dot{n} &= -x - \varphi(f(x, e^n)),\end{aligned}\tag{A.1}$$

which is defined for all (x, n) in R^2 . The steady state of (A1) is $(0, n^*)$, where $n^* = \log m^*$. Imagining that the phase diagram in Figure 1 also portraits the third and fourth quadrants, we can think of it as the phase diagram of (A1), interpreting m as n .

Consider a point on the vertical line $x = -r^*$ in Figure 1, say the point $z_1 = (-r^*, n_1)$, where $n_1 < n^*$. Let t_1 be fixed. Denote by $z(t)$ the vector function $(x(t), n(t))$, and let $z(t, z_1)$ be the solution of (A1) passing through z_1 at $t = t_1$, that is $z(t_1, z_1) = z_1$. The slope s of the corresponding trajectory in (x, n) -space is

$$s = s(x, n) = \frac{dn}{dx}(x, n) = \frac{\dot{n}}{\dot{x}} = \frac{-x}{b\varphi(f(x, e^n))} - \frac{1}{b},\tag{A.2}$$

which is negative for $(x, n) \in A \equiv \{(x, n) \in R^2 \mid \dot{x} < 0, \dot{n} > 0\}$. Thus the solution curve $z(t, z_1)$ at $(x, n) = z_1$ points to the North-West in Figure 1. Then, clearly $x(t) < -r^*$ for all $t > t_1$ for which the solution $z(t, z_1)$ is defined. Hence, to prove the proposition it is enough to prove that $z(t, z_1)$, when traced backward in time, crosses the half line $\{(x, n) \in R^2 \mid x = 0, n < n^*\}$.

Let $z_1 z_2$ be the line segment $\{(x, n) \in R^2 \mid -r^* \leq x \leq 0, n = n_1\}$. The continuous function $(x, n) \rightarrow s$ restricted to the nonempty compact set $z_1 z_2$ has a minimum, say \bar{s} , and $\bar{s} < 0$. Now, from (A.2)

$$\frac{\partial s}{\partial n} = \frac{x\varphi' f_m e^n}{b\varphi(f(x, e^n))^2} \leq 0$$

for $(x, n) \in Q \equiv \{(x, n) \in A \mid -r^* \leq x \leq 0, n \leq n_1\}$. Therefore $0 > s \geq \bar{s}$ for all (x, n) in Q , or $|s| \leq |\bar{s}|$ for all (x, n) in Q . The line through z_1 with slope \bar{s} crosses the vertical line $x = 0$ at some point z_3 . It follows that the solution $z(t, z_1)$ at some time $t_0 < t_1$ passes through some point, say $\bar{z} = (0, \bar{n})$, on the line segment $z_2 z_3$. Letting $\alpha = n^* - \bar{n}$ and $\partial = e^{-\alpha}$, the proposition is proved. Indeed, as $r(t) = i(t) - x(t) > -x(t) > -x(t_1) > r^*$ for all $t > t_1$, the solution for $y(t)$ is bounded away from y^* . Q.E.D.

7 Appendix B

Extension to the Cagan function and similar cases. (3.15a) is a special case of

$$\tilde{L}(y, i) = \begin{cases} L(y, i), & \text{if } i > 0, \\ [L(y, 0), \infty), & \text{if } i = 0, \end{cases} \quad (\text{B.1})$$

where $L(y, i)$ is single valued, $L_y > 0$, $L_i < 0$, and $\lim_{i \rightarrow 0} L(y, i) = L(y, 0) < \infty$. (B.1) implies that at $i = 0$ the amount of money which is willingly held (since no alternative is better) can be anything in the interval $[L(y, 0), \infty)$.

Equation (2.2) of our model is still valid when $m < L(y, 0)$, and in this case (2.9) is still implied by (2.2). To this we now add the case $m \geq L(y, 0)$, implying $i = 0$. That is, from equilibrium in the assets markets we have

$$i = H(y, m) = \begin{cases} h(y, m) & \text{if } m < L(y, 0), \\ 0 & \text{if } m \geq L(y, 0), \end{cases} \quad (\text{B.2})$$

where the function h obeys (2.9). H is continuous since $\lim_{m \rightarrow L(y, 0)} H(y, m) = \lim_{m \rightarrow L(y, 0)} h(y, m) = 0 = H(y, L(y, 0))$.

