Chapter 16

Natural resources and economic growth

In this course, up to now, the relationship between economic growth and the earth's finite natural resources has been touched upon in connection with: the discussion of returns to scale (Chapter 2), the transition from a pre-industrial to an industrial economy (in Chapter 7), the environmental problem of global warming (Chapter 8), and the resource curse (in Chapter 13.4.3). In a more systematic way the present chapter reviews how natural resources, including the environment, relate to economic growth.

The contents are:

- Classification of means of production.
- The notion of sustainable development.
- Renewable natural resources.
- Non-renewable natural resources and exogenous technology growth.
- Non-renewable natural resources and endogenous technology growth.
- Natural resources and the issue of limits to economic growth.

The first two sections aim at establishing a common terminology for the discussion.

16.1 Classification of means of production

We distinguish between different categories of production factors. First two broad categories:

1. *Producible* means of production, also called man-made inputs.
2. *Non-producible* means of production.

The first category includes:

1.1 *Physical inputs* like processed raw materials, other intermediate goods, machines, and buildings.

1.2 *Human inputs* of a produced character in the form of technical knowledge (available in books, USB sticks etc.) and human capital.

The second category includes:

2.1 Human inputs of a non-produced character, sometimes called "raw labor".[1]

2.2 Natural resources. By definition in limited supply on this earth.

Natural resources can be sub-divided into:

2.2.1 *Renewable resources*, that is, natural resources the stock of which can be replenished by a natural self-regeneration process. Hence, if the resource is not over-exploited, it can be sustained in a more or less constant amount. Examples: ground water, fertile soil, fish in the sea, clean air, national parks.

2.2.2 *Non-renewable resources*, that is, natural resources which have no natural regeneration process (at least not within a relevant time scale). The stock of a non-renewable resource is thus depletable. Examples: fossil fuels, many non-energy minerals, virgin wilderness and endangered species.

The climate change problem due to "greenhouse gasses" can be seen as belonging to somewhere between category 2.2.1 or 2.2.2 in that the quality of the atmosphere *has* a natural self-regeneration ability, but the speed of regeneration is very low.

Given the scarcity of natural resources and the pollution problems caused by economic activity, key issues are:

[1] Outside a slave society, biological reproduction is usually not considered as part of the economic sphere of society even though formation and maintainance of raw labor requires child rearing, health, food etc. and is thus conditioned on economic circumstances.

a. Is sustainable development possible?

b. Is sustainable economic growth (in a per capita welfare sense) possible?

c. How should a better "thermometer" for the evolution of the economy than measurement of GNP be designed?

But first: what does "sustainable" and "sustainability" really mean"?

16.2 The notion of sustainable development

The basic idea in the notion of sustainable development is to emphasize intergenerational responsibility. The Brundtland Commission (1987) defined sustainable development as "development that meets the needs of present generations without compromising the ability of future generations to meet theirs".

In more standard economic terms we may define *sustainable economic development* as a time path along which per capita "welfare" (somehow measured) remains non-decreasing across generations forever. An aspect of this is that current economic activities should not impose significant economic risks on future generations. The "forever" in the definition can not, of course, be taken literally, but as equivalent to "for a very long time horizon". We know that the sun will eventually (in some billion years) burn out and consequently life on earth will become extinct.

Our definition emphasizes *welfare,* which should be understood in a broad sense, that is, as more or less synonymous with "quality of life", "living conditions", or "well-being" (the term used in Smulders, 1995). What may matter is thus not only the per capita amount of marketable consumption goods, but also fundamental aspects like health, life expectancy, and enjoyment of services from the ecological system: capability to lead a worthwhile life.

To make this more specific, consider preferences as represented by the period utility function of a "typical individual". Suppose two variables enter as arguments, namely consumption, c, of a marketable produced good and some measure, q, of the quality of services from the eco-system. Suppose further that the period utility function is of CES form:[2]

$$u(c,q) = \left[\alpha c^{\beta} + (1-\alpha)q^{\beta}\right]^{1/\beta}, \quad 0 < \alpha < 1, \beta < 1. \tag{16.1}$$

The parameter β is called the *substitution parameter*. The elasticity of substitution between the two goods is $\sigma = 1/(1-\beta) > 0$, a constant. When $\beta \to 1$

[2]CES stands for Constant Elasticity of Substitution.

(from below), the two goods become perfect substitutes (in that $\sigma \to \infty$). The smaller is β, the less substitutable are the two goods. When $\beta < 0$, we have $\sigma < 1$, and as $\beta \to -\infty$, the indifference curves become near to right angled.[3] According to many environmental economists, there are good reasons to believe that $\sigma < 1$, since water, basic foodstuff, clean air, absence of catastrophic climate change, etc. are difficult to replace by produced goods and services. In this case there is a limit to the extent to which a rising c, obtainable through a rising per capita income, can compensate for falling q.

At the same time the techniques by which the consumption good is currently produced may be "dirty" and thereby *cause* a falling q. An obvious policy response is the introduction of pollution taxes that give an incentive for firms (or households) to replace these techniques (or goods) with cleaner ones. For certain forms of pollution (e.g., sulfur dioxide, SO_2, in the air) there is evidence of an inverted U-curve relationship between the degree of pollution and the level of economic development measured by GDP per capita − the *environmental Kuznets curve*.[4]

So an important element in *sustainable* economic development is that the economic activity of current generations does not spoil the environmental conditions for future generations. Living up to this requirement necessitates economic and environmental strategies consistent with the planet's endowments. This means recognizing the role of environmental constraints for economic development. A complicating factor is that specific abatement policies vis-a-vis particular environmental problems may face resistance from interest groups, thus raising political-economics issues.

As defined, a criterion for sustainable economic *development* to be present is that per capita welfare remains *non-decreasing* across generations. A subcategory of this is *sustainable* economic *growth* which is present if per capita welfare is *growing* across generations. Here we speak of growth in a *welfare* sense, not in a physical sense. Permanent exponential per capita output

[3] By L'Hôpital's rule for "0/0" it follows that, for fixed c and q,

$$\lim_{\beta \to 0, \beta \neq 0} \left[\alpha c^{\beta} + (1-\alpha) q^{\beta} \right]^{1/\beta} = c^{\alpha} q^{1-\alpha}.$$

So the Cobb-Douglas function, which has elasticity of substitution between the goods equal to 1, is an intermediate case, corresponding to $\beta = 0$. More technical details in Appendix A, albeit from the perspective of production rather than preferences.

[4] See, e.g., Grossman and Krueger (1995). Others (e.g., Perman and Stern, 2003) claim that when paying more serious attention to the statistical properties of the data, the environmental Kuznets curve is generally rejected. Important examples of pollutants accompanied by *absence* of an environmental Kuznets curve include waste storage, reduction of biodiversity, and emission of CO_2 to the atmosphere. A very serious problem with the latter is that emissions from a single country is spread all over the globe.

growth in a *physical* sense is of course not possible with limited natural resources (matter or energy). The issue about sustainable *growth* is whether, by combining the natural resources with man-made inputs (knowledge, human capital, and physical capital), an output stream of *increasing quality*, and therefore increasing *economic value,* can be maintained. In modern times capabilities of many digital electronic devices provide conspicuous examples of exponential growth in *quality* (or *efficiency*). Think of processing speed, memory capacity, and efficiency of electronic sensors. What is known as *Moore's Law* is the rule of thumb that there is a doubling of the efficiency of microprocessors within every 18 months. The evolution of the internet has provided much faster and widened dissemination of information and fine arts.

Of course there are intrinsic difficulties associated with measuring sustainability in terms of well-being. There now exists a large theoretical and applied literature dealing with these issues. A variety of extensions and modifications of the standard national income accounting GNP has been developed under the heading *Green NNP* (green net national product). An essential feature in the measurement of Green NNP is that from the conventional GDP (which essentially just measures the level of economic activity) is subtracted the depreciation of not only the physical capital but also the environmental assets. The latter depreciate due to pollution, overburdening of renewable natural resources, and depletion of reserves of non-renewable natural resources.[5] In some approaches the focus is on whether a comprehensive measure of *wealth* is maintained over time. Along with reproducible assets and natural assets (including the damage to the atmosphere from "greenhouse gasses"), Arrow et al. (2012) include health, human capital, and "knowledge capital" in their measure of "wealth". They apply this measure in a study of the United States, China, Brazil, India, and Venezuela over the period 1995-2000. They find that all five countries over this period satisfy the sustainability criterion of non-decreasing wealth in this broad sense. Indeed the wealth measure referred to is found to be growing in all five countries − in the terminology of the field positive "genuine saving" has taken place.[6] Note that it is sustainability that is claimed, not optimality.

[5] The depreciation of these environmental and natural assets is evaluated in terms of the social planner's shadow prices. See, e.g., Heal (1998), Weitzman (2001, 2003), and Stiglitz et al. (2010).

[6] Of course, many measurement uncertainties and disputable issues of weighting are involved; brief discussions, and questioning, of the study are contained in Solow (2012), Hamilton (2012), and Smulders (2012). Regarding Denmark 1990-2009, a study by Lind and Schou (2013), along lines somewhat similar to those of Arrow et al. (2012), also suggests sustainability to hold.

In the next two sections we will go more into detail with the challenge to sustainability and growth coming from renewable and non-renewable resources, respectively. We shall primarily deal with the issues from the point of view of technical feasibility of non-decreasing, and possibly rising, per-capita consumption. Concerning questions about appropriate institutional regulation the reader is referred to the specialized literature.

We begin with renewable resources.

16.3 Renewable resources

A useful analytical tool is the following simple model of the stock dynamics associated with a renewable resource.

Let $S_t \geq 0$ denote the *stock* of the renewable resource at time t (so in this chapter S is not our symbol for saving). Then we may write

$$\dot{S}_t \equiv \frac{dS_t}{dt} = G_t - R_t = G(S_t) - R_t, \qquad S_0 > 0 \text{ given}, \tag{16.2}$$

where G_t is the self-regeneration of the resource per time unit and $R_t \geq 0$ is the resource extraction (and use) per time unit at time t. If for instance the stock refers to the number of fish in the sea, the flow R_t represents the number of fish caught per time unit. And if, in a pollution context, the stock refers to "cleanness" of the air in cities, R_t measures, say, the emission of sulfur dioxide, SO_2, per time unit. The self-regenerated amount per time unit depends on the available stock through the function $G(S_t)$, known as a *self-regeneration function.*[7]

Let us briefly consider the example where S stands for the size of a fish population in the sea. Then the self-regeneration function will have a bell-shape as illustrated in the upper panel of Figure 16.1. Essentially, the self-regeneration ability is due to the flow of solar energy continuously entering the the eco-system of the earth. This flow of solar energy is constant and beyond human control.

The size of the stock at the lower intersection of the $G(S)$ curve with the horizontal axis is $\underline{S}(0) \geq 0$. Below this level, even with $R = 0$ there are too few female fish to generate offspring, and the population necessarily shrinks and eventually reaches zero. We may call $\underline{S}(0)$ the minimum sustainable stock.

At the other intersection of the $G(S)$ curve with the horizontal axis, $\bar{S}(0)$ represents the maximum sustainable stock. The eco-system cannot support

[7] The equation (16.2) also covers the case where S represents the stock of a *non-renewable* resource if we impose $G(S) \equiv 0$, i.e., there is no self-regeneration.

