Chapter 10

Knowledge creation and human capital in a growing economy

As a follow-up on the concept of a schooling technology presented in Chapter 9, Section 9.2, the present chapter considers aspects of the interplay between physical capital, human capital, and knowledge creation in a simple balanced growth framework. The aim is to get the structure of an economy with a central role for both R&D and human capital "right". We focus in this chapter on *technically feasible time paths*, that is, time paths that a feasible from the point of view of technology and initial resources. More precisely, we focus on technically feasible paths that are consistent with balanced growth. Institutions and incentives that may be needed for the economy to realize such a path is not the issue here.

10.1 The model

We consider a closed economy with education and two production sectors, manufacturing and R&D. Time is continuous. Postponing the modeling of education a little, at the aggregate level we have:

$$
\begin{aligned}
Y_t &= T_t K_t^{\alpha}(\bar{h}_t L_{Yt})^{1-\alpha}, \quad 0 < \alpha < 1, && (10.1)\\
\dot{K}_t &= Y_t - c_t N_t - \delta K_t, \quad \delta \geq 0, \quad K_0 > 0 \text{ given}, && (10.2)\\
T_t &= A_t^{\sigma}, \quad \sigma > 0, && (10.3)\\
\dot{A}_t &= \gamma A_t^{\varphi} \bar{h}_t L_{At}, \quad \gamma > 0, \varphi < 1, \quad A_0 > 0 \text{ given}, && (10.4)\\
0 &< L_{Yt} \leq L_{Yt} + L_{At} = L_t, && (10.5)
\end{aligned}
$$

where Y_t is manufacturing output (the value of which is less than GNP when $L_{At} > 0$), T_t is total factor productivity (TFP), K_t is physical capital input,

$\bar{h}_t$ is average human capital in the labor force, L_{Yt} and L_{At} are inputs of labor in manufacturing and R&D, respectively, C_t is aggregate consumption, A_t is the stock of technical knowledge, and L_t is aggregate labor input, all at time t. The size of population is denoted N_t and so per capita consumption is $c_t \equiv C_t/N_t$.

Comments: As to (10.5), $\bar{h}_t L_{At}$ is the total input of human capital per time unit in R&D and γA_t^{φ} is the productivity of this input at the aggregate level. The parameter φ measures the elasticity of research productivity w.r.t. the level of the available stock of technical knowledge. The case $0 < \varphi < 1$ represents the "standing on the shoulders" case where knowledge creation becomes easier the more knowledge there is already. In contrast, the case $\varphi < 0$ represents the "fishing out" case, also called the "easiest inventions are made first" case. This would reflect that it becomes more and more difficult to create the next advance in technical knowledge. Note also that the productivity of man-hours (L_{At}) in R&D depends on the level of human capital, $\bar{h}_t$. As to (10.5), the strict and weak inequalities are motivated by the view that for the system to be economically viable, there must be activity in the Y-sector whereas it is of interest to allow for − and compare − the cases $L_{At} > 0$ and $L_{At} = 0$ (active versus passive R&D sector).

The population growth rate is assumed constant:

$$N_t = N_0 e^{nt}, \qquad n \geq 0, \quad N_0 > 0 \text{ given.} \tag{10.6}$$

We assume a stationary age distribution in the population. Although details about schooling are postponed, we already here assume that schooling and retirement are consistent with the labor force being a constant fraction of the population:

$$L_t = (1 - \beta)N_t, \tag{10.7}$$

where $\beta \in (0, 1)$. Then, by (10.6) follows

$$L_t = L_0 e^{nt}, \qquad n \geq 0, \quad L_0 > 0. \tag{10.8}$$

We let the growth rate at time t of a variable $x > 0$ be denoted g_{xt}. When writing just g_x, without the time index t, it is understood that the growth rate of x is constant over time.

10.2 Productivity growth along a BGP with R&D

Let us first find an expression for the TFP growth rate. By log-differentiation w.r.t. t in (10.1), we have

$$g_{Yt} = g_{Tt} + \alpha g_{Kt} + (1 - \alpha)(g_{\bar{h}t} + g_{L_{Yt}}). \tag{10.9}$$

The current TFP growth rate is thus

$$g_{TFPt} \equiv g_{Yt} - (\alpha g_{Kt} + (1-\alpha)(g_{\bar{h}t} + g_{L_{Yt}})) = g_{Tt} = \sigma g_{At}, \tag{10.10}$$

where the last equality follows from (10.3). By (10.4), we get

$$g_{At} \equiv \frac{\dot{A}_t}{A_t} = \gamma A_t^{\varphi-1}\bar{h}_t L_{At} \geqq 0, \text{ with } > \text{ if and only if } L_{At} > 0. \tag{10.11}$$

We shall first consider the case of active R&D:

ASSUMPTION (A1): $L_{At} > 0$ for all $t \geq 0$.

