Written exam for the M. Sc. in Economics. Summer 2016

Economic Growth

Master's Course

May 31, 2016

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total including this page.

The weighting of the problems is: Problem 1: 70% and Problem 2: 30%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a horizontal innovations model for a closed economy. The household sector is described the following way. There is a fixed number of infinitely-lived households, all alike. Each household has $L_t = L_0 e^{nt}$ members, $n > 0$, and each member supplies inelastically one unit of labor per time unit. Households have perfect foresight. We normalize the number of households to be one. Given $\theta > 0$ and $\rho > 0$, the household's problem is to choose a plan $(c_t)_{t=0}^{\infty}$ so as to maximize

$$\begin{aligned} U_0 &= \int_0^{\infty} \frac{c_t{}^{1-\theta}}{1-\theta} e^{-(\rho-n)t} dt \quad \text{s.t.} \\ c_t &\geq 0, \\ \dot{a}_t &= (r_t - n)a_t + w_t - c_t, \qquad a_0 \text{ given}, \qquad (*) \\ \lim_{t\to\infty} a_t e^{-\int_0^t (r_s - n)ds} &\geq 0, \qquad (**) \end{aligned}$$

where c_t is per head consumption, a_t is per head financial wealth, r_t is the interest rate, and w_t is the real wage.

a) Briefly, interpret (*), (**), and the parameters θ and ρ.

b) The solution to the household's problem is characterized by two equations, the Keynes-Ramsey rule and the transversality condition. State these two equations and give a brief interpretation of them (if you do not remember them, you may derive at least the Keynes-Ramsey rule by using optimal control theory).

The production side of the economy is described by the following assumptions (the time index t on time-dependent variables is omitted unless needed for clarity; $\forall j$ means $j = 1, 2, \ldots, A$):

$$Y = L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

$$\frac{\partial Y}{\partial L} = (1-\alpha)\frac{Y}{L_Y} = w, \qquad \frac{\partial Y}{\partial x_j} = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} = p_j, \qquad \forall j, \tag{2}$$

$$Y = C + I_K = cL + \dot{K} + \delta K, \ \delta \geq 0, \qquad K_0 > 0 \text{ given}, \tag{3}$$

$$p_j = \frac{1}{\alpha}(r+\delta) \equiv p, \qquad \forall j, \tag{4}$$

$$\pi_j = (\frac{1}{\alpha} - 1)(r+\delta)x_j \equiv (\frac{1}{\alpha} - 1)(r+\delta)x \equiv \pi, \qquad \forall j, \tag{5}$$

$$\dot{A} = \bar{\eta} L_A \equiv \eta A^{\varphi} L_A^{1-\xi}, \qquad \eta > 0, \ \varphi < 1, \ 0 \leq \xi < 1, \qquad A_0 > 0 \text{ given}, \tag{6}$$

$$w \gtreqless P_A \partial \dot{A}/\partial L_A = P_A \bar{\eta}, \text{ with " = " if } L_A > 0. \tag{7}$$

Notation is standard.

c) Briefly explain the meaning of the symbol x_j and interpret (2), (5), (6), and (7).

In general equilibrium with active R&D ($L_A > 0$) we get:

$$\begin{aligned}
Ax &= K, && (8)\\
L_Y + L_A &= L = L_0 e^{nt}, && (9)\\
Y &= K^{\alpha}(AL_Y)^{1-\alpha}, && (10)\\
\frac{1}{\alpha}(r+\delta) &= \frac{\partial Y}{\partial x_j} = \alpha\frac{Y}{K}, && (11)\\
\pi &= (1-\alpha)\alpha\frac{Y}{A}, && (12)\\
w &= (1-\alpha)\frac{Y}{L_Y} = P_A \eta A^{\varphi} L_A^{-\xi}, && (13)\\
a &= \frac{K + P_A A}{L}, && (14)\\
P_A r &= \pi + \dot{P}_A. && (15)
\end{aligned}$$

d) Briefly interpret (8), (13), (14), and (15). Derive (10), (11), and (12).

Considering the equation (15) as a differential equation for P_A (as a function of time), the solution to this differential equation, presupposing that there are no bubbles, is

$$P_{At} = \int_t^{\infty} \pi_s e^{-\int_t^s r_u du} ds.$$

e) Interpret this formula in economic terms. Under certain conditions (that turn out to hold along a balanced growth path), the formula reduces to $P_{At} = \pi_t/(r-n)$. What are these conditions?