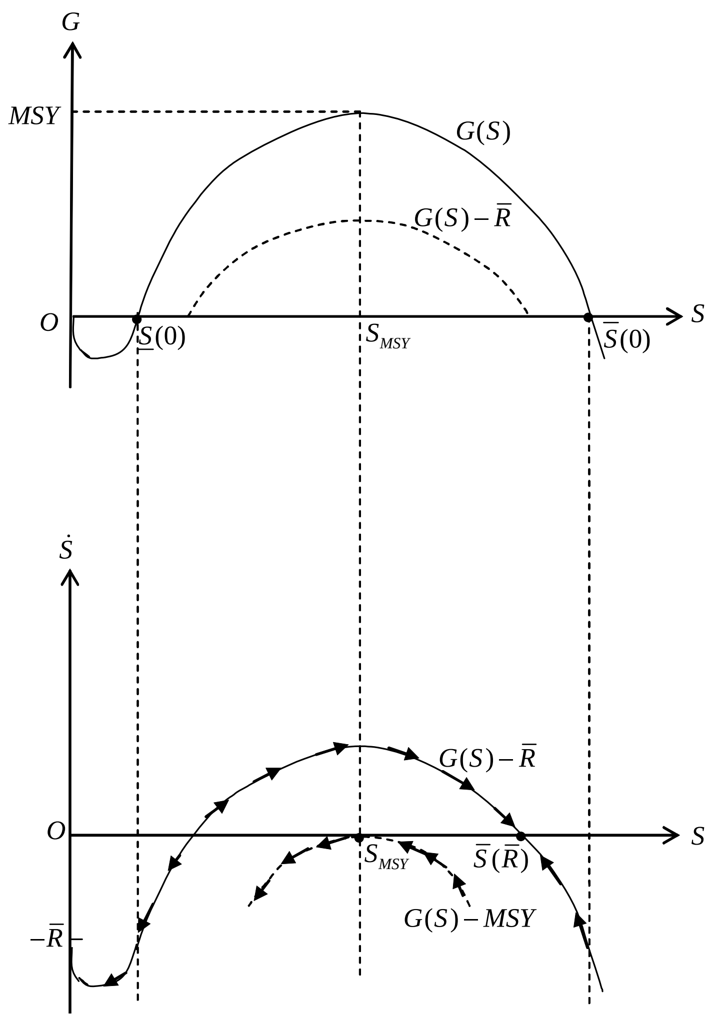

Figure 16.1: The self-generation function (upper panel) and stock dynamics for a given $R = \bar{R} \in (0, MSY]$ (lower panel).

further growth in the fish population. The reason may be food scarcity, spreading of diseases because of high population density, and easiness for predators to catch the considered fish species and themselves expand. Popular mathematical specifications of $G(\cdot)$ include the logistic function $G(S) = \alpha S(1 - S/\beta)$, where $\alpha > 0$, $\beta > 0$, and the quasi-logistic function $G(S) = \alpha S(1 - S/\beta)(S/\gamma - 1)$, where also $\gamma > 0$. In both cases $\bar{S}(0) = \beta$, but $\underline{S}(0)$ equals 0 in the first case and γ in the second.

The value MSY, indicated on the vertical axis in the upper panel, equals $\max_S g(S)$. This value is thus the *maximum sustainable yield* per time unit. This yield is sustainable from time t_0, provided the fish population is at time t_0 at least of size $S_{MSY} = \arg\max_S g(S)$ which is that value of S where $G(S) = MSY$. The size, S_{MSY}, of the fish population is consistent with maintaining the harvest MSY per time unit forever in a steady state.

The lower panel in Figure 16.1 illustrates the dynamics in the $(S, \dot{S})$ plane, given a fixed rate of resource extraction $R = \bar{R} \in (0, MSY]$. The arrows indicate the direction of movement in this plane. In the long run, if $R = \bar{R}$ for all t, the stock will settle down at the size $\bar{S}(\bar{R})$. The stippled curve in the upper panel indicates $G(S) - \bar{R}$, which is the same as $\dot{S}$ in the lower panel when $R = \bar{R}$. The stippled curve in the lower panel indicates the dynamics in case $R = MSY$. In this case the steady-state stock, $\bar{S}(MSY) = S_{MSY}$, is unstable. Indeed, a small negative shock to the stock will not lead to a gradual return but to a self-reinforcing reduction of the stock as long as the extraction $R = MSY$ is maintained.

Note that MSY is an *ecological maximum* and not necessarily in any sense an economic optimum. Indeed, since the search and extraction costs may be a decreasing function of the fish density in the sea, hence of the stock of fish, it may be worthwhile to increase the stock beyond S_{MSY}, thus settling for a smaller harvest per time unit. Moreover, a fishing industry cost-benefit analysis may consider maximization of the discounted expected aggregate profits per time unit, taking into account the expected evolution of the market price of fish, the cost function, and the dynamic relationship (16.2).

In addition to its importance for regeneration, the stock, S, may have amenity value and thus enter the instantaneous utility function. Then again some conservation of the stock over and above S_{MSY} may be motivated.

A dynamic model with a renewable resource and focus on technical feasibility Consider a simple model consisting of (16.2) together with

$$\begin{aligned} Y_t &= F(K_t, L_t, R_t, t), \qquad \partial F/\partial t \geq 0, \\ \dot{K}_t &= Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \quad K_0 > 0 \text{ given}, \\ L_t &= L_0 e^{nt}, \qquad n \geq 0, \quad L_0 > 0 \text{ given}, \end{aligned} \tag{16.3}$$

where Y_t is aggregate output and K_t, L_t, and R_t are inputs of capital, labor, and a renewable resource, respectively, per time unit at time t. Let the aggregate production function, F, be neoclassical[8] with constant returns to scale w.r.t. K, L, and R. The assumption $\partial F/\partial t \geq 0$ represents exogenous technical progress. Further, C_t is aggregate consumption ($\equiv c_t L_t$, where c_t is per capita consumption) and δ denotes a constant rate of capital depreciation. There is no distinction between employment and population, L_t. The population growth rate, n, is assumed constant.

Is sustainable economic development in this setting technically feasible? By definition, the answer will be yes if non-decreasing per capita consumption can be sustained forever. From economic history we know of examples of "tragedy of the commons", like over-grazing of unregulated common land. As our discussion is about technical feasibility, we assume this kind of problem is avoided by appropriate institutions.

Suppose the use of the renewable resource is kept constant at a sustainable level $\bar{R} \in (0, MSY)$. To begin with, suppose $n = 0$ so that $L_t = L$ for all $t \geq 0$. Assume that at $R = \bar{R}$, the system is "productive" in the sense that

$$\lim_{K \to 0} F_K(K, L, \bar{R}, 0) > \delta > \lim_{K \to \infty} F_K(K, L, \bar{R}, 0). \tag{A1}$$

This condition is satisfied in Figure 16.2 where the value $\bar{K}$ has the property $F(\bar{K}, L, \bar{R}, 0) = \bar{K}$. Given the circumstances, this value is the least upper bound for a sustainable capital stock in the sense that

$$\begin{aligned} &\text{if } K \geq \bar{K}, \text{ we have } \dot{K} < 0 \text{ for any } C > 0; \\ &\text{if } 0 < K < \bar{K}, \text{ we have } \dot{K} = 0 \text{ for } C = F(K, L, \bar{R}, 0) - \delta K > 0. \end{aligned}$$

For such a C, illustrated in Figure 16.2, a constant $Y = F(K, L, \bar{R}, 0)$ is maintained forever which implies non-decreasing per-capita income, $y \equiv Y/L$, forever. So, in spite of the limited availability of the natural resource, a *non-decreasing* level of consumption is technically feasible even without technical progress. A forever *growing* level of consumption will, of course, require sufficient technical progress capable of substituting for the natural resource.

[8] That is, marginal productivities of the production factors are positive, but diminishing, and the upper contour sets are strictly convex.

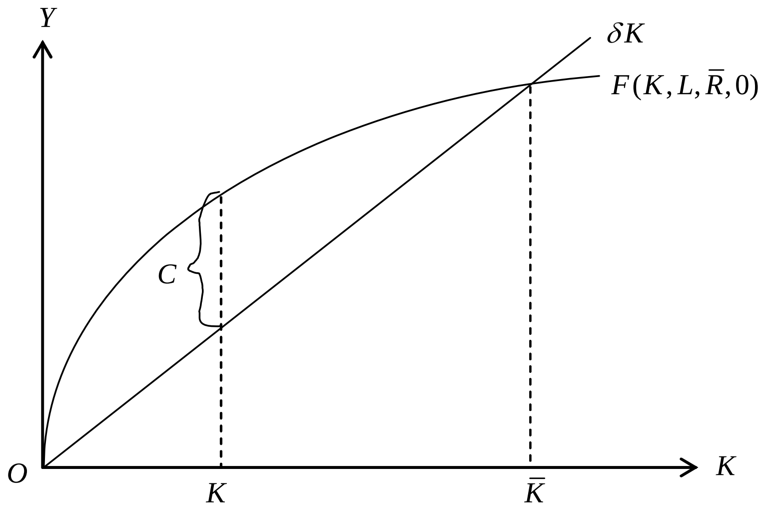

Figure 16.2: Sustainable consumption in the case of $n = 0$ and no technical progress (L and $\bar{R}$ fixed).

Now consider the case $n > 0$ and assume CRS w.r.t. K, L, and R. In view of CRS, we have

$$1 = F(\frac{K_t}{Y_t}, \frac{L_t}{Y_t}, \frac{\bar{R}}{Y_t}, t), \tag{16.4}$$

Along a balanced growth path with positive gross saving (if it exists) we know that K_t/Y_t and C_t/Y_t must be constant, cf. the balanced growth equivalence theorem of Chapter 4. Maintaining C_t/L_t $(= (C_t/Y_t)/(L_t/Y_t))$ constant along such a path, requires that L_t/Y_t is constant and thereby that Y_t grows at the rate n. But then $\bar{R}/Y_t$ will be declining over time. To compensate for this in (16.4), sufficient technical progress is necessary. This necessity of course is present, a fortiori, for sustained *growth* in per-capita consumption to occur.

As technical progress in the far future is by its very nature uncertain and unpredictable, there can be no guarantee for sustained per capita growth if there is sustained population growth.

Pollution As hinted at above, the concern that certain production methods involve pollution is commonly incorporated into economic analysis by subsuming environmental quality into the general notion of renewable resources. In that context S in (16.2) and Figure 16.1 will represent the "level of environmental quality" and R_t will be the amount of dirty emissions per time unit. Since the level of the environmental quality is likely to be an argument in both the utility function and the production function, again some limitation of the "extraction" (the pollution flow) is motivated. Pollution taxes may help to encourage abatement activities and make technical innovations towards cleaner production methods more profitable.

16.4 Non-renewable resources: The DHSS model

Whereas extraction and use of renewable resources can be sustained at a more or less constant level (if not too high), the situation is different with non-renewable resources. They have no natural regeneration process (at least not within a relevant time scale) and so continued extraction per time unit of these resources will inevitably have to decline and approach zero in the long run.

To get an idea of the implications, we will consider the Dasgupta-Heal-Solow-Stiglitz model (DHSS model) from the 1970s.[9] The production side of the model is described by:

$$\begin{aligned} Y_t &= F(K_t, L_t, R_t, t), \qquad \partial F/\partial t \geq 0, &(16.5)\\ \dot{K}_t &= Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \quad K_0 > 0 \text{ given}, &(16.6)\\ \dot{S}_t &= -R_t \equiv -u_t S_t, \qquad S_0 > 0 \text{ given}, &(16.7)\\ L_t &= L_0 e^{nt}, \qquad n \geq 0. &(16.8) \end{aligned}$$

The new element is the replacement of (16.2) with (16.7), where S_t is the *stock of the non-renewable* resource (e.g., oil reserves), and u_t is the depletion rate. Since we must have $S_t \geq 0$ for all t, there is a finite upper bound on cumulative resource extraction:

$$\int_0^\infty R_t dt \leq S_0. \tag{16.9}$$

Since the resource is non-renewable, no re-generation function appears in (16.7). Uncertainty is ignored and the extraction activity involves no costs.[10] As before, there is no distinction between employment and population, L_t.

The model was formulated as a response to the pessimistic Malthusian views expressed in the book *The Limits to Growth* written by MIT ecologists Meadows et al. (1972).[11] Stiglitz, and fellow economists, asked the

[9] See, e.g., Stiglitz, 1974.

[10] This simplified description of resource extraction is the reason that it is common to classify the model as a *one-sector* model, notwithstanding there are two productive activities in the economy, manufacturing and resource extraction.

[11] An up-date came in 2004, see Meadows at al. (2004).

question: what are the technological conditions needed to avoid falling per capita consumption in the long run in spite of the inevitable decline in the use of non-renewable resources? The answer is that there are three ways in which this decline in resource use may be counterbalanced:

1. input substitution;
2. resource-augmenting technical progress;
3. increasing returns to scale.

Let us consider each of them in turn (although in practice the three mechanisms tend to be intertwined).

16.4.1 Input substitution

By input substitution is here meant the gradual replacement of the input of the exhaustible natural resource by man-made input, capital. Substitution of fossil fuel energy by solar, wind, tidal and wave energy resources is an example. Similarly, more abundant lower-grade non-renewable resources can substitute for scarce higher-grade non-renewable resources - and this *will* happen when the scarcity price of these has become sufficiently high. A rise in the price of a mineral makes a synthetic substitute cost-efficient or lead to increased recycling of the mineral. Finally, the composition of final output can change toward goods with less material content. Overall, capital accumulation can be seen as the key background factor for such substitution processes (though also the arrival of new technical knowledge may be involved - we come back to this).

