IS-LM dynamics and the hypothesis of combined adaptive-forwardlooking expectations.

by

Christian Groth[1]

University of Copenhagen

I. Introduction

This paper studies the dynamic behaviour of an IS-LM model extended by the expectations augmented Phillips curve and what we call "combined adaptive-forwardlooking expectations".

The theory of IS-LM dynamics arose from the monetarist critique of Keynesian economics for being one-sidedly concerned with flows rather than stocks of assets and for attaching too much importance to short run effects rather than ultimate, cumulative effects (M. Friedman 1968, 1972). The "IS-LM cross" began to be seen as a momentary stage of a dynamic process. One source of the dynamics is the changes in stocks of money and bonds brought about by the financing of government budget deficits (Christ 1968, Blinder & Solow 1973, Tobin & Buiter 1976). Other sources of the dynamics are the change in prices, as derived from an expectations augmented Phillips curve, as well as the change in expectations, usually described by the adaptive expectations formula (Tobin 1975, Buiter & Lorie 1977, Scarth 1977, Taylor 1977, Yarrow 1977). The large body of studies dealing with these problems is surveyed in Currie 1978, Christ 1979, and Rau 1985.

One of the conclusions in this literature is that quite apart form the possible instability arising from the cumulative effects of financing budget deficits by bond issue, instability could arise from the dynamic interaction of output demand and inflation expectations. Given non-fulfilment of some generalized

form of the Cagan condition (Cagan 1956) an incipient recession will be self-enforcing; thereby an old Keynesian tenet is confirmed. This outcome rests on the possibility of the nominal and real rates of interest moving adversely, when actual and thereby expected inflation change.

However, one of the well known drawbacks of the adaptive expectations hypothesis used in these studies is that it assumes agents base their inflation forecast on a very limited set of information, i.e. the past development of the inflation rate. This implies that agents can be systematically wrong in their expectations, e.g. when the variable in question has a trend, and that they nevertheless continue to use the same method of forming expectations.

The radical alternative to adaptive expectations is the hypothesis of "rational expectations" (Muth 1961). Incorporating this hypothesis into IS-LM models, whether deterministic or stochastic, together with the expectations-augmented Phillips curve (or, what amounts to the same, the Lucas supply curve), represents, of course, a special case of the "new classical" theory. Output becomes independent of anticipated monetary change and, apart from erratic shoks, output tends towards the "natural level" (Lucas 1972, Sargent & Wallace 1975).[2]

Feeling that the rational-expectations hypothesis is too strong, Peel & Metcalfe (1979) developed an analysis of a simple "monetarist" model[3] in which expectations are formed in different ways by different groups of agents (e.g., adaptively by workers and rationally by firms and financial agents). Peel & Metcalfe showed that this being the case the condition for stability of production and inflation tends to become more stringent than in the case of uniformly held adaptive expectations.

Yet another hybrid assumption "the two-part expectations hypothesis", was studied within a similar "monetarist" framework by Frenkel (1975) and Peel (1979). According to this hypothesis changes in expected short-term inflation are based on two elements which are first, the deviation of expected short-term inflation from current inflation and second, the deviation of current inflation from expected long-term average ("normal") inflation. In its turn this expected long-term average inflation is formed according to the standard adaptive formula (Frenkel

1975) or to some kind of long-term "monetarist expectations"
(Peel 1979).

The hypothesis introduced below, which we call "combined adaptive-forwardlooking expectations", is akin to this. Both current inflation and the tendentially perceived long-term inflation (equal to monetary expansion) enter. While on the one hand being more crude (using only one exponentially distributed lag), on the other hand our hypothesis allows for a larger interval of parameter values than the Frenkel approach thus giving rise to cases with qualitatively different dynamic behaviour. We study the role of this hypothesis in an otherwise standard IS-LM (Keynesian income-expenditure) model, respecting the government budget constraint. The purpose is partly expository and partly to break new ground.

