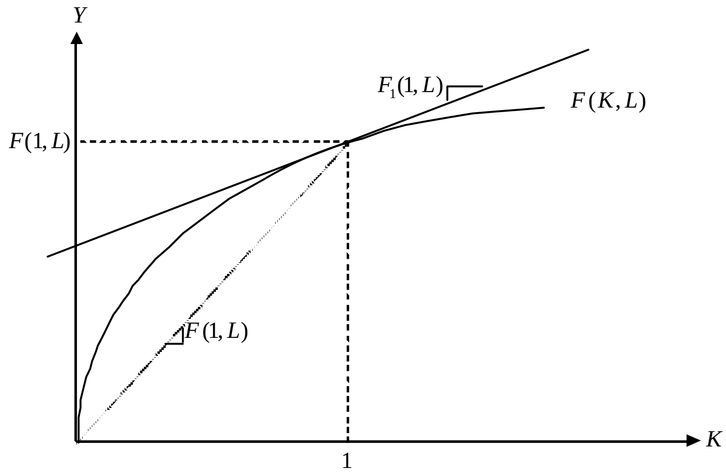

Figure 12.2: Illustration of the fact that for L given, $F(1, L) > F_1(1, L)$.

First, note that the dynamic resource constraint for the economy is

$$\dot{K} = Y - cL - \delta K = F(1, L)K - cL - \delta K,$$

or, in per-capita terms,

$$\dot{k} = [F(1, L) - \delta] k - c_0 e^{\gamma t}. \tag{12.25}$$

In this equation it is important that $F(1, L) - \delta - \gamma > 0$. To understand this inequality, note that, by (A2'), $F(1, L) - \delta - \gamma > F(1, L) - \delta - \bar{r} = F(1, L) - F_1(1, L) = F_2(1, L)L > 0$, where the first equality is due to $\bar{r} = F_1(1, L) - \delta$ and the second is due to the fact that since F is homogeneous of degree 1, we have, by Euler's theorem, $F(1, L) = F_1(1, L) \cdot 1 + F_2(1, L)L > F_1(1, L) > \delta$, in view of (A1'). The key property $F(1, L) - F_1(1, L) > 0$ is illustrated in Figure 12.2.

The solution of a general linear differential equation of the form $\dot{x}(t) + ax(t) = ce^{ht}$, with $h \neq -a$, is

$$x(t) = (x(0) - \frac{c}{a + h})e^{-at} + \frac{c}{a + h}e^{ht}. \tag{12.26}$$

Thus the solution to (12.25) is

$$k_t = (k_0 - \frac{c_0}{F(1, L) - \delta - \gamma})e^{(F(1,L)-\delta)t} + \frac{c_0}{F(1, L) - \delta - \gamma}e^{\gamma t}. \tag{12.27}$$