Whether capital accumulation can do the job depends crucially on the degree of substitutability between K and R. To see this, let the production function F be a three-factor CES production function. Suppressing the explicit dating of the variables when not needed for clarity, we have.

$$Y = \left(\alpha_1 K^\beta + \alpha_2 L^\beta + \alpha_3 R^\beta\right)^{1/\beta}, \ \alpha_1, \alpha_2, \alpha_3 > 0, \alpha_1 + \alpha_2 + \alpha_3 = 1, \beta < 1, \beta \neq 0. \tag{16.10}$$

We omit the the time index on Y, K, L, and R, when not needed for clarity. The important parameter is β, the *substitution parameter*. Let p_R denote the cost to the firm per unit of the resource flow and let $\hat{r}$ be the cost per unit of capital (generally, $\hat{r} = r + \delta$, where r is the real rate of interest). Then $p_R/\hat{r}$ is the relative factor price, which may be expected to increase as the resource becomes more scarce. The *elasticity of substitution* between K and R can be measured by $[d(K/R)/d(p_R/\hat{r})]\,(p_R/\hat{r}\ )/(K/R)$ evaluated along an isoquant curve, i.e., the percentage rise in the K-R ratio that a cost-minimizing firm will choose in response to a one-percent rise in the relative factor price, $p_R/\hat{r}$. Since we consider a CES production function, this elasticity is a constant $\sigma = 1/(1-\beta) > 0$. Indeed, the three-factor CES production function has the property that the elasticity of substitution between any pair of the three production factors is the same.

First, suppose $\sigma > 1$, i.e., $0 < \beta < 1$. Then, for fixed K and L, $Y \to \left(\alpha_1 K^\beta + \alpha_2 L^\beta\right)^{1/\beta} > 0$ when $R \to 0$. In this case of high substitutability the resource is seen to be *inessential* in the sense that it is not necessary for a positive output. That is, from a production perspective, conservation of the resource is not vital.

Suppose instead $\sigma < 1$, i.e., $\beta < 0$. Although increasing when R decreases, output per unit of the resource flow is then bounded from above. Consequently, the finiteness of the resource inevitably implies doomsday sooner or later if input substitution is the only salvage mechanism. To see this, keeping K and L fixed, we get

$$\frac{Y}{R} = Y(R^{-\beta})^{1/\beta} = \left[\alpha_1(\frac{K}{R})^\beta + \alpha_2(\frac{L}{R})^\beta + \alpha_3\right]^{1/\beta} \to {\alpha_3}^{1/\beta} \text{ for } R \to 0, \tag{16.11}$$

since $\beta < 0$. Even if K and L are increasing, $\lim_{R\to 0} Y = \lim_{R \to 0}(Y/R)R = \alpha_3^{1/\beta} \cdot 0 = 0$. Thus, when substitutability is low, the resource is *essential* in the sense that output is nil in the absence of the resource.

What about the intermediate case $\sigma = 1$? Although (16.10) is not defined for $\beta = 0$, using L'Hôpital's rule (as for the two-factor function, cf. Appendix), it can be shown that $\left(\alpha_1 K^\beta + \alpha_2 L^\beta + \alpha_3 R^\beta\right)^{1/\beta} \to K^{\alpha_1} L^{\alpha_2} R^{\alpha_3}$ for $\beta \to 0$. In the limit a three-factor Cobb-Douglas function thus appears. This function has $\sigma = 1$, corresponding to $\beta = 0$ in the formula $\sigma = 1/(1-\beta)$. The circumstances giving rise to the resource being essential thus include the Cobb-Douglas case $\sigma = 1$.

The interesting aspect of the Cobb-Douglas case is that it is the only case where the resource is essential while at the same time output per unit of the resource is unbounded from above (since $Y/R = K^{\alpha_1} L^{\alpha_2} R^{\alpha_3 - 1} \to \infty$ for $R \to 0$).[12] Under these circumstances it was considered an open question whether non-decreasing per capita consumption could be sustained. Therefore the Cobb-Douglas case was studied intensively. For example, Solow (1974) showed that if $n = \delta = 0$, then a necessary and sufficient condition that a constant positive level of consumption can be sustained is that $\alpha_1 > \alpha_3$.

[12] To avoid misunderstanding: by "Cobb-Douglas case" we refer to any function where R enters in a "Cobb-Douglas fashion", i.e., any function like $Y = \tilde{F}(K, L)^{1-\alpha_3} R^{\alpha_3}$.

This condition in itself seems fairly realistic, since, empirically, α_1 is many times the size of α_3 (Nordhaus and Tobin, 1972, Neumayer 2000). Solow added the observation that under competitive conditions, the *highest* sustainable level of consumption is obtained when investment in capital exactly equals the resource rent, $R \cdot \partial Y/\partial R$. This result was generalized in Hartwick (1977) and became known as *Hartwick's rule.* If there is population growth ($n > 0$), however, not even the Cobb-Douglas case allows sustainable per capita consumption unless there is sufficient technical progress, as equation (16.15) below will tell us.

Neumayer (2000) reports that the empirical evidence on the elasticity of substitution between capital and energy is inconclusive. Ecological economists tend to claim the poor substitution case to be much more realistic than the optimistic Cobb-Douglas case, not to speak of the case $\sigma > 1$. This invites considering the role of technical progress.

16.4.2 Technical progress

Solow (1974) and Stiglitz (1974) analyzed the theoretical possibility that resource-augmenting technological change can overcome the declining use of non-renewable resources that must be expected in the future. The focus is not only on whether a non-decreasing consumption level can be maintained, but also on the possibility of sustained per capita *growth* in consumption.

New production techniques may raise the efficiency of resource use. For example, Dasgupta (1993) reports that during the period 1900 to the 1960s, the quantity of coal required to generate a kilowatt-hour of electricity fell from nearly seven pounds to less than one pound.[13] Further, technological developments make extraction of lower quality ores cost-effective and make more durable forms of energy economical. On this background we incorporate resource-augmenting technical progress at the rate γ_3 and also allow labor-augmenting technical progress at the rate γ_2. So the CES production function now reads

$$Y_t = \left(\alpha_1 K_t^\beta + \alpha_2 (A_{2t} L_t)^\beta + \alpha_3 (A_{3t} R_t)^\beta\right)^{1/\beta}, \qquad (16.12)$$

where $A_{2t} = e^{\gamma_2 t}$ and $A_{3t} = e^{\gamma_3 t}$, considering $\gamma_2 \geq 0$ and $\gamma_3 > 0$ as exogenous constants. If the (proportionate) rate of decline of R_t is kept smaller than γ_3, then the "effective resource" input is no longer decreasing over time. As a consequence, even if $\sigma < 1$ (the poor substitution case), the finiteness of nature need not be an insurmountable obstacle to non-decreasing per capita consumption.

[13] For a historical account of energy technology, see Smil (1994).

Actually, a technology with $\sigma < 1$ *needs* a considerable amount of resource-augmenting technical progress to obtain compliance with the empirical fact that the income share of natural resources has *not* been rising (Jones, 2002). When $\sigma < 1$, market forces tend to increase the income share of the factor that is becoming relatively more scarce. Empirically, K/R and Y/R have increased systematically. However, with a sufficiently increasing A_3, the income share $p_R R/Y$ need not increase in spite of $\sigma < 1$. Compliance with Kaldor's "stylized facts" (more or less constant growth rates of K/L and Y/L and stationarity of the output-capital ratio, the income share of labor, and the rate of return on capital) can be maintained with moderate labor-augmenting technical change (A_2 growing over time). The motivation for not allowing a rising A_1 and replacing K in (16.12) by $A_1 K$, is that this would be at odds with Kaldor's "stylized facts", in particular the absence of a trend in the rate of return to capital.

With $\gamma_3 > \gamma_2 + n$, we end up with conditions allowing a *balanced growth path* (BGP for short), which we in the present context, with an essential resource, define as a path along which the quantities Y, C, K, R, and S are positive and change at constant proportionate rates (some or all of which may be negative). Given (16.12), it can be shown that along a BGP with positive gross saving, $Y/(A_2 L)$ is constant and so $g_y = \gamma_2$ (hence also $g_c = \gamma_2$).[14] There is thus scope for a positive g_y if $0 < \gamma_2 < \gamma_3 - n$.

Of course, one thing is that such a combination of assumptions allows for an upward trend in per capita consumption - which is what we have seen since the industrial revolution. Another thing is: will the needed assumptions be satisfied for a long time in the future? Since we have considered *exogenous* technical change, there is so far no hint from theory. But, even taking endogenous technical change into account, there will be many uncertainties about what kind of technological changes will come through in the future and how fast.

Balanced growth in the Cobb-Douglas case

The described results go through in a more straightforward way in the Cobb-Douglas case. So let us consider this. A convenience is that capital-augmenting, labor-augmenting, and resource-augmenting technical progress become indistinguishable and can thus be merged into one technology variable, the total factor productivity A_t :

$$Y_t = A_t K_t^{\alpha_1} L_t^{\alpha_2} R_t^{\alpha_3}, \qquad \alpha_1, \alpha_2, \alpha_3 > 0, \alpha_1 + \alpha_2 + \alpha_3 = 1, \tag{16.13}$$

[14] See Appendix B.

where we assume that A_t is growing at some constant rate $\tau > 0$. This, together with (16.6) - (16.8), is now the model under examination.

Log-differentiating w.r.t. time in (16.13) yields the growth-accounting relation

$$g_Y = \tau + \alpha_1 g_K + \alpha_2 n + \alpha_3 g_R. \tag{16.14}$$

In Appendix B it is shown that along a BGP with positive gross saving the following holds:

(i) $g_K = g_Y = g_C \equiv g_c + n$;
(ii) $g_R = g_S \equiv \dot{S}_t/S_t \equiv -R_t/S_t \equiv -\bar{u} =$ constant < 0;
(iii) nothing of the resource is left unutilized forever.

With constant depletion rate, R_t/S_t, denoted $\bar{u}$, along the BGP, (16.14) thus implies

$$g_c = g_y = \frac{1}{1-\alpha_1}(\tau - \alpha_3 n - \alpha_3 \bar{u}), \tag{16.15}$$

since $\alpha_1 + \alpha_2 - 1 = -\alpha_3$.

Absent the need for input of limited natural resources, we would have $\alpha_3 = 0$ and so $g_c = \tau/(1-\alpha_1)$. But with $\alpha_3 > 0$, the non-renewable resource is essential and implies a *drag on per capita growth* equal to $\alpha_3(n+\bar{u})/(1-\alpha_1)$. We get $g_c > 0$ if and only if $\tau > \alpha_3(n + \bar{u})$ (where, the constant depletion rate, $\bar{u}$, can in principle, from a social point of view, be chosen very small if we want a strict conservation policy).

It is noteworthy that in spite of per-capita growth being due to exogenous technical progress, (16.15) shows that there is scope for policy affecting the long-run per-capita growth rate to the extent that policy can affect the depletion rate u in the opposite direction.[15]

"Sustained growth" in K_t and c_t should not be understood in a narrow physical sense. As alluded to earlier, we have to understand K_t broadly as "produced means of production" of rising quality and falling material intensity; similarly, c_t must be seen as a composite of consumer "goods" with declining material intensity over time (see, e.g., Fagnart and Germain, 2011). This accords with the empirical fact that as income rises, the share of consumption expenditures devoted to agricultural and industrial products declines and the share devoted to services, hobbies, sports, and amusement increases. Although "economic development" is perhaps a more appropriate term (suggesting qualitative and structural change), we retain standard terminology and speak of "economic growth".

In any event, simple aggregate models like the present one should be seen as no more than a frame of reference, a tool for thought experiments.

[15] Cf. Section 13.5.1 of Chapter 13.

At best such models might have some validity as an approximate summary description of a certain period of time. One should be aware that an economy in which the ratio of capital to resource input grows without limit might well enter a phase where technological relations (including the elasticity of factor substitution) will be very different from now. For example, along *any* economic development path, the aggregate input of non-renewable resources must in the long run asymptotically approach zero. From a physical point of view, however, there must be some minimum amount of the resource below which it can not fulfil its role as a productive input. Thus, strictly speaking, sustainability requires that in the "very long run", non-renewable resources become inessential.