II. The model

The symbols are

y = real output

y_d = perceived real disposable income

P = price of output

M = nominal money supply (equal to monetary base)

m = real money supply, M/P

$\dot{P}/P$ = actual rate of inflation

x = expected rate of inflation

i = nominal rate of interest (after tax)

T = nominal taxes net of transfer payments

g = real government purchases

a = rate of monetary expansion, $\dot{M}/M$

A dot denotes a time derivative, e.g., $\dot{x} = dx/dt$, time, t, being treated as continuous.

The hypothesis of "combined adaptive-forwardlooking expectations" is:

$$\dot{x} = b\left(\left(c\frac{\dot{M}}{M} + (1 - c)\frac{\dot{P}}{P}\right) - x\right), \qquad b > 0, \ c \geq 0 \qquad (1)$$

This says that expected inflation is lagged on an indicator, which is a weighted average of the tendentially perceived long term inflation rate (equal to monetary expansion) and the current inflation rate. The case $c = 0$ is the simple, one-sidedly backwardlooking, adaptive expectations formula. The case $c = 1$ is considered in Stein (1982) and may be called "monetarist expectations"[4] – corresponding to the monetarist belief in stability of the market economy and convergence towards a steady state. The intermediate case $0 < c < 1$ is a mixture of these two extremes and contains both a forwardlooking and an adaptive element. The case $c > 1$ is included, although we do not find it plausible. It corresponds to a variant (introduced by Peel 1979) of Frenkel's above-mentioned "two-part expectations hypothesis"; the presumption is that people strongly believe in a certain long-term rate of inflation – not only as an ultimate level at which inflation would tend to settle, but also as an average rate of inflation over shorter time intervals.

The remaining seven equations of our short-to-medium run IS-LM model are given below. The derivative of a function, say F, with one argument is denoted F'. The partial derivative with respect to the ith argument of a function, say C, with more than one argument is denoted C_i.

$$y = C(y_d, m) + I(y, i-x) + g, \qquad (2)$$
$$0 < C_1 < C_1 + I_1 < 1, \ C_2 \geq 0, \ I_2 < 0, \ g \geq 0$$

$$y_d = y - T/P - xm \qquad (3)$$

$$T = Pg - \dot{M} \qquad (4)$$

$$m = L(y, i, m), \qquad L_1 > 0, \ L_2 < 0, \ 0 \leq L_3 < 1 \qquad (5)$$

$$\dot{M}/M = a, \qquad a \geq 0 \qquad (6)$$

$$\dot{P}/P = F(y) + x, \quad F' \;) \; 0, \text{ and for some } y* \;) \; 0,$$ (7)
$$F(y*) = 0$$

$$m \equiv M/P$$ (8)

a, b, c, and g are given constants. From the point of view of the dynamic analysis of the model there are eight endogenous variables, y, i, y_d, T, P, M, m, and x, to be determined from equations (1) - (8) and given initial values of x, P, and M.

y* is the "natural" level of output defined as that level of output which is consistent with actual and expected inflation being equal. It is understood that both the real wage, the capital stock, and the labour supply allow profitable production to take place above the natural level, provided there is sufficient demand. (3) gives perceived disposable income, i.e., real gross income corrected for real net taxes and the perceiced capital loss on the real value of the public's net claims on the government, the socalled inflation tax. (4) is the government budget constraint. It states that government purchases not financed by money creation are financed by taxes, that is by adjusting the tax and transfer instruments. There are no government bonds in the model. Thus, the problem whether government bonds are part of private net wealth or not (see Barro 1974, McCallum 1978, Tobin 1980) does not arise. The remaining equations are self-explanatory.

Short run equilibrium

Inserting (3) in (2), and using (4) and (6) gives

$$y = C(y + (a - x)m - g, m) + I(y, i - x) + g.$$ (9)

In the short run the money supply, output price and expectations are historically given. Thus m and x are given for fixed t, and the "IS equation", (9), together with the "LM equation", (5), determine the output level and the rate of interest. Thus short run y can be written as a function

$$y = f(x, m; a, g)$$ (10)

with continuous partial derivatives (impact multipliers)

$$f_1 = \frac{-I_2 - C_1 m}{A}, \quad f_2 = \frac{(1-L_3)I_2/L_2 + C_1(a-x) + C_2}{A},$$ (11)

where

$$A \equiv 1 - C_1 - I_1 + L_1 I_2/L_2 \rangle 0,$$ (12)

cf. (2) and (5). The signs of f_1 and f_2 are in general ambiguous. However, for $x \leq a$, $f_2 \rangle 0$, a fact which will be useful in what follows.[5]

