Chapter 7

Michael Kremer's population-breeds-ideas model

This chapter relates to Section 4.2 of Acemoglu's book where two special cases of the *population-breeds-ideas model* (Kremer 1993) are presented. Here we start with a more general version of the model. The point of the model is to show that under certain conditions, the cumulative and nonrival character of technical knowledge makes it likely that the Malthusian regime of stagnating income per capita, close to existence minimum and caused by scarcity of land, will sooner or later in the historical evolution be surpassed.

7.1 The general model

Suppose a pre-industrial economy can be described by:

$$Y_t = A_t^{\sigma} L_t^{\alpha} Z^{1-\alpha}, \qquad \sigma > 0, 0 < \alpha < 1, \tag{7.1}$$

$$\dot{A}_t = \lambda A_t^{\varepsilon} L_t, \qquad \lambda > 0, \varepsilon \lesseqgtr 0, \quad A_0 > 0 \text{ given}, \tag{7.2}$$

$$L_t = \frac{Y_t}{\bar{y}}, \qquad \bar{y} > 0, \tag{7.3}$$

where Y is aggregate output, A the level of technical knowledge, L the labor force (= population), Z the amount of land (fixed), and $\bar{y}$ subsistence minimum (so the φ in Acemoglu's equation (4.2) is simply the inverse of the subsistence minimum). Both Z and $\bar{y}$ are considered as constant parameters. Time is continuous and it is understood that a kind of Malthusian population mechanism (see below) is operative behind the scene.

The exclusion of capital from the aggregate production function, (7.1), reflects the presumption that capital (tools etc.) is quantitatively of minor importance in a pre-industrial economy. In accordance with the replication argument, the production function has CRS w.r.t. the rival inputs, labor and land. The factor A_t^σ measures total factor productivity. As the right-hand side of (7.2) is positive, the technology level, A_t, is rising over time (although far back in time very very slowly). The increase in A_t per time unit is seen to be an increasing function of the size of the population. This reflects the hypothesis that population breeds ideas; these are *nonrival* and enter the pool of technical knowledge available for society as a whole. Indeed, the use of an idea by one agent does not preclude others' use of the same idea. Dividing through by L in (7.1) we see that $y \equiv Y_t/L_t = A_t^\sigma(Z/L_t)^{1-\alpha}$. The nonrival character is displayed by labor productivity being dependent on the total stock of knowledge, not on this stock per worker. In contrast, labor productivity depends on *land per worker*.

The rate per capita by which population breeds ideas is , λA^ε. In case $\varepsilon > 0$, this rate is an increasing function of the already existing level of technical knowledge. This case reflects the hypothesis that the larger is the stock of ideas the easier do new ideas arise (perhaps by combination of existing ideas). The opposite case, $\varepsilon < 0$, is the one where "the easiest ideas are found first" or "the low-hanging fruits are picked first".

Equation (7.3) is a shortcut description of a Malthusian population mechanism. Suppose the true mechanism is

$$\dot{L}_t = \beta(y_t - \bar{y})L_t \gtreqless 0 \quad \text{for} \quad y_t \gtreqless \bar{y}, \tag{7.4}$$

where $\beta > 0$ is the speed of adjustment, y_t is per capita income, and $\bar{y} > 0$ is subsistence minimum. A rise in y_t above $\bar{y}$ will lead to increases in L_t, thereby generating downward pressure on Y_t/L_t and perhaps end up pushing y_t below $\bar{y}$. When this happens, population will be decreasing for a while and so return towards its sustainable level, $Y_t/\bar{y}$. Equation (7.3) treats this mechanism as if the population instantaneously adjusts to its sustainable level (as if $\beta \to \infty$). The model hereby gives a long-run picture, ignoring the Malthusian ups and downs in population and per capita income about the subsistence minimum. The important feature is that the technology level, and thereby Y_t, as well as the sustainable population will be *rising* over time. This speeds up the arrival of new ideas and so Y_t is raised even faster although per-capita income remains at its long-run level, $\bar{y}$.[1]

For simplicity, we now normalize the constant Z to be 1.