A backstop technology We end this sub-section by a remark on a rather different way of modeling resource-augmenting technical change. Dasgupta and Heal (1974) present a model of resource-augmenting technical change, considering it not as a smooth gradual process, but as something arriving in a discrete once-for-all manner with economy-wide consequences. The authors envision a future major discovery of, say, how to harness a lasting energy source such that a hitherto essential resource like fossil fuel becomes inessential. The contour of such a *backstop technology* might be currently known, but its practical applicability still awaits a technological breakthrough. The time until the arrival of this breakthrough is uncertain and may well be long. In Dasgupta, Heal and Majumdar (1977) and Dasgupta, Heal and Pand (1980) the idea is pursued further, by incorporating costly R&D. The likelihood of the technological breakthrough to appear in a given time interval depends positively on the accumulated R&D as well as the current R&D. It is shown that under certain conditions an index reflecting the probability that the resource becomes unimportant acts like an addition to the utility discount rate and that R&D expenditure begins to decline after some time. This is an interesting example of an early study of *endogenous* technological change.[16]

16.4.3 Increasing returns to scale

The third circumstance that might help overcoming the finiteness of nature is increasing returns to scale. For the CES function with poor substitution ($\sigma < 1$), however, increasing returns to scale, though helping, are not by themselves sufficient to avoid doomsday. For details, see, e.g., Groth (2007).

[16] A similar problem has been investigated by Kamien and Schwartz (1978) and Just et al. (2005), using somewhat different approaches.

16.4.4 Summary on the DHSS model

Apart from a few remarks by Stiglitz, the focus of the fathers of the DHSS model is on constant returns to scale; and, as in the simple Solow and Ramsey growth models, only *exogenous* technical progress is considered. For our purposes we may summarize the DHSS results in the following way. Non-renewable resources do not really matter seriously if the elasticity of substitution between them and man-made inputs is above one. If not, however, then:

(a) absent technical progress, if $\sigma = 1$, sustainable per capita consumption requires $\alpha_1 > \alpha_3$ and $n = 0 = \delta$; otherwise, declining per capita consumption is inevitable and this is definitely the prospect, if $\sigma < 1$;

(b) on the other hand, if there is enough resource-augmenting and labor-augmenting technical progress, non-decreasing per capita consumption and even growing per capita consumption may be sustained;

(c) population growth, implying more mouths to feed from limited natural resources, exacerbates the drag on growth implied by a declining resource input; indeed, as seen from (16.15), the drag on growth is $\alpha_3(n+u)/(1-\alpha_1)$ along a BGP.

16.4.5 An extended DHSS model

The above discussion of sustainable economic development in the presence of non-renewable resources was carried out on the basis of the original DHSS model with only capital, labor, and a non-renewable resource as inputs. In practice the issues of input substitution and technological change are to a large extent interweaved into the question of substitutability of non-renewable with renewable resources. A more natural point of departure for the discussion may therefore be an extended DHSS model of the form:

$$
\begin{aligned}
Y_t &= F(K_t, L_t, R_{rt}, R_{nt}, t), \qquad \partial F/\partial t \geq 0, \\
\dot{K}_t &= Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \quad K_0 > 0 \text{ given}, \\
\dot{S}_{rt} &= G(S_{rt}) - R_{rt}, \qquad S_{r0} > 0 \text{ given}, \\
\dot{S}_{nt} &= -R_{nt}, \qquad S_{n0} > 0 \text{ given}, \\
\int_0^\infty R_{nt} dt &\leq S_{n0},
\end{aligned}
$$

where R_{rt} is input of the renewable resource and S_{rt} the corresponding stock, while R_{nt} is input of the non-renewable resource to which corresponds the stock S_{nt}. Only the non-renewable resource is subject to the constraint of a finite upper bound on cumulative resource extraction.

Within such a framework a more or less gradual transition from use of non-renewable energy forms to renewable energy forms (hydro-power, wind energy, solar energy, biomass, and geothermal energy), likely speeded up learning by doing as well as R&D, can be studied (see for instance Tahvonen and Salo, 2001).

16.5 Endogenous technical change

The obvious next step is to examine how *endogenizing* technical change may throw new light on the issues relating to non-renewable resources, in particular the visions (b) and (c). Because of the non-rival character of technical knowledge, endogenizing knowledge creation may have profound implications, in particular concerning point (c). Indeed, the relationship between population growth and economic growth may be circumvented when endogenous creation of ideas (implying a form of increasing returns to scale) is considered.

16.5.1 A two-sector R&D-based model

We shall look at the economy from the perspective of a fictional social planner who cares about finding a resource allocation so as to maximize the intertemporal utility function of a representative household subject to technical feasibility as given from the initial technology and initial resources.

In addition to cost-free resource extraction, there are two "production" sectors, the manufacturing sector and the R&D sector. In the manufacturing sector the aggregate production function is

$$Y_t = A_t^{\varepsilon} K_t^{\alpha} L_t^{\beta} R_t^{\gamma} J^{\lambda}, \qquad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma + \lambda = 1, \tag{16.16}$$

whereY_t is output of manufacturing goods, while K_t, L_t, R_t, and J are inputs of capital, labor, a non-renewable resource, and land (a renewable resource), respectively, per time unit at time t. The amount of land is an exogenous constant. Total factor productivity is A_t^{ε} where the variable A_t is assumed proportional to the stock of technical knowledge accumulated through R&D investment. Due to this proportionality we can simply identify A_t with the stock of knowledge at time t.

Aggregate manufacturing output is used for consumption, C_t, investment, I_{Kt}, in physical capital, and investment, I_{At}, in R&D,

$$C_t + I_{Kt} + I_{At} = Y_t.$$

Accumulation of capital occurs according to

$$\dot{K}_t = I_{Kt} - \delta_K K_t = Y_t - C_t - I_{At} - \delta_K K_t, \qquad \delta_K \geq 0, \qquad K_0 > 0 \text{ given,} \tag{16.17}$$

where δ_K is the (exogenous) rate of depreciation (decay) of capital.

In the R&D sector additions to the stock of technical knowledge are created through R&D investment, I_A :

$$\dot{A}_t = I_{At} - \delta_A A_t, \qquad \delta_A \geq 0 \qquad A_0 > 0 \text{ given.} \tag{16.18}$$

We allow for a positive depreciation rate, δ_A, to take into account the possibility that as technology advances, old knowledge becomes obsolete and then over time gradually becomes useless in production.

Extraction of the non-renewable resource is again given by

$$\dot{S}_t = -R_t \equiv -u_t S_t, \qquad S_0 > 0 \text{ given,} \tag{16.19}$$

where S_t is the stock of the non-renewable resource (e.g., oil reserves) and u_t is the depletion rate. Since we must have $S_t \geq 0$ for all t, there is a finite upper bound on cumulative resource extraction:

$$\int_0^\infty R_t dt \leq S_0. \tag{16.20}$$

Finally, population (= labor force) grows according to

$$L_t = L_0 e^{nt}, \qquad n \geq 0, \qquad L_0 > 0 \text{ given.}$$

Uncertainty is ignored and the extraction activity involves no costs.

We see that the setup is elementarily related to the simple lab-equipment model (Chapter 14) since by investing part of the manufacturing output new knowledge is directly generated without intervention by researchers and similar.[17] Note also that there are no intertemporal knowledge-spillovers.

16.5.2 Analysis

We now skip the explicit dating of the variables where not needed for clarity. The model has *three* state variables, the stock, K, of physical capital, the stock, S, of non-renewable resources, and the stock, A, of technical knowledge. To simplify the dynamics, we will concentrate on the special case

[17] An interpretation is that part of the activity in the manufacturing sector is directly R&D activity using the same technology (production function) as is used in the production of consumption goods and capital goods.

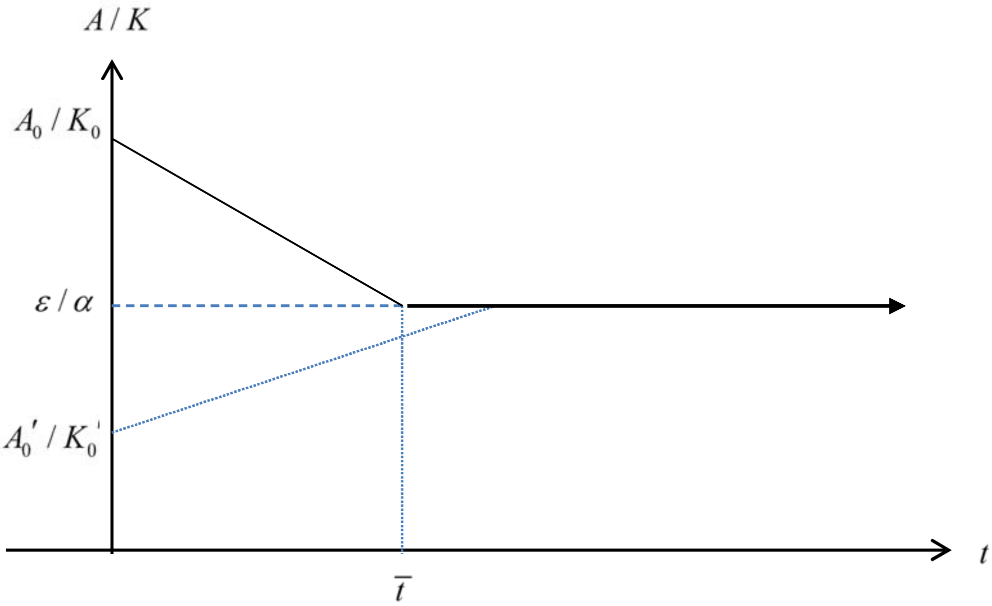

Figure 16.3: Initial complete specialization followed by balanced growth.

$\delta_A = \delta_K = \delta$. In this case, as we shall see, after an initial adjustment period, the economy behaves in many respects similarly to a reduced-form AK model.

Let us first consider *efficient paths*, i.e., paths such that aggregate consumption can not be increased in some time interval without being decreased in another time interval. The net marginal productivities of A and K are equal if and only if $\varepsilon Y/A - \delta = \alpha Y/K - \delta$, i.e., if and only if

$$A/K = \varepsilon/\alpha.$$

The initial stocks, A_0 and K_0 are historically given. Suppose $A_0/K_0 > \varepsilon/\alpha$ as in Figure 16.3. Then, initially, the net marginal productivity of capital is larger than that of knowledge, i.e., capital is relatively scarce. An efficient economy will therefore for a while invest only in capital, i.e., there will be a phase where $I_A = 0$. This phase of complete specialization lasts until $A/K = \varepsilon/\alpha$, a state reached in finite time, say at time $\bar{t}$, cf. the figure. Hereafter, there is investment in both assets so that their ratio remains equal to the efficient ratio ε/α forever. Similarly, if initially $A_0/K_0 < \varepsilon/\alpha$, then there will be a phase of complete specialization in R&D, and after a finite time interval the efficient ratio $A/K = \varepsilon/\alpha$ is achieved and maintained forever.

For $t > \bar{t}$ it is thus as if there were only one kind of capital, which we may call "broad capital" and define as $\tilde{K} = K + A = (\alpha + \varepsilon)K/\alpha$. Indeed, substitution of $A = \varepsilon K/\alpha$ and $K = \alpha\tilde{K}/(\varepsilon + \alpha)$ into (16.16) gives

$$Y = \frac{\varepsilon^{\varepsilon}\alpha^{\alpha}}{(\varepsilon + \alpha)^{\varepsilon+\alpha}}\tilde{K}^{\varepsilon+\alpha}L^{\beta}R^{\gamma} \equiv B\tilde{K}^{\tilde{\alpha}}L^{\beta}R^{\gamma}, \qquad \tilde{\alpha} \equiv \alpha + \varepsilon, \tag{16.21}$$

so that $\tilde{\alpha} + \beta + \gamma > 1$. Further, adding (16.18) and (16.17) gives

$$\dot{\tilde{K}} = \dot{A} + \dot{K} = Y - cL - \delta\tilde{K}, \tag{16.22}$$

where c is per capita consumption.

We now proceed with a model based on broad capital, using (16.21), (16.22) and the usual resource depletion equation (16.19). Essentially, this model amounts to an extended DHSS model allowing increasing returns to scale, thereby offering a simple framework for studying *endogenous* growth with essential non-renewable resources.

We shall focus on questions like:

1 Is sustainable development (possibly even growth) possible within the model?

2 Can the utilitarian principle of discounted utility maximizing possibly clash with a requirement of sustainability? If so, under what conditions?

3 How can environmental policy be designed so as to enhance the prospects of sustainable development or even sustainable economic growth?