Dynamics

In order to study the evolution of short run equilibria through time we derive he differential equations implied by the model. From (1), (6), (7), and (10) we obtain

$$\dot{x} = b[c(a-x) + (1-c) F (f(x,m; a,g))].$$ (13)

Combining (6), (7), (8) and (10) gives

$$\dot{m} = [a - x - F(f(x,m; a,g))]m.$$ (14)

A steady state is a time path along which x and m (and therefore y and i) are constant. In a steady state the nominal variables, M, P, and T, are changing at the same constant rate, a. Since $m \rangle 0$, $\dot{x} = \dot{m} = 0$ implies, by (13) and (14), that the steady state value of x equals a, the rate of mometary expansion. Hence, in a steady state, $f(x,m; a,g) = y*$ by (14) and (7), i.e. the steady state value of output is $y*$, the natural rate. The steady state value of m, $m*$, is the solution in m to the equation $f(a,m; a,g) = y*.$[6] Observe that none of the parameters a, b, c, and g influences $y*$, that only a influences $x*$ and only a and g influence $m*$.

The phase portrait of the system (13), (14) is depicted in Figure 1. The sloping dotted curves in the diagram represent

iso-production-loci obtained from (10). Moving North in the
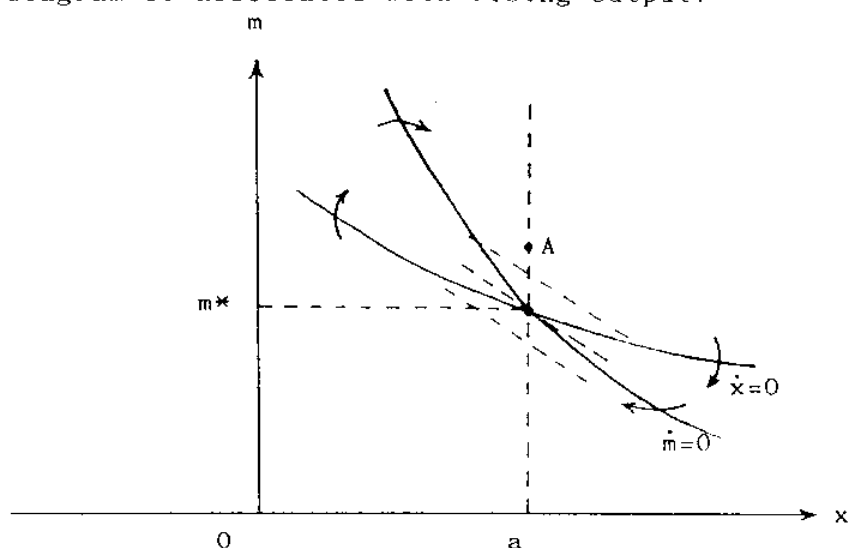diagram is associated with rising output.

Figure 1. Phase portrait of (13), (14): the case $f^*_1 > c/((1-c)F'^*)$.

III. Expectations and stability

The Jacobian matrix of (13), (14) evaluated at the steady state
point (a, m^*) is

$$
\begin{bmatrix} \dfrac{\delta \dot{x}}{\delta x} & \dfrac{\delta \dot{x}}{\delta m} \\[2em] \dfrac{\delta \dot{m}}{\delta x} & \dfrac{\delta \dot{m}}{\delta m} \end{bmatrix}_{(x,m)=(a,m^*)} = \begin{bmatrix} b[-c+(1-c)F'^* f^*_1] & b(1-c)F'^* f^*_2 \\[2em] (-1-F'^* f^*_1)m^* & -F'^* f^*_2 m^* \end{bmatrix} \quad (15)
$$

The determinant is $bF'^* f^*_2 m^*$, which is positive because $x = a$
implies $f^*_2 > 0$, by (11). The trace is $b[-c+(1-c)\ F'^* f^*_1]-F'^* f^*_2 m^*$,
the sign of which is ambiguous. It follows that the steady state
is <u>locally asymptotically stable if and only if</u>[7]

$$
b[(1-c)F'^* f^*_1 -c] < F'^* f^*_2 m^* . \quad (16)
$$

The possibly destabilizing force comes from the endogenuous inflation expectations, which may, through the feedback in the economy, be self-fortifying such that $\delta \dot{x}/\delta x > 0$.