[1] Extending the model with the institution of private ownership and competitive markets, the absence of a growing standard of living corresponds to the doctrine from classical economics called the *iron law of wages.* This is the theory (from Malthus and Ricardo) that scarce natural resources and the pressure from population growth causes real wages to remain at subsistende level.

Comparison with the two special cases in Acemoglu

At pp. 113-14 Acemoglu presents two versions of this framework, both of which assume $\sigma = 1 - \alpha$. This assumption is arbitrary; it is included as a special case in our formulation above. As to the other parameter relating to the role of knowledge, ε, Acemuglu assumes $\varepsilon = 0$ in his first version of the framework. This leads to constant population growth but stagnating standard of living (Acemoglu, p. 113). In his second version, Acemoglu assumes $\varepsilon = 1$. This leads to many centuries of slow but (weakly) accelerating population growth and then ultimately a "takeoff" with sustained rise in the standard of living, to be followed by the "demographic transition" (outside the model). This latter outcome arises for a much larger set of parameter values and is therefore theoretically more robust than appears in Acemoglu's exposition.

7.2 Law of motion

The dynamics of the model can be reduced to one differential equation, the law of motion of technical knowledge. By (7.3), $L_t = Y_t/\bar{y} = A_t^\sigma L_t^\alpha/\bar{y}$. Consequently $L_t^{1-\alpha} = A_t^\sigma/\bar{y}$ so that

$$L_t = \bar{y}^{\frac{1}{\alpha-1}} A_t^{\frac{\sigma}{1-\alpha}}. \tag{7.5}$$

Substituting this into (7.2) gives the law of motion of technical knowledge:

$$\dot{A}_t = \lambda \bar{y}^{\frac{1}{\alpha-1}} A_t^{\varepsilon+\frac{\sigma}{1-\alpha}} \equiv \hat{\lambda} A_t^\mu, \tag{7.6}$$

where we have defined $\hat{\lambda} \equiv \lambda \bar{y}^{1/(\alpha-1)}$ and $\mu \equiv \varepsilon + \sigma/(1-\alpha)$. As will appear in the remainder, the "feedback parameter" μ is of key importance for the dynamics. We immediately see that if $\mu = 1$, the differential equation (7.6) is linear, while otherwise it is nonlinear.

The case $\mu = 1$: When $\mu = 1$, there will be a constant growth rate $g_A = \hat{\lambda}$ in technical knowledge. By (7.5), this results in a constant population growth rate $g_L = [\sigma/(1-\alpha)]\,\hat{\lambda}$, which is also the growth rate of output in view of (7.3). By the definition of $\hat{\lambda}$ in (7.6), we see that, as expected, the population and output growth rate is an increasing function of the creativity parameter λ and a decreasing function of the subsistence minimum.[2]

These classical economists did not recognize any tendency to sustained technical progress and therefore missed the immanent tendency to population growth at the pre-industrial stage of economic development. Karl Marx was the first among the classical economists to really see and emphasize sustained technical progress.

[2]If $\sigma = 1 - \alpha$ as in Acemoglu's analysis, $\mu = 1$ requires $\varepsilon = 0$, and in this case L and Y grow at the same rate as knowledge.

In this case the economy never leaves the Malthusian regime of a more or less constant standard of living close to existence minimum. Takeoff never occurs.

