Economic Growth

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Saddle-point stability

A concept which perplexes many is the concept of saddle-point stability.

Consider a *two-dimensional* dynamic system (two coupled first-order differential equations with two endogenous time-dependent variables). Suppose the system has a steady state which is a *saddle point* (which is the case if and only if the two eigenvalues of the associated Jacobi matrix are of opposite sign). Then, so far, either presence or absence of saddle-point stability is possible. And which of the two cases occur can not be diagnosed from the two differential equations in isolation. One has to consider the *boundary conditions.*

Here is a complete definition of (local) saddle-point stability. A steady state of a two-dimensional dynamic system is (locally) *saddle-point stable* if:

1. the steady state is a saddle point;
2. one of the two endogenous variables is predetermined while the other is a jump variable;
3. the saddle path is not parallel to the jump variable axis; and
4. there is a boundary condition on the system such that the diverging paths are ruled out as solutions.

Thus, to establish saddle-point stability all four properties must be verified. If for instance point 1 and 2 hold but, contrary to point 3, the saddle path is parallel to the jump variable axis, then saddle-point stability does not obtain. Indeed, given that the predetermined variable initially deviated from its steady-state value, it would not be possible to find any initial value of the jump variable such that the solution of the system would converge to the steady state for $t \to \infty$.

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