Before commenting further on (16) we make the reasonable assumption

A1. $C_1^* m^* < - I_2^*$,   i.e., $f_1^* > 0$.

Thus we assume that the negative impact on consumption demand of increased expected inflation, due to the inflation tax, is more than counterbalanced by the positive impact on investment demand. Given this assumption the price mechanism is easily seen to have an ambiguous role in relation to stability. While higher prices (or more generally, higher inflation) decrease output demand, the expectation of rising prices (or more generally, of rising inflation) increases demand.

To interpret (16), suppose the economy has for some time been in a steady state. Then some disturbance takes place. One might prefer to look at a contractionary disturbance creating excess capacity - feeling that this is the case for which the IS-LM model is most suitable. It is also the case with which Keynesian theory in general is primarily concerned. However, from the point of view of interpretation of the mathematical logic of (16), it is most straightforward to speak of the effects of increases in actual and expected inflation, that is to start with the case of an expansionary disturbance. When this case has been explained, it is easy to translate to the case of a contractionary disturbance by reversing all signs.

Now, suppose the steady state at time zero is disturbed by, say, an upward shift in the investment function. The immediate effect of this is an upward jumb in short run output. Refering to Figure 1, the initial situation is that x and m are at the historically given values (the old steady state), while the new steady state value of m is below the old (to compensate for the higher propensity to invest). Thus, immediately after the disturbance the economy is at some point, say A, above the new steady state point, cf. Figure 1. The Phillips curve now tells us that the rate of inflation immediately jumbs to a higher level,

i.e. higher than a, the rate of monetary expansion. Real cash balances begin to decrease while expected inflation begins to increase. The question then is whether the further interaction between these dynamic forces is deviation-amplifying or deviation-counteracting. It is convenient to distinguish between four cases according to the value of the parameter c.

Case 1: c = 0. This is the standard case of simple adaptive expectations. The stability condition (16) reduces to

$$bf_1^* < f_2^* m^*$$ (16.i)

which is essentially the condition Tobin (1975) found in a related model. The term on the righthand side of (16.i) measures the <u>price level effect</u> on output which is stabilizing. The term on the lefthand side measures the <u>expectations effect</u> which is potentially destabilizing. Still, one gets more insight, if condition (16.i) is expressed in terms of the basic behavioural functions of the model. Inserting (11) ,with x = a, in (16.i) and rearranging gives

$$(|I_2^*| - C_1 m^*)\, b \qquad < \qquad - I_2^*(1-L_3^*)\, \frac{m^*}{L_2^*} + C_2^* m^* .$$ (16.ii)

↑	↑	↑	↑
The potentially destabilizing expectations effect on investment	Inflation tax effect	Keynes effect	Pigou effect

When written in this way we see that the stability condition displays the famous "effects" from the old "Keynes versus the Classics" debate of the late 1930s and the 1940s. Furthermore, if $L_3^* = 0$,

$$b < - \frac{m^*}{L_2^*}$$ (16.iii)

is sufficient, though not necessary for stability. (16.iii) may also be expressed as b α < 1, where $\alpha \equiv -L_2^*/m^*$, the "sensitivity"

of money demand with respect to the rate of interest. In this form we recognize the condition Cagan found to be necessary and sufficient for stability in his pioneering "monetary dynamics" model developed for the study of historical cases of rapid inflation (Cagan 1956).[8]

Case 2: $0 < c < 1$. In this case expectations are formed as a convex combination of adaptive and "monetarist" expectations. The parameter c may be called the "degree of monetarist faith". Dividing through by F'^* in (16) we see that the stability condition becomes less stingent, the higher is c. Thus, stability is more likely, the more people tend to believe in stability. This is an example of a socalled bootstrap effect of expectations. On the other hand, as long as $c < 1$, i.e., in their expectations formation people put some weight to current inflation, instability cannot be excluded.