Balanced growth

Log-differentiating (16.21) w.r.t. t gives the "growth-accounting equation"

$$g_Y = \tilde{\alpha}g_{\tilde{K}} + \beta n + \gamma g_R. \tag{16.23}$$

Hence, along a BGP we get

$$(1 - \tilde{\alpha})g_c + \gamma u = (\tilde{\alpha} + \beta - 1)n. \tag{16.24}$$

Since $u > 0$, it follows immediately that:

Result (i) A BGP has $g_c > 0$ if and only if

$$(\tilde{\alpha} + \beta - 1)n > 0 \qquad \text{or} \qquad \tilde{\alpha} > 1. \tag{16.25}$$

Proof. Since $\gamma u > 0$, (*) implies $(1-\tilde{\alpha})g_c < (\tilde{\alpha}+\beta-1)n$. Hence, if $g_c > 0$, either $\tilde{\alpha} > 1$ or ($\tilde{\alpha} \leq 1$ and $(\tilde{\alpha}+\beta-1)n > 0$). This proves "only if". The "if" part is more involved but follows from Proposition 2 in Groth (2004). □

Result (i) tells us that endogenous growth is theoretically possible, if there are either increasing returns to the capital-*cum*-labor input combined with population growth *or* increasing returns to capital (broad capital) itself. At least one of these conditions is required in order for capital accumulation to offset the effects of the inescapable waning of resource use over time. Based on Nordhaus (1992), $\alpha \approx 0.2, \beta \approx 0.6, \gamma \approx 0.1$, and $\lambda \approx 0.1$ seem reasonable. Given these numbers,

(i) semi-endogenous growth requires $(\varepsilon+\alpha+\beta-1)n > 0$, hence $\varepsilon > 0.20$;

(ii) fully endogenous growth requires $\varepsilon + \alpha > 1$, hence $\varepsilon > 0.80$.

We have defined *fully endogenous growth* to be present if the long-run growth rate in per capita output is positive without the support of growth in any exogenous factor. Result (i) shows that only if $\tilde{\alpha} > 1$, is *fully* endogenous growth possible. Although the case $\tilde{\alpha} > 1$ has potentially explosive effects on the economy, if $\tilde{\alpha}$ is not too much above 1, these effects can be held back by the strain on the economy imposed by the declining resource input.

In some sense this is "good news": fully endogenous steady growth is theoretically possible and no knife-edge assumption is needed. As we have seen in earlier chapters, in the conventional framework without non-renewable resources, fully endogenous growth requires constant returns to the producible input(s) in the growth engine. In our one-sector model the growth engine is the manufacturing sector itself, and without the essential non-renewable resource, fully endogenous growth would require the knife-edge condition $\tilde{\alpha} = 1$ ($\tilde{\alpha}$ being above 1 is excluded in this case, because it would lead to explosive growth in a setting without some countervailing factor). When non-renewable resources are an essential input in the growth engine, a drag on the growth potential is imposed. To be able to offset this drag, fully endogenous growth requires *increasing* returns to capital.

The "bad news" is, however, that even in combination with essential non-renewable resources, an assumption of increasing returns to capital seems too strong and too optimistic. A technology having $\tilde{\alpha}$ just slightly above 1 can sustain *any* per capita growth rate − there is no upper bound on g_c.[18] This appears overly optimistic.

This leaves us with *semi-endogenous* growth as the only plausible form of endogenous growth (as long as n is not endogenous). Indeed, Result (i)

[18] This is shown in Groth (2004).

indicates that semi-endogenous growth corresponds to the case $1-\beta < \tilde{\alpha} \leq 1$. In this case sustained positive per capita growth driven by some internal mechanism is possible, but only if supported by $n > 0$, that is, by growth in an exogenous factor, here population size.

Growth policy and conservation

Result (i) is only about whether the technology as such allows a positive per capita growth rate or not. What about the *size* of the growth rate? Can the growth rate temporarily or perhaps permanently be affected by economic policy? The simple growth-accounting relation (16.24) immediately shows:

Result (ii) Along a BGP, policies that decrease (increase) the depletion rate u (and only such policies) will increase (decrease) the per capita growth rate (here we presuppose $\tilde{\alpha} < 1$, the plausible case).

This observation is of particular interest in view of the fact that changing the perspective from exogenous to endogenous technical progress implies bringing a source of numerous market failures to light. On the face of it, the result seems to run against common sense. Does high growth not imply *fast* depletion (high u)? Indeed, the answer is affirmative, but with the addition that exactly because of the fast depletion such high growth will only be temporary – it carries the seeds to its own obliteration. For faster sustained growth there must be sustained slower depletion. The reason for this is that with protracted depletion, the rate of decline in resource input becomes smaller. Hence, so does the drag on growth caused by this decline.

As a statement about policy and long-run growth, (ii) is a surprisingly succinct conclusion. It can be clarified in the following way. For policy to affect long-run growth, it must affect a linear differential equation linked to the basic goods sector in the model. In the present framework the resource depletion relation,

$$\dot{S} = -uS,$$

is such an equation. In balanced growth $g_S = -R/S \equiv -u$ is constant, so that the proportionate rate of decline in R must comply with, indeed be equal to, that of S. Through the growth accounting relation (16.23), given u, this fixes g_Y and $g_{\tilde{K}}$ (equal in balanced growth), hence also $g_c = g_Y - n$.

The conventional wisdom in the endogenous growth literature is that interest income taxes impede economic growth and investment subsidies promote economic growth. Interestingly, this need not be so when non-renewable resources are an essential input in the growth engine (which is here the manufacturing sector itself). At least, starting from a Cobb-Douglas aggregate production function as in (16.16), it can be shown that only those policies that interfere with the depletion rate u in the long run (like a profits tax on resource-extracting companies or a time-dependent tax on resource use) can affect long-run growth. It is noteworthy that this long-run policy result holds whether $g_c > 0$ or not and whether growth is exogenous, semi-endogenous or fully endogenous.[19] The general conclusion is that with non-renewable resources entering the growth-generating sector in an essential way, conventional policy tools receive a different role and there is a role for new tools (affecting long-run growth through affecting the depletion rate).[20]

Introducing preferences

To be more specific we introduce household preferences and a social planner. The resulting resource allocation will coincide with that of a decentralized competitive economy if agents have perfect foresight and the government has introduced appropriate subsidies and taxes. As in Stiglitz (1974a), let the utilitarian social planner choose a path $(c_t, R_t)_{t=0}^{\infty}$ so as to optimize

$$U_0 = \int_0^{\infty} \frac{{c_t}^{1-\theta}}{1-\theta} L_t e^{-\rho t} dt, \qquad \theta > 0, \tag{16.26}$$

subject to the constraints given by technology, i.e., (16.21), (16.22), and (16.19), and initial conditions. The parameter $\theta > 0$ is the (absolute) elasticity of marginal utility of consumption (reflecting the strength of the desire for consumption smoothing) and ρ is a constant rate of time preference.[21]

Using the Maximum Principle, the first-order conditions for this problem lead to, first, the social planner's *Keynes-Ramsey* rule,

$$g_c = \frac{1}{\theta}(\frac{\partial Y}{\partial \tilde{K}} - \delta - \rho) = \frac{1}{\theta}(\tilde{\alpha}\frac{Y}{\tilde{K}} - \delta - \rho), \tag{16.27}$$

second, the social planner's *Hotelling rule,*

$$\frac{d(\partial Y/\partial R)}{dt} = \frac{\partial Y}{\partial R}(\frac{\partial Y}{\partial \tilde{K}} - \delta) = \gamma\frac{Y}{R}(\tilde{\alpha}\frac{Y}{\tilde{K}} - \delta). \tag{16.28}$$

The Keynes-Ramsey rule says: as long as the net return on investment in capital is higher than the rate of time preference, one should let current c be

[19] This is a reminder that the distinction between fully endogenous growth and semi-endogenous growth is not the same as the distinction between policy-dependent and policy-invariant growth.

[20] These aspects are further explored in Groth and Schou (2006).

[21] For simplicity we have here ignored (as does Stiglitz) that also environmental quality should enter the utility function.

low enough to allow positive net saving (investment) and thereby higher consumption in the future. The Hotelling rule is a no-arbitrage condition saying that the return ("capital gain") on leaving the marginal unit of the resource in the ground must equal the return on extracting and using it in production and then investing the proceeds in the alternative asset (reproducible capital).[22]

After the initial phase of complete specialization described above, we have, due to the proportionality between K, A and $\tilde{K}$, that $\partial Y/\partial K = \partial Y/\partial A = \partial Y/\partial \tilde{K} = \tilde{\alpha}Y/\tilde{K}$. Notice that the Hotelling rule is independent of preferences; *any* path that is *efficient* must satisfy the Hotelling rule (as well as the exhaustion condition $\lim_{t\to\infty} S(t) = 0$).

Using the Cobb-Douglas specification, we may rewrite the Hotelling rule as $g_Y - g_R = \tilde{\alpha}Y/\tilde{K} - \delta$. Along a BGP $g_Y = g_C = g_c + n$ and $g_R = -u$, so that the Hotelling rule combined with the Ramsey rule gives

$$(1-\theta)g_c + u = \rho - n. \tag{16.29}$$

This linear equation in g_c and u combined with the growth-accounting relationship (16.24) constitutes a linear two-equation system in the growth rate and the depletion rate. The determinant of this system is $D \equiv 1 - \tilde{\alpha} - \gamma + \theta\gamma$. We assume $D > 0$, which seems realistic and is in any case necessary (and sufficient) for stability.[23] Then

$$g_c = \frac{(\tilde{\alpha}+\beta+\gamma-1)n - \gamma\rho}{D}, \quad \text{and} \tag{16.30}$$

$$u = \frac{[(\tilde{\alpha}+\beta-1)\theta - \beta]\, n + (1-\tilde{\alpha})\rho}{D}. \tag{16.31}$$

To ensure boundedness from above of the utility integral (16.26) we need the parameter restriction $\rho - n > (1-\theta)g_c$, which we assume satisfied for g_c as given in (16.30).

Interesting implications are:

[22] After Hotelling (1931), who considered the rule in a market economy. Assuming perfect competition, the real resource price is $p_R = \partial Y/\partial R$ and the real rate of interest is $r = \partial Y/\partial K - \delta$. Then the rule takes the form $\dot{p}_R/p_R = r$. If there are extraction costs at rate $C(R, S, t)$, then the rule takes the form $\dot{p}_S - \partial C/\partial S = rp_S$, where p_S is the price of the unextracted resource (whereas $p_R = p_S + \partial C/\partial R$).

It is another matter that the rise in resource prices and the predicted decline in resource use have not yet shown up in the data (Krautkraemer 1998, Smil 2003); this may be due to better extraction technology and discovery of new deposits. But in the long run, if non-renewable resources *are* essential, this tendency inevitably will be reversed.

[23] As argued above, $\tilde{\alpha} < 1$ seems plausible. Generally, θ is estimated to be greater than one (see, e.g., Attanasio and Weber 1995); hence $D > 0$. The stability result as well as other findings reported here are documented in Groth and Schou (2002).

Result (iii) If there is impatience ($\rho > 0$), then even when a non-negative g_c is technically feasible (i.e., (16.25) satisfied), a negative g_c can be optimal and stable.

Result (iv) Population growth is *good* for economic growth. In its absence, when $\rho > 0$, we get $g_c < 0$ along an optimal BGP; if $\rho = 0$, $g_c = 0$ when $n = 0$.

Result (v) There is never a scale effect on the growth rate.

Result (iii) reflects that utility discounting and consumption smoothing weaken the "growth incentive".

Result (iv) is completely contrary to the conventional (Malthusian) view and the learning from the DHSS model. The point is that two offsetting forces are in play. On the one hand, higher n means more mouths to feed and thus implies a drag on per capita growth (Malthus). On the other hand, a growing labour force is exactly what is needed in order to exploit the benefits of increasing returns to scale (anti-Malthus). And at least in the present framework this dominates the first effect. This feature might seem to be contradicted by the empirical finding that there is no robust correlation between g_c and population growth in cross-country regressions (Barro and Sala-i-Martin 2004, Ch. 12). However, the proper unit of observation in this context is not the individual country. Indeed, in an internationalized world with technology diffusion a positive association between n and g_c as in (16.30) should not be seen as a prediction about individual countries, but rather as pertaining to larger regions, perhaps the global economy. In any event, the second part of Result (iv) is a dismal part − in view of the projected long-run stationarity of world population (United Nations 2005).