The case $\mu \neq 1$. Then (7.6) can be written

$$\dot{A}_t = \hat{\lambda} A_t^{\mu}, \tag{7.7}$$

which is a nonlinear differential equation in A.[3] Let $x \equiv A^{1-\mu}$. Then

$$\dot{x}_t = (1-\mu) A_t^{-\mu} \hat{\lambda} A_t^{\mu} = (1-\mu)\hat{\lambda}, \tag{7.8}$$

a constant. To find x_t from this, we only need simple integration:

$$x_t = x_0 + \int_0^t \dot{x}_\tau d\tau = x_0 + (1-\mu)\hat{\lambda} t.$$

As $A = x^{\frac{1}{1-\mu}}$ and $x_0 = A_0^{1-\mu}$, this implies

$$A_t = x_t^{\frac{1}{1-\mu}} = \left[A_0^{1-\mu} + (1-\mu)\hat{\lambda} t\right]^{\frac{1}{1-\mu}} = \frac{1}{\left[A_0^{1-\mu} - (\mu-1)\hat{\lambda} t\right]^{\frac{1}{\mu-1}}}. \tag{7.9}$$

There are now two sub-cases, $\mu > 1$ and $\mu < 1$. The latter sub-case leads to permanent but decelerating growth in knowledge and population and the Malthusian regime is never transcended (see Exercise III.3). The former sub-case is the interesting one.

7.3 The inevitable ending of the Malthusian regime when $\mu > 1$

Assume $\mu > 1$. In this case the result (7.9) implies that the Malthusian regime *must* come to an end.

Although to begin with, A_t may grow extremely slowly, the growth in A_t will be *accelerating* because of the *positive feedback* (visible in (7.2)) from both rising population and rising A_t. Indeed, since $\mu > 1$, the denominator in (7.9) will be decreasing over time and approach zero in finite time, namely as t approaches the finite value $t^* = A_0^{1-\mu}/((\mu-1)\hat{\lambda})$. As an implication, A_t goes towards *infinity* in *finite* time. The stylized graph in Fig. 7.1 illustrates. The evolution of technical knowledge becomes *explosive* as t approaches t^*.

[3]The differential equation, (7.7), is a special case of what is known as the *Bernoulli equation*. In spite of being a non-linear differential equation, the Bernoulli equation always has an explicit solution.

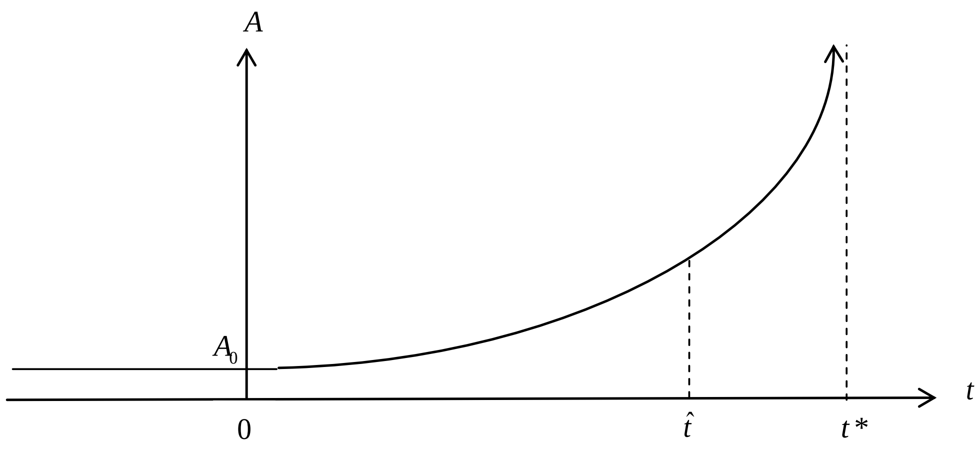

Figure 7.1: Accelerating growth in A when the feedback parameter μ exceeds one.

It follows from (7.5) and (7.1) that explosive growth in A implies explosive growth in L and Y, respectively. The acceleration in the evolution of Y will sooner or later make Y rise fast enough so that the Malthusian population mechanism (which for biological reasons has to be slow) can not catch up. Then, what was in the Malthusian population mechanism, equation (7.4), only a transitory excess of y_t over $\bar{y}$, will at some $t = \hat{t} < t^*$ become a permanent excess and take the form of sustained growth in y_t. This is known as the *takeoff*.