The explanation of the stabilizing role of monetarist faith is that increasing c dampens the self-stimulation of x by dampening $\delta\dot{x}/\delta x$. Indeed, the higher c is, the smaller is 1-c, i.e., the weight given to the positive feedback on $\dot{x}$, via actual inflation. $\dot{P}/P = F(f(x,m;a,g)) +x$, from a higher expected inflation. Furthermore, when $1-c < 1$, the negative feedback on $\dot{x}$, via the term -x in (1), is no longer neutralized by the term +x in (7), the net impact being $b[(1-c)x-x] = -bcx$. This explains why, when $c > 0$, the slope, F'^*, of the short run Phillips curve matters for stability, the likelyhood of stability being less, the higher is this slope. When $c = 0$, the size of F'^* does not matter for stability because in this case F'^* affects the two sides of (16) equally much, proportionately. When $c > 0$, this is no longer so. Indeed, increasing F'^* now affects the self-stimulation of x, $\delta\dot{x}/\delta x$, by more, proportionately, than it affects $\delta\dot{m}/\delta m$, due to the above-mentioned term -bcx, which is independent of F'^*.

Case 3: $c = 1$. In this case (used in Stein 1982) we have $\dot{x} = b(\dot{M}/M-x)$, that is pure "monetarist" expectations - corresponding to the monetarist belief in the convergence towards the steady state where the rate of inflation equals the rate of monetary

expansion. (16) shows that when c = 1 stability is certain. Thus, monetarist expectations are, at least ultimately, self-validating.

Case 4: c) 1. This case is a variant of Frenkel's "two-part expectations hypothesis" (Frenkel 1975, Peel 1979). The presumption is that people strongly believe in a certain long-term rate of inflation - not only as an ultimate level at which inflation would tend to settle, but also as an average rate of inflation over shorter time intervals. The implied negative weight to current inflation - which we do not find plausible and which was rejected by the empirical test by Jacobs & Jones (1980) - was wanted by Frenkel in order to revise the Cagan model (Cagan 1956) to make it consistent with the empirical evidence. According to this evidence a shift to a higher monetary growth rate initially raises and eventually lowers real holdings of cash balances.[9]

Now, whatever the plausibility of the case c) 1, if people have such expectations, stability is certain, cf. (16). On the other hand, all these results presuppose asumption Al. If the opposite of Al holds, then a "reversal of stability" takes place. Stability becomes certain when $0 \leq c \leq 1$ and uncertain when c)1.

Finally, we mention a contrast with respect to stabilization policy between the limit case c = 0 (adaptive expectations) and the case c) 0. It can be shown that, e.g., a fiscal policy of the form $g = \varphi(y)$, φ' (0, when c = 0, is neutral as to stability, while being favourable to stability, when c) 0.[10]

IV. Rational expectations

For comparison a few comments on the case of rational expectations are added. In our deterministic framework rational expectations means perfect foresight. i.e.,

$$x = \frac{\dot{p}}{p} \qquad \text{for all } t,$$ (17)

which now replaces (1) in our model. Then, by (7) $y = y^*$ for all t. This is the wellknown fact that rational expectations in this

type of models stabilize output. On the other hand, a complete
solution to the model (including assumption A1) shows that
rational expectations, by "speeding up" the adjustment of
expected inflation to actual inflation, makes the inflationary
process definitely instable, if either the initial price level or
the initial rate of inflation is regarded as historically
given.[11] There is, however, also another approach to the
question of stability under perfect foresight or rational
expectations. This is to add to the model the postulate that
under perfect foresight the economy <u>must</u> always be on a
convergent path. This approach, suggested by Sargent & Wallace
(1973), and often used in the "new classial macroeconomics"
literature, corresponds, in our set up, to postulating $m = m^*$
immediately after any disturbance. This requires that the output
price behaves as an asset price and instantaneously adjusts (by
arbitrage) to keep real balances at m^*. In our opinion this begs
the stability question.