A somewhat surprising result appears if we imagine (unrealistically) that $\tilde{\alpha}$ is sufficiently above one to make D a negative number. If population growth is absent, $D < 0$ is in fact needed for $g_c > 0$ along a BGP. However, $D < 0$ implies instability. Hence this would be a case of an instable BGP with fully endogenous growth.[24]

As to Result (v), it is noteworthy that the absence of a scale effect on growth holds for *any* value of $\tilde{\alpha}$, including $\tilde{\alpha} = 1$.[25]

[24] Thus, if we do not require $D > 0$ in the first place, (iv) could be reformulated as: existence of a *stable* optimal BGP with $g_c > 0$ *requires* $n > 0$. This is not to say that reducing n from positive to zero renders an otherwise stable BGP instable. Stability-instability is governed solely by the sign of D. Given $D > 0$, letting n decrease from a level above the critical value, $\gamma\rho/(\tilde{\alpha} + \beta + \gamma - 1)$, given from (16.30), to a level below, changes g_c from positive to negative, i.e., growth comes to an end.

[25] More commonplace observations are that increased impatience leads to faster depletion

A pertinent question is: are the above results just an artifact of the very simplified reduced-form AK-style set-up applied here? The answer turns out to be no. For models with a distinct research technology and intertemporal knowledge spillovers, this is shown in Groth (2007).

16.6 Natural resources and the issue of limits to economic growth

Two distinguished professors were asked by a journalist: Are there limits to economic growth?
The answers received were:[26]

Clearly YES:

- A finite planet!
- The amount of cement, oil, steel, and water that we can use is limited!

Clearly NO:

- Human creativity has no bounds!
- The quality of wine, TV transmission of concerts, computer games, and medical treatment knows no limits!

An aim of this chapter has been to bring to mind that it would be strange if there were no limits to growth. So a better question is:

What determines the limits to economic growth?

The answer suggested is that these limits are determined by the capability of the economic system to substitute limited natural resources by man-made goods the variety and quality of which are expanded by creation of new ideas. In this endeavour frontier countries, first the U.K. and Europe, next the United States, have succeeded at a high rate for two and a half century. To what extent this will continue in the future nobody knows. Some economists, e.g. Gordon (2012), argue there is an enduring tendency to slowing down of innovation and economic growth (the low-hanging fruits have been taken).

and lower growth (in the plausible case $\tilde{a} < 1$). Further, in the log-utility case ($\theta = 1$) the depletion rate u equals the effective rate of impatience, $\rho - n$.

[26] Inspired by Sterner (2008).

Others, e.g. Brynjolfsson and McAfee (2014), disagree. They reason that the potentials of information technology and digital communication are on the verge of the point of ubiquity and flexible application. For these authors the prospect is "The Second Machine Age" (the title of their book), by which they mean a new innovative epoch where smart machines and new ideas are combined and recombined - with pervasive influence on society.

16.7 Bibliographic notes

It is not always recognized that the research of the 1970s on macro implications of essential natural resources in limited supply already laid the groundwork for a theory of endogenous and policy-dependent growth with natural resources. Actually, by extending the DHSS model, Suzuki (1976), Robson (1980) and Takayama (1980) studied how endogenous innovation may affect the prospect of overcoming the finiteness of natural resources.

Suzuki's (1976) article contains an additional model, involving a resource externality. Interpreting the externality as a "greenhouse effect", Sinclair (1992, 1994) and Groth and Schou (2006) pursue this issue further. In the latter paper a configuration similar to the model in Section 16.5 is studied. The source of increasing returns to scale is not intentional creation of knowledge, however, but learning as a by-product of investing as in Arrow (1962a) and Romer (1986). Empirically, the evidence furnished by, e.g., Hall (1990) and Caballero and Lyons (1992) suggests that there are quantitatively significant increasing returns to scale w.r.t. capital and labour or external effects in US and European manufacturing. Similarly, Antweiler and Trefler (2002) examine trade data for goods-producing sectors and find evidence for increasing returns to scale.

Concerning Result (i), note that if some irreducibly exogenous element in the technological development is allowed in the model by replacing the constant B in (16.21) by $e^{\tau t}$, where $\tau \geq 0$, then (16.25) is replaced by $\tau + (\tilde{\alpha}+\beta-1)n > 0$ or $\tilde{\alpha} > 1$. Both Stiglitz (1974a, p. 131) and Withagen (1990, p. 391) ignore implicitly the possibility $\tilde{\alpha} > 1$. Hence, from the outset they preclude fully endogenous growth.

16.8 Appendix

16.8.1 A. The CES function

The CES (Constant Elasticity of Substitution) function is used in consumer theory as a specification of preferences and in production theory as a specification of a production function. Here we consider it as a production function.

It can be shown[27] that if a neoclassical production function with CRS has a constant elasticity of (factor) substitution different from one, it must be of the form

$$Y = A\left[\alpha K^{\beta} + (1-\alpha)L^{\beta}\right]^{\frac{1}{\beta}}, \tag{16.32}$$

where A, α, and β are parameters satisfying $A > 0$, $0 < \alpha < 1$, and $\beta < 1$, $\beta \neq 0$. This function has been used intensively in empirical studies and is called a *CES production function.* For a given choice of measurement units, the parameter A reflects efficiency and is thus called the *efficiency parameter.* The parameters α and β are called the *distribution parameter* and the *substitution parameter,* respectively. The restriction $\beta < 1$ ensures that the isoquants are strictly convex to the origin. Note that if $\beta < 0$, the right-hand side of (16.32) is not defined when either K or L (or both) equal 0. We can circumvent this problem by extending the domain of the CES function and assign the function value 0 to these points when $\beta < 0$. Continuity is maintained in the extended domain.

By taking partial derivatives in (16.32) and substituting back we get

$$\frac{\partial Y}{\partial K} = \alpha A^{\beta}\left(\frac{Y}{K}\right)^{1-\beta} \quad \text{and} \quad \frac{\partial Y}{\partial L} = (1-\alpha)A^{\beta}\left(\frac{Y}{L}\right)^{1-\beta}, \tag{16.33}$$

where $Y/K = A\left[\alpha + (1-\alpha)k^{-\beta}\right]^{\frac{1}{\beta}}$ and $Y/L = A\left[\alpha k^{\beta} + 1 - \alpha\right]^{\frac{1}{\beta}}$. The marginal rate of substitution of K for L therefore is

$$MRS = \frac{\partial Y/\partial L}{\partial Y/\partial K} = \frac{1-\alpha}{\alpha}k^{1-\beta} > 0.$$

Consequently,

$$\frac{dMRS}{dk} = \frac{1-\alpha}{\alpha}(1-\beta)k^{-\beta},$$

where the inverse of the right-hand side is the value of $dk/dMRS$. Substituting these expressions into the general definition of the *elasticity of substitution* between capital and labor, evaluated at the point (K, L),

$$\tilde{\sigma}(K,L) = \frac{MRS}{K/L}\frac{d(K/L)}{dMRS}_{|Y=\bar{Y}} = \frac{\frac{d(K/L)}{K/L}}{\frac{dMRS}{MRS}}_{|Y=\bar{Y}}. \tag{16.34}$$

gives

$$\tilde{\sigma}(K,L) = \frac{1}{1-\beta} \equiv \sigma, \tag{16.35}$$

[27] See, e.g., Arrow et al. (1961).

confirming the constancy of the elasticity of substitution, given (16.33). Since $\beta < 1$, $\sigma > 0$ always. A higher substitution parameter, β, results in a higher elasticity of substitution, σ. And $\sigma \lesseqgtr 1$ for $\beta \lesseqgtr 0$, respectively.

Since $\beta = 0$ is not allowed in (16.32), at first sight we cannot get $\sigma = 1$ from this formula. Yet, $\sigma = 1$ can be introduced as the *limiting* case of (16.32) when $\beta \rightarrow 0$, which turns out to be the Cobb-Douglas function. Indeed, one can show[28] that, for fixed K and L,

$$A\left[\alpha K^{\beta} + (1-\alpha)L^{\beta}\right]^{\frac{1}{\beta}} \rightarrow AK^{\alpha}L^{1-\alpha}, \text{ for } \beta \rightarrow 0.$$

By a similar procedure as above we find that a Cobb-Douglas function always has elasticity of substitution equal to 1; this is exactly the value taken by σ in (16.35) when $\beta = 0$. In addition, the Cobb-Douglas function is the *only* production function that has unit elasticity of substitution everywhere.

Another interesting limiting case of the CES function appears when, for fixed K and L, we let $\beta \rightarrow -\infty$ so that $\sigma \rightarrow 0$. We get

$$A\left[\alpha K^{\beta} + (1-\alpha)L^{\beta}\right]^{\frac{1}{\beta}} \rightarrow A\min(K, L), \text{ for } \beta \rightarrow -\infty. \tag{16.36}$$

So in this case the CES function approaches a Leontief production function, the isoquants of which form a right angle, cf. Figure 16.4. In the limit there is *no* possibility of substitution between capital and labor. In accordance with this the elasticity of substitution calculated from (16.35) approaches zero when β goes to $-\infty$.

Finally, let us consider the "opposite" transition. For fixed K and L we let the substitution parameter rise towards 1 and get

$$A\left[\alpha K^{\beta} + (1-\alpha)L^{\beta}\right]^{\frac{1}{\beta}} \rightarrow A\left[\alpha K + (1-\alpha)L\right], \text{ for } \beta \rightarrow 1.$$

Here the elasticity of substitution calculated from (16.35) tends to ∞ and the isoquants tend to straight lines with slope $-(1-\alpha)/\alpha$. In the limit, the production function thus becomes linear and capital and labor become *perfect substitutes.*

Figure 16.4 depicts isoquants for alternative CES production functions and their limiting cases. In the Cobb-Douglas case, $\sigma = 1$, the horizontal and vertical asymptotes of the isoquant coincide with the coordinate axes. When $\sigma < 1$, the horizontal and vertical asymptotes of the isoquant belong to the interior of the positive quadrant. This implies that both capital and labor are essential inputs. When $\sigma > 1$, the isoquant terminates in points

[28]For proofs of this and the further claims below, see Appendix E of Chapter 4 in Groth (2014).

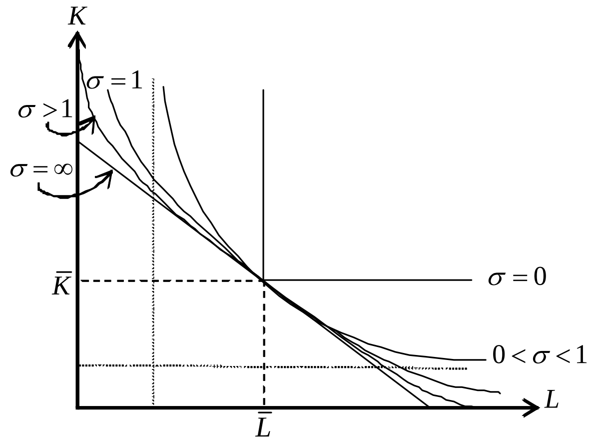

Figure 16.4: Isoquants for the CES production function for alternative values of $\sigma = 1/(1-\beta)$.

on the coordinate axes. Then neither capital, nor labor are essential inputs. Empirically there is not complete agreement about the "normal" size of the elasticity of factor substitution for industrialized economies. The elasticity also differs across the production sectors. A recent thorough econometric study (Antràs, 2004) of U.S. data indicate the aggregate elasticity of substitution to be in the interval $(0.5, 1.0)$.