Note that Fig. 7.1 illustrates only what the process (7.7), with $\mu > 1$, implies *as long as it rules*, namely that knowledge goes towards infinity in *finite* time. The process necessarily ceases to rule long before time t^* is reached, however. This is because the process presupposes that the Malthusian population mechanism keeps track with output growth which at some point before t^* becomes impossible because of the acceleration in the latter.

In a neighborhood of this point the takeoff will occur, featuring sustained growth in output per capita. According to equation (7.4) the takeoff should also feature a permanently rising population growth rate. As economic history has testified, however, along with the rising standard of living the demographics changed radically (in the U.K. during the 19th century). The *demographic transition* took place with fertility declining faster than mortality. This results in completely different dynamics about which the present model has nothing to say.[4] As to the demographic transition as such, ex-

[4] Kremer (1993), however, also includes an extended model taking some of these changed dynamics into account.

planations suggested by economists include: higher real wages mean higher opportunity costs of raising children instead of producing; reduced use of child labor; the trade-off between "quality" (educational level) of the offspring and their "quantity" (Becker, Galor)[5]; skill-biased technical change; and improved contraception technology.

7.4 Closing remarks

The population-breeds-ideas model is about dynamics in the Malthusian regime of the pre-industrial epoch. The story told by the model is the following. When the feedback parameter, μ, is above one, the Malthusian regime has to come to an end because the battle between scarcity of land (or natural resources more generally) and technological progress will inevitably be won by the latter. The reason is the cumulative and nonrival character of technical knowledge. This nonrivalry implies economies of scale. Moreover, the stock of knowledge is growing *endogenously.* This knowledge growth generates output growth and, through the demographic mechanism (7.3), growth in the stock of people, which implies a *positive feedback* to the growth of knowledge and so on. On top of this, if $\varepsilon > 0$, knowledge growth has a direct positive feedback on itself through (7.2). When the total positiv feedback is strong enough ($\mu > 1$), it generates an explosive process.[6]

On the basis of demographers' estimates of the growth in global population over most of human history, Kremer (1993) finds empirical support for $\mu > 1$. Indeed, in the opposite case, $\mu \leq 1$, there would *not* have been a rising world population growth rate since one million years B.C. to the industrial revolution. The data in Kremer (1993, p. 682) indicates that the world population growth rate has been more or less proportional to the size of population until recently.

Final remark. In the formulation of the model, I have made one simplification relative to Kremer's setup. Kremer starts from a slightly more general ideas-creation equation, namely $\dot{A}_t = \lambda A_t^{\varepsilon} L_t^{\psi}$ with $\psi > 0$, while in our (7.2) we have assumed $\psi = 1$. If $\psi > 1$, the ideas-creating brains reinforce one another. This only fortifies the acceleration in knowledge creation and thereby "supports" the case $\mu > 1$.[7] If on the other hand $0 < \psi < 1$, the idea-creating brains partly offset one another, for instance by simultaneously coming up with more or less the same ideas (the case of "overlap"). This generalization does not change the qualitative results. By assuming that the

[5]See Acemoglu, Section 21.2.

[6]In the appendix the explosion result is considered in a general mathematical context.

[7]Kremer's calibration suggests $\psi \approx 6/5$.

number of new ideas per time unit is proportional to the stock of brains, we have chosen to focus on an intermediate case in order to avoid secondary factors blurring the main mechanism.

7.5 Appendix

Mathematically, the background for the explosion result is that the solution to a first-order differential equation of the form $\dot{x}(t) = \alpha + bx(t)^c$, $c > 1$, $b \neq 0$, $x(0) = x_0$ given, is always explosive. Indeed, the solution, $x = x(t)$, will have the property that $x(t) \to \pm\infty$ for $t \to t^*$ for some $t^* > 0$ where t^* depends on the initial conditions; and thereby the solution is defined only on a bounded time interval which depends on the initial condition.

Take the differential equation $\dot{x}(t) = 1 + x(t)^2$ as an example. As is well-known, the solution is $x(t) = \tan t = \sin t / \cos t$, defined on the interval $(-\pi/2, \pi/2)$.

7.6 References

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