V. Conclusion

The main part of this paper has dealt with the hypothesis that
inflation expectations are formed according to a convex
combination of an adaptive and a forwardlooking ("monetarist")
formula. The role of this hypothesis as to the question of
stability of production and inflation in a Keynesian IS-LM model
has been investigated. The existence of bootstrap effects of
expectations was observed. The more people believe in the
"monetarist" formula, the more likely it is that this formula
will be corroborated by the course of events. On the other hand,
as long as people, in their expectation formation, put some
weight to current inflation, instability of production and
inflation cannot be excluded.

It was also shown that just as sluggishness in the
adjustment of expectations (small b and small 1-c) promotes
stability, sluggishness in price adjustment (small slope of the
short run Phillips curve) is in the regular case $(0 < c < 1)$,
favourable to stability. In the pure adaptive case (c=0),
however, the size of the slope of the short run Phillips curve is
neutral as to stability.

We may add finally, that in practice an important "automatic stabilizer" is the tendency to downward rigidty of prices which seems to be a typical feature of the real world. Thus, if the restriction $\dot{p} \geq 0$ is added to the model and the money growth rate is positive, then any depression will tend to be only temporary. The explanation is that the contractionary fall of expected inflation ceases sooner or later, while the expansionary increase in real balances continues. Once again we see that sluggishness promotes stability.

Notes:

1) I am grateful to Jorgen H. Gelting, Hans Jorgen Jacobsen, Soren Bo Nielsen, Christian Schultz, Peter Skott, Peter Birch Sorensen, and Troels O. Sorensen for discussions and comments on an earlier draft of this paper.

2) Nevertheless, nominal instability in no way disappears. Indeed, the whole price path is instable if the initial price is given, and is otherwise indeterminate (see section IV).

3) "Monetarist" - because the real rate of interest is assumed fixed.

4) Or "asymptotically rational expectations" (Stein 1982, p. 52).

5) Thus, respecting the government budget constraint implies that in a neighbourhood of the steady state (where $x \simeq a$), $f_2 > 0$ unambiguously, and therefore the problems with the "Pigou argument", stressed by Sargent (1979, pp. 64-65), disappear.

6) Sufficient conditions for the existence of a steady state and for the existence of economically meaningful short run equilibria in a neighbourhood of the steady state are given in Groth (1986). These conditions are not too demanding and need not occupy us here.

7) The "only if" part of this statement is only true, of cource, when we ignore the possibility of the singular case: trace = 0. This is so because a two-dimensional non-linear dynamic system with trace, evaluated at the steady state point, equal to zero may be stable. For simplicity, we abstract from this formal detail throughout the text.

8) Cagan (1956) tested the model on monthly data from seven European countries during hyperinflation using the fact that the adaptive expectations assumption implies that the expected rate of inflation is an exponentially-weighted average of all past actual rates of inflation. Cagan obtained a resonable fit for his model and found that the stability condition was met in five or six out of the seven examined cases of hyperinflation. His procedure and results are, however, rejected in Jacobs (1975) and B. Friedman (1978).

9) Interestingly, it can easily be shown that the Keynesian IS-LM model does not need c > 1 to be consistent with the empirical evidence (Groth 1986 p. 27).

10) This corresponds to the absence of I_1 in (16.ii) versus the presence of I_1, with a positive weight, on the left hand side of (16), when inserting (11) and multiplying through by A. cf. (12).

11) This is familiar from the money and growth literature (Tobin 1965, Sidrausky 1967) and other literature on multiple assets (Hahn 1966). If on the other hand neither the initial price or the initial rate of inflation is historically given, then an infinity of solutions to the model exists, an example of the general indeterminacy problem in self-fulfilling expectations models (see, e.g., Shiller 1978, Geanakoplos & Polemarchakis 1986).

REFERENCES

Barro, Robert J: Are government bonds net wealth? Journal of Political Economy 82, 1095-1117, 1974.

Blinder, Alan S., & Robert M. Solow: Does fiscal policy matter? Journal of Public Econimics 2, 319-337, 1973.

Buiter, Willem H., & Henri R. Lorie: Some unfamiliar properties of a familiar macroeconomic model. Economic Journal 87, 743-754, 1977.

Cagan, Phillip: The monetary dynamics of hyperinflation. In Milton Friedman (ed.): Studies in the quantity theory of money. Chicago, 1977.