The CES production function in intensive form

Dividing through by L on both sides of (16.32), we obtain the CES production function in intensive form,

$$y \equiv \frac{Y}{L} = A(\alpha k^{\beta} + 1 - \alpha)^{\frac{1}{\beta}}, \tag{16.37}$$

where $k \equiv K/L$. The marginal productivity of capital can be written

$$MPK = \frac{dy}{dk} = \alpha A \left[\alpha + (1-\alpha)k^{-\beta}\right]^{\frac{1-\beta}{\beta}} = \alpha A^{\beta}\left(\frac{y}{k}\right)^{1-\beta},$$

which of course equals $\partial Y/\partial K$ in (16.33). We see that the CES function violates either the lower or the upper Inada condition for MPK, depending on the sign of β. Indeed, when $\beta < 0$ (i.e., $\sigma < 1$), then for $k \to 0$ both y/k and dy/dk approach an upper bound equal to $A\alpha^{1/\beta} < \infty$, thus violating the lower Inada condition for MPK (see the right-hand panel of Figure 2.3 in Chapter 2). It is also noteworthy that in this case, for $k \to \infty$, y approaches

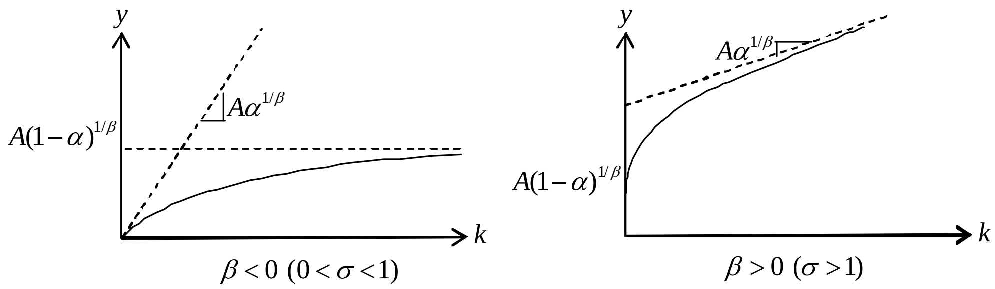

Figure 16.5: The CES production function for $\sigma < 1$ (left panel) and $\sigma > 1$ (right panel).

an upper bound equal to $A(1-\alpha)^{1/\beta} < \infty$. These features reflect the low degree of substitutability when $\beta < 0$.

When instead $\beta > 0$, there is a high degree of substitutability ($\sigma > 1$). Then, for $k \to \infty$ both y/k and $dy/dk \to A\alpha^{1/\beta} > 0$, thus violating the upper Inada condition for MPK (see the right-hand panel of Figure 16.5). It is also noteworthy that for $k \to 0$, y approaches a positive lower bound equal to $A(1-\alpha)^{1/\beta} > 0$. Thus, in this case capital is not essential. At the same time $dy/dk \to \infty$ for $k \to 0$ (so the lower Inada condition for the marginal productivity of capital holds).

The marginal productivity of labor is

$$MPL = \frac{\partial Y}{\partial L} = (1-\alpha)A(\alpha k^{\beta} + 1 - \alpha)^{(1-\beta)/\beta} \equiv w(k),$$

from (16.33).

Since (16.32) is symmetric in K and L, we get a series of symmetric results by considering output per unit of capital as $x \equiv Y/K = A\left[\alpha + (1-\alpha)(L/K)^{\beta}\right]^{1/\beta}$. In total, therefore, when there is low substitutability ($\beta < 0$), for fixed input of either of the production factors, there is an upper bound for how much an unlimited input of the other production factor can increase output. And when there is high substitutability ($\beta > 0$), there is no such bound and an unlimited input of either production factor take output to infinity.

The Cobb-Douglas case, i.e., the limiting case for $\beta \to 0$, constitutes in several respects an intermediate case in that *all* four Inada conditions are satisfied and we have $y \to 0$ for $k \to 0$, and $y \to \infty$ for $k \to \infty$.

Generalizations

The CES production function considered above has CRS. By adding an elasticity of scale parameter, γ, we get the generalized form

$$Y = A\left[\alpha K^{\beta} + (1-\alpha)L^{\beta}\right]^{\frac{\gamma}{\beta}}, \qquad \gamma > 0. \tag{16.38}$$

In this form the CES function is homogeneous of degree γ. For $0 < \gamma < 1$, there are DRS, for $\gamma = 1$ CRS, and for $\gamma > 1$ IRS. If $\gamma \neq 1$, it may be convenient to consider $Q \equiv Y^{1/\gamma} = A^{1/\gamma}\left[\alpha K^{\beta} + (1-\alpha)L^{\beta}\right]^{1/\beta}$ and $q \equiv Q/L$ $= A^{1/\gamma}(\alpha k^{\beta} + 1 - \alpha)^{1/\beta}$.

The elasticity of substitution between K and L is $\sigma = 1/(1-\beta)$ whatever the value of γ. So including the limiting cases as well as non-constant returns to scale in the "family" of production functions with constant elasticity of substitution, we have the simple classification displayed in Table 16.1.

Table 16.1 The family of production functions with constant elasticity of substitution.

$\sigma = 0$	$0 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
Leontief	CES	Cobb-Douglas	CES

Note that only for $\gamma \leq 1$ is (16.38) a *neoclassical* production function. This is because, when $\gamma > 1$, the conditions $F_{KK} < 0$ and $F_{NN} < 0$ do not hold everywhere.

We may generalize further by assuming there are n inputs, in the amounts $X_1, X_2, ..., X_n$. Then the CES production function takes the form

$$Y = A\left[\alpha_1 X_1{}^{\beta} + \alpha_2 X_2{}^{\beta} + ...\alpha_n X_n{}^{\beta}\right]^{\frac{\gamma}{\beta}}, \ \alpha_i > 0 \text{ for all } i, \sum_i \alpha_i = 1, \gamma > 0. \tag{16.39}$$

In analogy with (16.34), for an n-factor production function the *partial elasticity of substitution* between factor i and factor j is defined as

$$\sigma_{ij} = \frac{MRS_{ij}}{X_i/X_j}\frac{d(X_i/X_j)}{dMRS_{ij}}_{|Y=\bar{Y}},$$

where it is understood that not only the output level but also all X_k, $k \neq i, j$, are kept constant. Note that $\sigma_{ji} = \sigma_{ij}$. In the CES case considered in (16.39), all the partial elasticities of substitution take the same value, $1/(1-\beta)$.

16.8.2 B. Balanced growth with an essential non-renewable resource

The production side of the DHSS model with CES production function is described by:

$$\begin{aligned} Y_t &= \tilde{F}(K_t, L_t, R_t, t), \qquad \partial \tilde{F}/\partial t \geq 0, & (16.40)\\ \dot{K}_t &= Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \quad K_0 > 0 \text{ given}, & (16.41)\\ \dot{S}_t &= -R_t \equiv -u_t S_t, \qquad S_0 > 0 \text{ given}, & (16.42)\\ L_t &= L_0 e^{nt}, \qquad n \geq 0, & (16.43)\\ \int_0^\infty R_t dt &\leq S_0. & (16.44) \end{aligned}$$

We will assume that the non-renewable resource is essential, i.e.,

$$R_t = 0 \text{ implies } Y_t = 0. \tag{16.45}$$

From now we omit the dating of the time-dependent variables where not needed for clarity. Recall that in the context of an essential non-renewable resource, we define a *balanced growth path* (BGP for short) as a path along which the quantities Y, C, K, R, and S are positive and change at constant proportionate rates (some or all of which may be negative).

Lemma 1 Along a BGP the following holds: (a) $g_S = g_R < 0$; (b) $R(0) = -g_R S(0)$, and

$$\lim_{t\to\infty} S = 0. \tag{16.46}$$

Proof Consider a BGP. (a) From (16.42), $g_S = -R/S$; differentiating with respect to time gives

$$\dot{g}_S = -(g_R - g_S)R/S = 0,$$

by definition of a BGP. Hence, $g_S = g_R$ since $R > 0$ by definition. For any constant g_R we have $\int_0^\infty R_t dt = \int_0^\infty R_0 e^{g_R t} dt$. If $g_R \geq 0$, (16.44) would thus be violated. Hence, $g_R < 0$. (b) With $t = 0$ in (16.42), we get $\dot{S}_0/S_0 = -R_0/S_0 = g_R$, the last equality following from (a). Hence, $R_0 = -g_R S_0$. Finally, the solution to (16.42) can be written $S_t = S_0 e^{g_S t}$. Then, since g_S is a negative constant, $S_t \to 0$ for $t \to \infty$. □

Define

$$z \equiv \frac{Y}{K}, \qquad x \equiv \frac{C}{K}, \qquad \text{and} \qquad u \equiv \frac{R}{S}. \tag{16.47}$$

We may write (16.42) as

$$g_K = z - x - \delta. \tag{16.48}$$

Similarly, by (16.42),

$$g_S \equiv -u. \tag{16.49}$$

Lemma 2 Along a BGP, $g_R = g_S = -u < 0$ is constant and $g_Y = g_C$. If gross saving is positive in some time interval, we have along the BGP in addition that $g_K = g_Y$, both constant, and that z and x are constant.

Proof Consider a BGP. Since g_S is constant by definition of a BGP, u must also be constant in view of (16.49). Then, by Lemma 1, $g_R = g_S = -u$ is constant and $u > 0$. Differentiating in (16.48) with respect to t gives $\dot{g}_K = \dot{z} - \dot{x} = (g_Y - g_K)z - (g_C - g_K)x = 0$ since g_K is constant along a BGP. Dividing through by z, which is positive along a BGP, and reordering gives

$$g_Y - g_K = (g_C - g_K)\frac{x}{z}. \tag{16.50}$$

But this is a contradiction unless $g_Y = g_C$; indeed, if $g_Y \neq g_C$, then $g_Y - g_K \neq g_C - g_K$ at the same time as $x/z = C/Y \to 0$ if $g_Y > g_C$, and $x/z = C/Y \to \infty$ if $g_Y < g_C$, both cases being incompatible with (16.50) and the presumed constancy of g_Y, g_K, and g_C, hence constancy of both $g_Y - g_K$, and $g_C - g_K$. So $g_Y = g_C$ along a BGP. Suppose gross saving is positive in some time interval and that at the same time $g_K \neq g_Y = g_C$, then (16.50) implies $x/z \equiv 1$, i.e., $C = Y$ for all t or gross saving $= 0$ for all t, a contradiction. Hence, $g_K = g_Y = g_C$. It follows by (16.47) that z and x are constant. □

Consider the case where the production function is neoclassical with CRS, and technical progress is labor- and resource-augmenting:

$$\begin{aligned} Y_t &= F(K_t, A_{2t}L_t, A_{3t}R_t), \\ A_{2t} &= e^{\gamma_2 t}, \quad \gamma_2 \geq 0, \qquad A_{3t} = e^{\gamma_3 t}, \quad \gamma_3 \geq 0. \end{aligned} \tag{16.51}$$

Let $\hat{L} \equiv A_2 L$ and $\hat{R} \equiv A_3 R$. Let ε_K, $\varepsilon_{\hat{L}}$, and $\varepsilon_{\hat{R}}$ denote the output elasticities w.r.t. K, $\hat{L}$, and $\hat{R}$, i.e.,

$$\varepsilon_K \equiv \frac{K}{Y}\frac{\partial Y}{\partial K}, \qquad \varepsilon_{\hat{L}} \equiv \frac{A_2 L}{Y}\frac{\partial Y}{\partial (A_2 L)}, \qquad \varepsilon_{\hat{R}} \equiv \frac{A_3 R}{Y}\frac{\partial Y}{\partial (A_3 R)}.$$

Differentiating in (16.51) w.r.t. t and dividing through by Y (as in growth-accounting), we then have

$$\begin{aligned} g_Y &\equiv \frac{\dot{Y}}{Y} = \varepsilon_K g_K + \varepsilon_{\hat{L}}(\gamma_2 + n) + \varepsilon_{\hat{R}}(\gamma_3 + g_R) \\ &= \varepsilon_K g_K + \varepsilon_{\hat{L}}(\gamma_2 + n) + (1 - \varepsilon_K - \varepsilon_{\hat{L}})(\gamma_3 + g_R), \end{aligned} \tag{16.52}$$

the last equality being implied by the CRS property in (16.51).

Suppose the economy follows a BGP with positive gross saving. Then, by Lemma 2, $g_K = g_Y$ and $g_R = -u < 0$. Hence, (16.52) can be written

$$(1-\varepsilon_K)(g_Y - (\gamma_3 - u)) = \varepsilon_{\hat{L}}(\gamma_2 + n - (\gamma_3 - u)). \tag{16.53}$$

Consider the special case where F is CES:

$$Y_t = \left(\alpha_1 K_t^\beta + \alpha_2(A_{2t}L_t)^\beta + \alpha_3(A_{3t}R_t)^\beta\right)^{1/\beta}, \quad \alpha_1, \alpha_2, \alpha_3 > 0, \sum_i \alpha_i = 1, \beta < 1. \tag{16.54}$$

As we know from Appendix A, for $\beta = 0$, the CES formula can be interpreted as the Cobb-Douglas formula (16.13). Applying (16.33) from Appendix A, the output elasticities w.r.t. K, $\hat{L}$, and $\hat{R}$ are

$$\varepsilon_K = \alpha_1\left(\frac{Y}{K}\right)^{-\beta}, \quad \varepsilon_{\hat{L}} = \alpha_2\left(\frac{Y}{A_2L}\right)^{-\beta}, \quad \text{and} \quad \varepsilon_{\hat{R}} = \alpha_3\left(\frac{Y}{A_3R}\right)^{-\beta}, \tag{16.55}$$

respectively.