This assumption implies $g_{At} > 0$ and so the growth rate of g_{At} is well-defined. By log-differentiation w.r.t. t in (10.11) we have

$$\frac{\dot{g}_{At}}{g_{At}} = (\varphi - 1)g_{At} + g_{\bar{h}t} + g_{L_A t}. \tag{10.12}$$

10.2.1 Balanced growth with R&D

In the present context we define a *balanced growth path* (BGP) as a path along which g_{Yt}, g_{Ct}, g_{Kt}, g_{At}, and $g_{\bar{h}t}$ are constant (not necessarily equal and not necessarily positive). With y denoting per capita manufacturing output, i.e., $y \equiv Y/L$, let us find the growth rate of y in balanced growth with active R&D. We introduce the following additional assumptions:

ASSUMPTION (A2): The economy follows a BGP.

ASSUMPTION (A3): $Y_t - c_t N_t > 0$ for all $t \geq 0$.

By imposing (A3), we rule out the degenerate case where $g_K = -\delta$.

Along a BGP, by definition, g_{At} is a constant, g_A. Since thereby $\dot{g}_A = 0$, solving for g_A in (10.12) gives

$$g_A = \frac{g_{\bar{h}t} + g_{L_A t}}{1-\varphi} > 0, \tag{10.13}$$

where the positivity is due to the assumption (A1). For the formula (10.13) to be consistent with balanced growth, $g_{L_A t}$ must be a constant, g_{L_A}, since otherwise g_A and $g_{\bar{h}t}$ could *not* both be constant as they must in balanced growth, by definition. Moreover, we must have $g_{L_A} = n$. To see this, imagine that $g_{L_A} < n$. Then, in order for the growth rate of the sum $L_{Yt} + L_{At}$ to accord with (10.8), we would need $g_{L_Y t} > n$ *forever*, which would imply $L_{Yt} + L_{At} > L_t$ sooner or later. This is a contradiction. And if instead we imagine that $g_{L_A} > n$ while still being constant, we would, at least after some time, have $L_{At} > L_t$, again a contradiction. We conclude that $g_{L_A} = n$. For $L_{Yt} + L_{At}$ to accord with (10.8), it then follows that also $g_{L_Y t}$ must be a constant, g_{L_Y}, and equal to n. We have hereby proved that along a BGP with R&D,

$$g_{L_A} = g_{L_Y} = n. \tag{10.14}$$

It follows that L_A/L is constant along a BGP with R&D.

Given the accumulation equation (10.2) and the assumption (A3), it follows by the Balanced Growth Equivalence Theorem of Chapter 4 that

$$g_C = g_Y = g_K$$

along a BGP. From (10.9), together with (10.7) and the definition $c \equiv C/N$, then follows that along a BGP,

$$\begin{aligned} g_c &= g_y = g_Y - n = g_T + \alpha g_K + (1-\alpha)(g_{\bar{h}} + n) - n \\ &= g_T + \alpha(g_K - n) + (1-\alpha) g_{\bar{h}} = g_T + \alpha g_k + (1-\alpha) g_{\bar{h}}, \quad (10.15) \end{aligned}$$

where the last equality comes from $k \equiv K/L_Y$ and $g_{L_Y} = n$. As $g_K = g_Y$ and $g_{L_Y} = g_L$, we have $g_k = g_y$. Then (10.15) gives

$$g_y = \frac{g_T}{1-\alpha} + g_{\bar{h}} = \frac{\sigma g_A}{1-\alpha} + g_{\bar{h}}, \tag{10.16}$$

in view of (10.10).