Inserting (B.2) in (2.7) gives

$$0 = y - C(y) - I(y, H(y, m) - x) \equiv D(y, x, m), \quad (\text{B.3})$$

where $D_y = 1 - C_y - I_y - I_r H_y \geq 1 - \beta > 0$ by assumption A1. There are two “regimes”, the positive interest rate regime and the zero interest rate regime. The solution in y to (B.3) can be written

$$y = F(x, m) = \begin{cases} f(x, m) & \text{if } i > 0, \\ \bar{f}(x, m) & \text{if } i = 0, \end{cases}$$

where $f(x, m)$ obeys (2.11), while

$$\bar{f}_x = \frac{-I_r}{D_y} > f_x > 0, \quad \bar{f}_m = \frac{I_r H_m}{D_y} = 0$$

by (B.2). The boundary line between the two regimes is called B and is given by the equation $m = L(f(x, m), 0)$. Its slope is

$$\left. \frac{dm}{dx} \right|_B = \frac{-L_y(y, 0)I_r}{1 - C_y - I_y} > 0.$$

The phase portrait is given in Figure 2. The argument of Appendix A still goes through, and the existence of a corridor if and only if $ab < 1$ follows.

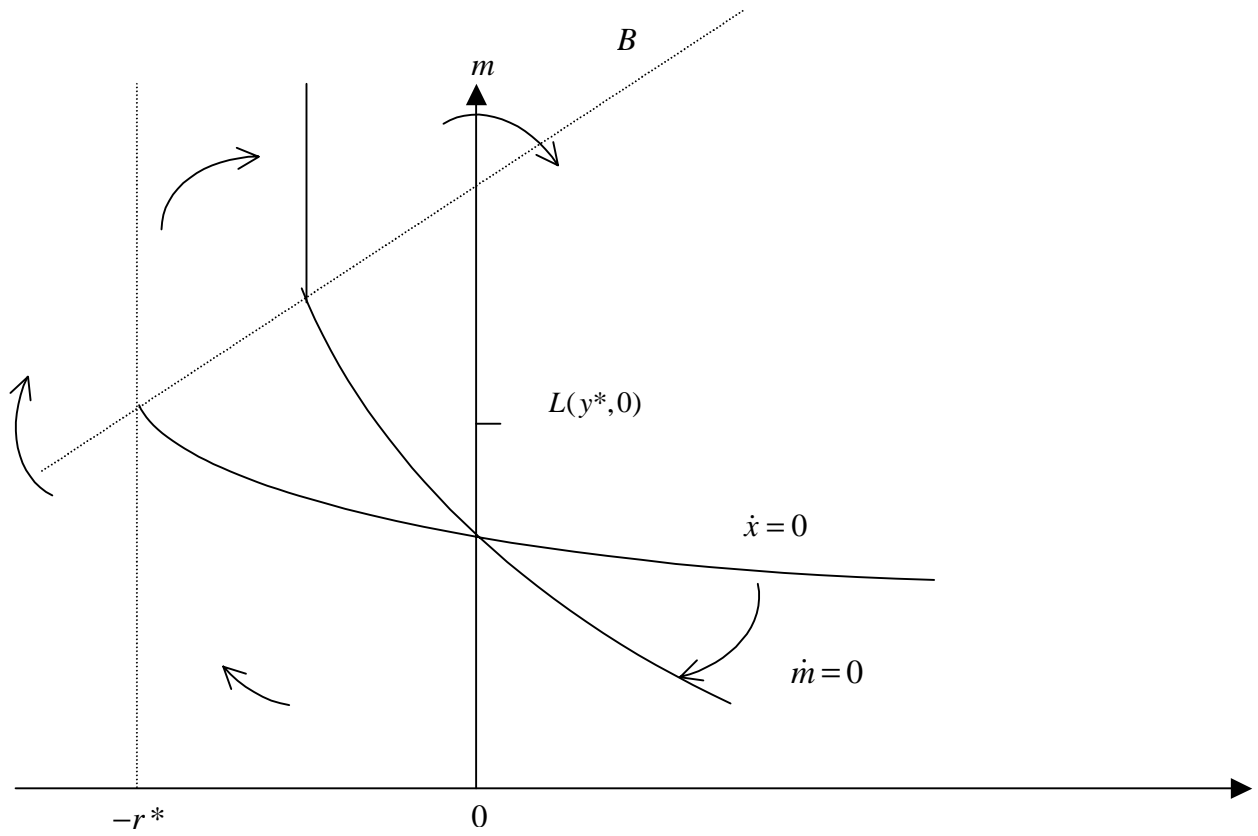


Figure 2:

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