Christ, Carl F.: A simple macroeconomic model with a government budget constraint. Journal of Political Economy 76, 53-67, 1968.

Christ, Carl F.: On fiscal and monetary policies and the government budget restraint. American Economic Review 69, 526-38, 1979.

Currie, David A.: Macroeconomic policy and government financing. In M.J. Artis & A.R. Nobay (eds.): _Contemporary econmic analysis_. London. 1978, 65-99.

Frenkel, Jacob A.: Inflation and the formation of expectations. _Journal of Monetary Economics_ 1, 403-421, 1975.

Friedman, Benjamin M.: Stability and rationality in models of hyperinflation. _International Economic Review_ 19, 45-64, 1978.

Friedman, Milton: The role of monetary policy. _American Economic Review_ 58, 1.17, 1968.

Friedman, Milton: Comments on the critics. _Journal of Political Economy_ 80, 906-950, 1972.

Geanakoplos, J.D., & H.M. Polemarchakis: Walrasian indeterminacy and Keynesian macroeconomics. _R.E.S._ 53, 1986.

Groth, C.: Centripetal and centrifugal forces in IS-LM dynamics. A reconsideration. Memo nr. 149, University of Copenhagen, Institute of Economics. 1986.

Hahn, Frank H.: Equilibrium dynamics with heterogenous capital goods. _Quarterly Journal of Economics_ 80, 633-646, 1966.

Jacobs, Rodney L.: A difficulty with monetarist models of hyperinflation. _Economic Inquiry_ 13, 1975.

Jacobs, Rodney L., & Robert A. Jones: Price expectations in the United States: 1947-75. _American Economic Review_ 70, 269-277, 1980.

Lucas, Robert E. Jr.: Expectations and the neutrality of money. _Journal of Economic Theory_ 4, 1972.

McCallum, Bennet T.: On macroeconomic instability from a monetarist policy rule. _Economic Letters_ 1, 121-124, 1978.

Muth, John F.: Rational expectations and the theory of price movements. _Econometrica_ 29, 1961.

Peel, D.A.: On dynamic stability in monetary models which incorporate short- and long-run expectations of inflation in the demand for money function. _Economics Letters_ 2, 131-136, 1979.

Peel, D.A. & J.S. Metcalfe: Divergent expectations and the dynamic stability of some simple macroeconomic models. _Economic Journal_ 89, 789-798, 1979.

Rau, Nicholas: Simplifying the theory of the government budget restraint. _Oxford Economic Papers_ 37, 210-229, 1985.

Sargent, Thomas J.: _Macroeconomic theory_. New York, 1979.

Sargent, Thomas J., & Neil Wallace: The stability of models of money and growth with perfect foresight. _Econometrica_ 41, 1043-1048, 1973.

Sargent Thomas J., & Neil Wallace: "Rational" expectations, the optimal monetary instrument, and the optimal money supply rule. _Journal of Political Economy_ 83, 1975.

Scarth, William M.: Expectations and the wage-price spiral in a simple monetary model. _The Manchester School of Economic an Social Studies_ 45, 1977.

Shiller, Robert, J: Rational expectations and the dynamic structure of macroeconomic models. _Journal of Monetary Economics_ 4, 1-44, 1978.

Sidrausky, Miguel: Inflation and economic growth. _Journal of Political Economy_ 75, 796-810, 1967.

Taylor, Dean: A simple model of monetary dynamics. _Journal of Money, Credit and Banking_ 9, 107-111, 1977.

Tobin, James: Money and economic growth. _Econometrica_ 33, 671-684, 1965.

Tobin, James: Keynesian models of recession and depression. _American Economic Review_ 65, Proceedings, 195-202, 1975.

Tobin, James: _Asset accumulation and economic activity_. Oxford 1980.

Tobin, James, & Willem H. Buiter: Long-run effects of fiscal and monetary policy on aggregate demand. In J.R. Stein (ed.): _Monetarism: Studies in monetary economics_, vol. 1. Amsterdam, 1976, 274336.

Yarrow, G.K.: The demand for money function and the stability of monetary equilibrium. _Economic Journal_ 87, 114-123, 1977.