From now on, assume the economy follows a balanced growth path (BGP) with $L_A > 0$. Let the growth rate of any time-dependent variable, z, be denoted g_z.

f) Derive the growth rate of A, $y \equiv Y/L$, and π along the BGP. Comment.

g) r must be constant along the BGP. Why? *Hint:* use one of the above equations and your general knowledge about BGPs in an economy satisfying (3) and having positive gross capital investment.

h) Show that $g_{P_A} = n$. *Hint:* an easy approach is to use (15) together with e), f), and g).

The fraction of the labor force allocated to R&D along the BGP can be shown to satisfy

$$s_R = \frac{1}{1 + \frac{r-n}{\alpha g_A^*}} \equiv s_R^*, \qquad (***)$$

where g_A^* is the result for g_A in f).

i) How does s_R depend on r, n, and g_A^*, respectively? Give the intuition in each case.

We now introduce a "social planner" with the same criterion function as that of the representative household. The social planner's problem can be decomposed into two problems. The first is the static optimization problem:

$$\begin{aligned}\max_{x_1,\dots,x_A} Y &= \bar{L}_Y^{1-\alpha}\sum_{j=1}^{A} x_j^{\alpha} \text{ s.t.} \\ \sum_{j=1}^{A} x_j &= \bar{K},\end{aligned}$$

where $\bar{L}_Y$ and $\bar{K}$ are given positive numbers. The second problem is the dynamic problem to choose a plan $(c_t, L_{Yt})_{t=0}^{\infty}$ so as to maximize U_0 above, subject to the constraints

$$\begin{aligned} c_t &\geq 0, 0 \leq L_{Yt} \leq L_t, \\ \dot{K}_t &= K_t^{\alpha}(A_t L_{Yt})^{1-\alpha} - c_t L_t - \delta K_t, \qquad K_0 > 0 \text{ given}, \\ \dot{A}_t &= \eta A_t^{\varphi}(L_t - L_{Yt})^{1-\xi}, \qquad A_0 > 0 \text{ given}, \\ K_t &\geq 0 \quad \text{for all } t > 0. \end{aligned}$$

j) Explain (in words or by derivation) why the social planner's solution to the static problem must be as indirectly displayed in the dynamic problem.

Assuming $\rho - n > (1-\theta)(1-\xi)n/(1-\varphi)$, the solution to the social planner's dynamic problem will, along a BGP, have $g_A = g_A^*$ from f) and satisfy

$$s_R = \frac{1}{1 + \frac{1}{1-\xi}(\frac{\rho-n}{g_A^*} + \theta - \varphi)} \equiv s_R^{SP}.$$

k) Why *must* the solution to the social planner's problem, along a BGP, in this economy have $g_A = g_A^*$ from f)?

We shall now compare s_R^{SP} and s_R^*.

ℓ) To prepare for that, insert into the formula for s_R^* in (***), the solution in the laissez-faire market economy for r along a BGP. *Hint:* the Keynes-Ramsey rule is useful here.

From now on, assume that the duplication externality is absent and that $\varphi > 0$.

m) Compare under these conditions s_R^{SP} and s_R^*. Sign the difference and give an intuitive interpretation.

n) What kind of policy instruments would you suggest for a government that wanted to implement the social planner's solution in the market economy?

Problem 2 *Short questions*

a) Capital per worker, K/L, tends to be much lower in poor countries than in rich countries. Can we, while *accepting* the neoclassical assumption of diminishing marginal productivity of capital, explain why the capital flows from rich to poor countries are not much larger than they actually are? Why or why not?

b) Briefly, give an account of important conceptual differences between the three kinds of capital: physical capital, human capital, and "knowledge capital" (the stock of technical knowledge).

c) Some models for advanced economies assume an aggregate production function of the following form:

$$Y = F(K, H),$$

where Y is output, K is input of physical capital, H is a measure of the input of human capital, and F is a neoclassical production function with constant returns to scale. Moreover, $H \equiv hL$, where h is a measure of average human capital, and L is working hours per time unit. This is combined with the relations

$$\begin{aligned} Y &= cL + I_K + I_H, \\ \dot{K} &= I_K - \delta_K K, \\ \dot{H} &= I_H - \delta_H H, \end{aligned}$$

where I_K and I_H are gross investment in physical and human capital, respectively, and δ_K and δ_H are the corresponding depreciation rates. Perfect competition in all markets is assumed.

Give a brief evaluation of this approach.

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