Education

Let the time unit be one year. Suppose an individual "born" at time v (v for "vintage") spends the first S years of life in school and then enters the labor market with a human capital equal to $h(S)$, where $h' > 0$. We ignore the role of teachers and schooling equipment in the formation of human capital. The role of work experience for human capital later in life is likewise ignored. Moreover, we assume that S is the same for all members of a given cohort and also – until further notice – the same across cohorts. So

$$\bar{h} = h(S), \qquad h' > 0. \tag{10.17}$$

After leaving school, individuals work full-time until either death before age R or retirement at age R where $R > S$, of course; life expectancy is assumed the same for all cohorts. Assuming a stationary age distribution in the population, we see that β in (10.7) represents the constant fraction of the population consisting of people either below age S, i.e., under education, or above age R, i.e., retired people (β will be an increasing function of S and a decreasing function of R).[1]

[1]A complete model would treat S as endogenous in general equilibrium. In a partial

Sustained productivity growth along a BGP

It follows that average human capital is constant. Thus $g_{\bar{h}} = 0$ and (10.16) reduces to

$$g_y = \frac{\sigma g_A}{1-\alpha} > 0, \tag{10.18}$$

In equation (10.15) productivity growth, g_y, is decomposed into a contribution from technical change, a contribution from "capital deepening" (growth in k), and a contribution from human capital growth if any. As long as S in (10.17) is assumed constant over time, there is no human capital growth. So we can re-write (10.15):

$$g_y = g_T + \alpha g_k = g_{TFP} + \alpha g_k, \tag{10.19}$$

in view of $g_{TFP} = g_T = \sigma g_A$ from (10.10). This equation decomposes the productivity growth rate into a direct contribution from technical change and a direct contribution from capital deepening. Digging deeper, (10.18) tells us that both these direct contributions rest on sustained knowledge growth. The correct interpretation of (10.19) is that it just displays the two factors behind the *current* increase in y, while (10.18) takes into account that both TFP growth and capital deepening are in a long-run perspective themselves driven by knowledge growth.

10.2.2 A precondition for sustained productivity growth when $g_{\bar{h}} = 0$: population growth

We saw that along a BGP with R&D, $g_{L_A} = n$. By (10.13) and $g_{\bar{h}} = 0$ then follows that along a BGP with R&D,

$$g_A = \frac{n}{1-\varphi} > 0. \tag{10.20}$$

From this inequality we see that existence of a BGP with R&D requires

ASSUMPTION (A4): $n > 0$

to hold.

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equilibrium analysis one could possibly use an approach similar to the one in Chapter 9, Section 9.3. We shall not enter into that, however, because the next step, determination of the real rate of interest in general equilibrium, is a complex problem and requires a lot of additional specifications of households' characteristica and market structure. Fortunately, it is not necessary to determine S as long as the focus is only on determining the productivity growth rate along a BGP.

On the basis of (A4) and (10.18) we finally conclude that

$$g_y = \frac{\sigma n}{(1-\varphi)(1-\alpha)} > 0. \tag{10.21}$$

Here we have taken into account that also knowledge growth is endogenous in that it is determined by allocation of resources (research workers) to R&D activity. The result (10.21) tells us that not only is population growth necessary for sustained productivity growth but productivity growth is faster the faster is population growth.

Why does population growth ultimately help productivity growth (at least in this model)? The explanation is that productivity growth is driven by knowledge creation. Knowledge is a *nonrival* good – its use by one agent does not, in itself, limit its simultaneous use by other agents. The value of a piece of technical knowledge – a technical idea – is proportional to the number of users. Considering the producible T in (10.1) as an additional production factor along with capital and labor, (10.1) displays *increasing returns to scale* in manufacturing w.r.t. these three production factors. Although there are diminishing marginal returns to capital, there are increasing returns to scale w.r.t. capital, labor, *and* the accumulative technology level. For the increasing returns to unfold in the long run, growth in the labor force (hence in population) is needed. Growth in the labor force and T not only counterbalances the falling marginal productivity of capital,[2] but actually upholds sustained per capita growth – the more so the faster is population growth.

The growth-promoting role of the exogenous rate of population growth reflects the presence of what is called a *weak scale effect* in the model. A *scale effect* is said to be present in an economic system if there is an advantage of scale measured by population size. This advantage of scale is in the present case due to the productivity-enhancing role of a nonrival good, technical knowledge, that is produced by the research workers in the idea-creating R&D sector. Thereby higher population growth results in higher per capita growth in the long run. On the other hand, a large population is not in itself, when $\varphi < 1$, sufficient to generate sustained positive per capita growth. This is why we talk of a *weak* scale effect. In contrast, what is known as a *strong* scale effect (associated with the case $\varphi \geq 1$) is present if a larger population as such (without population growth) would be enough to generate higher per capita growth in the long run.

In view of cross-border diffusion of ideas and technology, the result (10.21) should not be seen as a prediction about individual countries. It should

[2] This counter-balancing role reflects the direct complementarity between the production factors in (10.1).

rather be seen as pertaining to larger regions, nowadays probably the total industrialized part of the world. So the single country is not the relevant unit of observation and cross-country regression analysis thereby not the right framework for testing such a link from n to g_y.