Lemma 3 Let $y \equiv Y/L$ and $c \equiv C/L$. Given (16.43) and (16.54), along a BGP with positive gross saving, ε_K and $\varepsilon_{\hat{L}}$ are constant, and $g_c = g_y = \gamma_2$. In turn, such a BGP exists if and only if

$$u = \gamma_3 - (\gamma_2 + n) > 0. \tag{16.56}$$

Proof Consider a BGP with positive gross saving. By Lemma 2, $0 > g_R = -u$ is constant and $Y/K \equiv z$ is constant, hence so is ε_K. The left hand side of (16.53) is thus constant and so must the right-hand side therefore be. Suppose that, contrary to (16.56), $u \neq \gamma_3 - (\gamma_2 + n)$. Then constancy of the right-hand of (16.53) requires that $\varepsilon_{\hat{L}}$ is constant. In turn, by (16.55), this requires that $Y/(A_2L) \equiv y/A_2$ is constant. Consequently, $g_y = \gamma_2 = g_c$, where the second equality is implied by the claim in Lemma 2 that along a BGP with positive gross saving in some time interval, $g_C = g_Y$. As Y/K is constant, it follows that $g_K = g_Y \equiv g_y + n = \gamma_2 + n$. Inserting this into (16.52) and rearranging, we get

$$(1 - \varepsilon_K - \varepsilon_{\hat{L}})(\gamma_2 + n) = (1 - \varepsilon_K - \varepsilon_{\hat{L}})(\gamma_3 - u)$$

where the last equality follows from $g_R = -u$. Isolating u gives the equality in (16.56). Thereby, our assumption $u \neq \gamma_3 - (\gamma_2 + n)$ leads to a contradiction. Hence, given (16.43) and (16.54), if a BGP with positive gross saving exists, then $u = \gamma_3 - (\gamma_2 + n) > 0$. This shows the necessity of (16.56).

The sufficiency of (16.56) follows by construction, starting by fixing u, and thereby $-g_R$, in accordance with (16.56) and moving "backward", showing consistency with (16.52) for $g_K = g_Y = \gamma_2 + n$. □

16.9 References

Antràs, 2004, ...

Arrow, K. J., 1962, The economic implications of learning by doing, Review of Economic Studies 29, 153-73.

Arrow, K. J., P. Dasgupta, L. Goulder, G. Daily, P. Ehrlich, G. Heal, S. Levin, K.-G. Mäler, S. Schneider, D. Starrett, and B. Walker, 2004, Are we consuming too much?, *J. of Economic Perspectives 18*, no. 3, 147-172.

Arrow, K.J, P. Dasgupta, L. Goulder, K. Mumford, and K.L.L. Oleson, 2012, Sustainability and the measurement of wealth, *Environment and Development Economics 17*, no. 3, 317–353.

Brock, W. A., and M. Scott Taylor, 2005. Economic Growth and the Environment: A review of Theory and Empirics. In: *Handbook of Economic Growth*, vol. 1.B, ed. by ...

Brundtland Commission, 1987, ...

Brynjolfsson, E., and A. McAfee, 2014, *The Second Machine Age*, Norton.

Caballero, R. J., and R. K. Lyons, The case of external economies. In: Political Economy, Growth, and Business Cycles, ed. by A. Cukierman, Z. Hercowitz and L. Leiderman, MIT Press, Cambridge, MA, 1992, pp. 117-139.

Dasgupta, P., 1993, Natural resources in an age of substitutability. In: *Handbook of Natural Resource and Energy Economics*, vol. 3, ed. by A. V. Kneese and J. L. Sweeney, Elsevier: Amsterdam, 1111-1130.

Dasgupta, P., and G. M. Heal, 1974, The optimal depletion of exhaustible resources, *Review of Economic Studies,* vol. 41, Symposium, 3-28.

Dasgupta, P., and G. M. Heal, 1979, Economic Theory and Exhaustible Resources, Cambridge University Press, Cambridge.

Dasgupta, P., G. M. Heal and M. K. Majumdar, 1977, Resource depletion and research and development. In: Frontiers of Quantitative Economics, vol. IIIB, ed. by M. D. Intrilligator, North-Holland, Amsterdam, 483-505.

Dasgupta, P., G. M. Heal and A. K. Pand, 1980, Funding research and development, Applied Mathematical Modeling 4, 87-94.

Fagnart, J.-F., and M. Germain, 2011, Quantitative vs. qualitative growth with recyclable resources, *Ecological Economics*, vol. 70, 929-941.

Gordon, R.J., 2012, Is US economic growth over? Faltering innovation confronts the six headwinds, NBER WP no. 18315.

Grimaud, A., and L. Rougé, 2003, Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policies, Journal of Environmental Economics and Management 45, 433-453.

Grossman, G.M., and A.B. Krueger, 1995, , QJE, vol. 90 (2).

Groth, C., 2004, Strictly endogenous growth with non-renewable resources implies an unbounded growth rate, Topics in Macroeconomics 4 (1), 1-13.

Groth, C., 2007, A New-Growth Perspective on Non-renewable Resources. In: *Sustainable Resource Use and Economic Dynamics*, ed. by L. Bretschger and S. Smulders, Springer Verlag: Dordrecht (available at the course website).

Groth, C., 2014, Lecture notes in macroeconomics (mimeo), Department of Economics, University of Copenhagen.

Groth, C., and P. Schou, 2002, Can nonrenewable resources alleviate the knife-edge character of endogenous growth? Oxford Economic Papers 54, 386-411.

−− , 2006, Growth and non-renewable resources: The different roles of capital and resource taxes, Journal of Environmental Economics and Management, forthcoming.

Hall, R. E., 1990, Invariance properties of Solow's residual. In: Growth/Productivity/Unemployment. Essays to Celebrate Bob Solow's Birthday, ed. by P. Diamond, MIT Press, Cambridge (Mass.), 71-112.

Hamilton, K., 2012, Comments on Arrow et al., ‘Sustainability and the measurement of wealth’, *Environment and Development Economics 17*, no. 3, 356 - 361.

Handbook of Natural Resource and Energy Economics, vol. III, ed. by A. V. Kneese and J. L. Sweeney, Elsevier: Amsterdam, 1993.

Hartwick, J. M., 1977, Intergenerational equity and the investing of rents from exhaustible resources, American Economic Review 67, 972-974.

Heal, G., 1998. *Valuing the Future. Economic Theory and Sustainability.* Columbia University Press: New York.

Hotelling, H., 1931, The economics of exhaustible resources, Journal of Political Economy 39, 137-175.

Jones, C., 2002, *Introduction to Economic Growth*, 2. ed., W.W. Norton: New York.

Kamien, M. I., and N. L. Schwartz, 1978, Optimal exhaustible resource depletion with endogenous technical change, Review of Economic Studies 45 (1), 179-196.

Krautkraemer, J., 1985, Optimal growth, resource amenities and the preservation of natural environments, Review of Economic Studies 52, 153-170.

−− , 1998, Nonrenewable resource scarcity, Journal of Economic Literature 36 (4), 2065-2107.

Lind, S., og P. Schou, P., 2013, Ægte opsparing, *Arbejdspapir 2013:1*, De Økonomiske Råds Sekretariat, København.

Meadows, D. H., D. Meadows, J. Randers, and W. W. Behrens III, 1972, *The Limits to Growth*, Universe Books: New York.

Meadows, D. H., D. L. Meadows and J. Randers, 1992, Beyond the Limits, Chelsea Green Publ. Com., Post Mills.

Meadows, D. H., D. Meadows, and J. Randers, 2004, *Limits to Growth: The 30-Year Update,...*

Neumayer, E., 2000, Scarce or abundant? The economics of natural resource availability, Journal of Economic Surveys 14 (3), 307-335.

Nordhaus, W., 1969, An economic theory of technological change, American Economic Review 59 (2), 18-28.

Nordhaus, W. , 1992, Lethal Model 2: The limits to growth revisited, Brookings Papers on Economic Activities, (2), 1-43.

Nordhaus, W.D., 1999, Quality change in price indices, *J. of Economic Perspectives*, vol. 12 (1), 59-68.

Nordhaus, W., and J. Tobin, 1972, Is growth obsolete? In: Economic Growth: Fiftieth Anniversary Colloquium V, Columbia University Press, New York. Reprinted in: J. Tobin, Essays in Economics. Theory and Policy, The MIT Press, Cambridge (Mass.) 1985, 360-450.

Popp, D., 2002, Induced innovation and energy prices, American Economic Review 92 (1), 160-180.

Robson, A. J., 1980, Costly innovation and natural resources, International Economic Review 21, 17-30.

Schou, P., 2000, Polluting nonrenewable resources and growth, Environmental and Resource Economics 16, 211-27.

—— , 2002, When environmental policy is superfluous: Growth and polluting resources, Scandinavian Journal of Economics 104, 605-620.

Seierstad, A., and K. Sydsaeter, 1987, Optimal Control Theory with Economic Applications, North Holland, Amsterdam.

Sinclair, P., 1994, On the optimum trend of fossil fuel taxation, Oxford Economic Papers 46, 869-877.

Smil, V., 2003, *Energy at the crossroads. Global perspectives and uncertainties*, Cambridge Mass.: MIT Press.

Smulders, S., 1995, Entropy, environment, and endogenous growth, *International Tax and Public Finance*, vol. 2, 319-340.

Smulders, S., 1999,...

Smulders, S., 2012, An arrow in the Achilles' heel of sustainability and wealth accounting, *Environment and Development Economics 17*, no. 3, 368-372.

Smulders, S., and M. de Nooij, 2003, The impact of energy conservation on technology and economic growth, Resource and Energy Economics 25, 59-79.

Solow, R.M., 1973, Is the end of the world at hand? *Challenge*, vol. 16, no. 1, 39-50.

Solow, R. M., 1974a, Intergenerational equity and exhaustible resources, Review of Economic Studies 41, Symposium Issue, 29-45.

−− , 1974b, The economics of resources or the resources of economics, American Economic Review 64, Papers and Proceedings, 1-14.

Solow, R.M., 2012, A few comments on 'Sustainability and the measurement of wealth, *Environment and Development Economics 17*, no. 3, 354-355.

Squires and Vestergaard, 2011,

Sterner, T., 2008, The costs and benefits of climate policy: The impact of the discount rate and changing relative prices. Lecture at Miljøøkonomisk konference 2008 in Skodsborg, see http://www.dors.dk/graphics/Synkron-Library/Konference%202008/

Keynote/Sterner_Copenhagen_SEPT08.pdf

Stiglitz, J., 1974a, Growth with exhaustible natural resources: Efficient and optimal growth paths, Review of Economic Studies 41, Symposium Issue, 123-137.

−− , 1974b, Growth with exhaustible natural resources: The competitive economy, Review of Economic Studies 41, Symposium Issue, 139-52.

Stiglitz, J., A. Sen, and J.-P. Fitoussi, 2009, Report by the Commission on the Measurement of Economic Performance and Social Progress, www.stiglitz-sen-fitoussi.fr.

Stiglitz, J., A. Sen, and J.-P. Fitoussi, 2010, *Mismeasuring Our Lives. Why the GDP Doesn't Add Up*, The New Press: New York.

Suzuki, H., 1976, On the possibility of steadily growing per capita consumption in an economy with a wasting and non-replenishable resource, Review of Economic Studies 43, 527-35.

Tahvonen, O., and S. Salo, 2001, Economic growth and transitions between renewable and nonrenewable energy resources, European Economic Review 45, 1379-1398.

Takayama, A., 1980, Optimal technical progress with exhaustible resources. In: Exhaustible Resources, Optimality, and Trade, ed. by M. C. Kemp and N. V. Long, North-Holland, Amsterdam, 95-110.

United Nations, 2005, World Population Prospects. The 2004 Revision, Population Division, New York.

van Kooten, G. C., and E. H. Bulte, 2000. *The Economics of Nature. Managing Biological Assets*. Blackwell: Oxford.

Weitzman, M., 2001, A contribution to the theory of welfare accounting, *Scandinavian J. of Economics 103* (1), 1-23.

Weitzman, M., 2003, *Income, Wealth, and the Maximum Principle*, Harvard Univ. Press: Cambridge (Mass.).

Withagen, C., 1990, Topics in resource economics. In: Advanced Lectures in Quantitative Economics, ed. by F. van der Ploeg, Academic Press, London, 381-419.