Written re-exam for the M. Sc. in Economics. Summer 2012

Economic Growth

Master's Course

August 15, 2012

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is: Problem 1: 25%, Problem 2: 50%, and Problem 3: 25%.[1]

---

[1]The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1   Consider a closed economy with a manufacturing sector and an R&D sector. Aggregate output of the manufacturing sector at time t is

$$Y_t = \tilde{F}(K_t, L_{Yt}, t),$$

where K_t is capital input and L_{Yt} is labor input. The function $\tilde{F}$ is a neoclassical production function w.r.t. these two arguments. W.r.t. the third argument, we have $\partial \tilde{F}/\partial t \equiv \tilde{F}_t > 0$.

The only input in the R&D sector is research labor, L_{Rt}, and we have $L_{Yt} + L_{Rt} = L_t$, where L_t is aggregate employment. Time is continuous. We apply the convenient notation: $g_z \equiv \dot{z}/z$.

We first consider the manufacturing sector separately.

a) By the standard growth accounting method, decompose the output growth rate into its three basic components.

b) How is the TFP growth rate, x_t, defined? What is its economic interpretation? What data is usually applied as an approximate measure of the two key weights in the decomposition? What is the argument behind the approximation?

From now on assume $\tilde{F}$ has CRS w.r.t. K_t and L_{Yt} and can be written

$$\tilde{F}(K_t, L_{Yt}, t) = F(K_t, A_t L_{Yt}),$$

where A_t represents the "technology level", which due to the activity in the R&D sector is growing over time. The increase in capital per time unit is given by

$$\dot{K}_t = S_t - \delta K_t \equiv Y_t - C_t - \delta K_t, \qquad \delta \geq 0,$$

where C_t is aggregate consumption; gross saving, S_t, is positive for all t.

c) Let $y_t \equiv Y_t/L_{Yt}$. Suppose A grows at a constant rate $g > 0$. Given this g, determine g_y under balanced growth in the manufacturing sector. *Hint:* in view of the information given, we know something about the relationship between Y and K under balanced growth.

d) Show that the TFP growth rate is constant along a balanced growth path with L_{Yt} growing at a constant rate n. *Hint:* in view of the information given, we know something about the sum of the output elasticities w.r.t. K_t and L_{Yt}, respectively.

e) We now consider the economy as a whole. Is there a sense in which technical progress explains more than what the growth accounting suggests? Explain.

Problem 2   An economy described by the Lab-Equipment Model will under laissez-faire suffer from a certain kind of inefficiency.

a) Briefly describe the kind of inefficiency hinted at.

We now imagine that there is a "social planner" with the same criterion function as that of the representative household. The social planner's decision problem can be split into a static problem and a dynamic problem. The static problem is to ensure that Sector 1 (the basic-goods sector) uses the "right" quantity of intermediate goods. Output in the sector is

$$Y_t = A \left(\sum_{i=1}^{N_t} x_{it}^{1-\beta} \right) L^\beta, \qquad A > 0, \quad 0 < \beta < 1,$$

where Y_t is output per time unit, x_{it} is input of intermediate good i, N_t is the number of different types of intermediate goods available at time t, and L is labor input $=$ the exogenous and constant labor supply. The output of basic goods is used partly as input, X_t, in Sector 2, partly for consumption, $C_t \equiv c_t L$, and partly for R&D investment, Z_t, in Sector 3:

$$Y_t = X_t + C_t + Z_t = X_t + C_t + \frac{\dot{N}_t}{\eta}, \qquad \eta > 0,$$

and

$$X_t = \sum_{i=1}^{N_t} \psi x_{it}, \qquad \psi > 0.$$

b) Interpret the parameters η and ψ.

Although the textbook by Acemoglu focuses on the special case $A = (1-\beta)^{-1}$ and $\psi = 1 - \beta$, we shall here study the general case which does not rely on such arbitrary parameter links.

The social planner first solves the following static problem (the dating of the variables is suppressed for convenience):

$$\max_{x,\dots,x_N} Y - X = A \left(\sum_{i=1}^{N} x_i^{1-\beta} \right) L^\beta - \sum_{i=1}^{N} \psi x_i.$$

c) Why is this problem of relevance from an efficiency point of view? Let x_{SP} denote the solution for x_i (the solution will be the same for all i). Find x_{SP}.

d) The market outcome under laissez-faire is $x_i = \left(\frac{A(1-\beta)^2}{\psi} \right)^{1/\beta} L \equiv x$ for all i. Compare x_{SP} with this market outcome. What is the economic explanation of the difference?

e) Show that net output of basic goods can be written $Y - X = \tilde{A} N$, where $\tilde{A}$ is a positive constant.

The dynamic problem faced by the social planner is to choose $(c_t)_{t=0}^{\infty}$ so as to:

$$\max U_0 = \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt \qquad \text{s.t.}$$

$$0 \le c_t \le \frac{\tilde{A} N_t}{L}, \qquad\qquad\qquad\qquad\qquad\qquad (*)$$

$$\dot{N}_t = \eta(\tilde{A} N_t - L c_t), \qquad N_0 > 0 \text{ given}, \qquad (**)$$

$$N_t \ge 0 \text{ for all } t \ge 0, \qquad\qquad\qquad\qquad\quad (***)$$

where θ and ρ are given parameters, $\theta > 0$. In (**) indivisibilities are ignored and N is regarded as a continuous and differentiable function of time t. In view of (*) and (**), condition (***) will automatically hold and can therefore be deleted.

f) Explain why in (*) the control variable is bounded above; and explain how the dynamic constraint (**) arises.

g) Derive the first-order conditions for an interior solution of the social planner's dynamic problem. Determine the implied growth rate of c_t. Next, apply your general knowledge about reduced-form AK models to determine the time paths of N_t and Y_t (a brief verbal account is enough).

h) Write down the parameter restrictions required for positive growth and boundedness of the utility integral, respectively.

i) Recalling that the equilibrium interest rate in the laissez-faire market economy is r^* $= \eta\pi = \eta(\frac{\psi}{1-\beta} - \psi)x$, with x given as in d), compare the social planner's consumption growth rate with that of the laissez-faire market economy. *Hint:* an answer can be reached by showing that $\tilde{A} = (\frac{\psi}{1-\beta} - \psi)x_{SP}$ and bearing in mind the result from d).

j) Now consider a government that attempts to implement the social planner's allocation in a decentralized way. The government pays a subsidy at constant rate, σ, to purchases of intermediate goods such that the net price of intermediate good i is $(1 - \sigma)p_i$, where $p_i = \psi/(1 - \beta)$ is the price set by the monopolist supplier of good i. The government finances this subsidy by taxing consumption at a constant rate τ. It can be shown that a proper choice of σ and τ is sufficient to obtain the social planner's allocation in a decentralized way. Derive the required value of the subsidy rate σ. Comment.

Problem 3   *Short questions*

a) Briefly describe Acemoglu's distinction between proximate and fundamental determinants of differences in economic performance.

b) Define the concepts of fully endogenous growth and semi-endogenous growth. Many fully endogenous growth models rely on knife-edge conditions; give at least three examples. Why may knife-edge conditions be considered problematic?

c) "Economic policy will have no effect on the long-term economic performance of an economy described by a semi-endogenous growth model". True or false? Why?

———