The reason that in (10.21), a higher σ promotes productivity growth is that σ indicates the sensitivity (elasticity) of TFP w.r.t. accumulative knowledge. Indeed, the larger is σ, the larger is the percentage increase in manufacturing output that results from a one-percentage increase in the stock of knowledge.

The intuition behind the growth-enhancing role of α in (10.21) follows from (10.1) which indicates that α measures the elasticity of manufacturing output w.r.t. another accumulative input, physical capital. The larger is α, the larger is the percentage increase in manufacturing output resulting from a one-percentage increase in the stock of capital.

Finally, the intuition behind the growth-enhancing role of φ in (10.21) can be obtained from the equation (10.4) which describes the creation of new knowledge. The equation shows that the larger is φ, the larger is the percentage increase in the time-derivative of technical knowledge resulting from a one-percentage increase in the stock of knowledge.

10.2.3 The concept of endogenous growth

The above analysis provides an example of *endogenous growth* in the sense that the positive sustained per capita growth rate is generated through an economic mechanism within the model, allocation of resources to R&D; by an "economic mechanism" is meant a process involving economic decisions, either directly or indirectly. This is in contrast to the Solow or standard Ramsey model where technical progress is exogenous, given as manna from heaven.

There are basically two types of endogenous growth. One is called *semi-endogenous* growth and is present if growth is endogenous but a positive per capita growth rate can not be sustained in the long run without the support from growth in some exogenous factor (for example growth in the labor force). As $n > 0$ is needed for sustained per capita growth in the above model, growth is here driven by R&D in a semi-endogenous way.

The other type of endogenous growth is called *fully endogenous* growth and occurs if the long-run growth rate of Y/L is positive without the support from growth in any exogenous factor (for example growth in the labor force).

10.3 Permanent level effects

In the result (10.21), there is no trace of the *size* of the fraction, L_A/L, of the labor force allocated to R&D. This is due to the assumption that $\varphi < 1$. This assumption implies diminishing *marginal* productivity of knowledge in the creation of new knowledge. Indeed, when $\varphi < 1$, $\partial \dot{A}/\partial A = \gamma\varphi A^{\varphi-1}\bar{h}L_A$ is a decreasing function of the stock of knowledge already obtained. A shift of L_A/L to a higher level can *temporarily* generate faster knowledge growth and thereby faster productivity growth, but due to the diminishing *marginal* productivity of knowledge in the creation of new knowledge, in the long run g_A and g_y will be back at their balanced-growth level given in (10.20) and (10.21), respectively.

It can be shown, however, that a marginally higher L_A/L generally has a permanent *level* effect, that is, a permanent effect on y along a BGP. If initially L_A/L is "small", this level effect tends to be positive. This is like in the Solow growth model where a shift to a higher saving-income ratio, s, has a temporary positive growth effect and a permanent positive level effect on y. In contrast to the Solow model, however, if L_A/L is already "large", the level effect on y of a marginal increase in L_A/L may be negative. This is because Y is produced by $L_Y = (1 - L_A/L)L$, not L.[3]

Human capital affects the productivity of man-hours in both manufacturing and R&D. As mentioned we treat the number of years in school and average human capital, $\bar{h}$, as exogenous. Then it is straightforward to study the comparative-dynamic effect of a higher level of average human capital, $\bar{h}$. In the present model there will be a permanent level effect on y but no permanent growth effect.

Yet, a complicating aspect is that, given the model, a higher value of $\bar{h}$ will cost a higher number of years in school, i.e., a higher S. A higher S implies that a smaller fraction of the population will be in the labor force, cf. (10.7) where β is an increasing function of S. This implies that there is no longer a one-to-one relationship between a positive level effect on $y \equiv Y/L$ and a positive level effect on per capita consumption, $c \equiv C/N = (C/Y) \cdot (Y/L) \cdot (L/N)$. We will not go into detail with this kind of trade-off here.

10.4 The case of no R&D

As an alternative to (A1) we now consider the case of no R&D:

ASSUMPTION (A5): $L_{At} = 0$ for all $t \geq 0$.

[3] In Exercise VII.7 you are asked to analyze this kind of problems in a more precise way.

Under this assumption the whole labor force is employed in manufacturing, i.e., $L_{Yt} = L_t$ for all $t \geq 0$. There is no growth in knowledge and therefore no TFP growth. Whether $n > 0$ or $n = 0$, along a BGP satisfying (A3), (10.19) is still valid but reduces to $g_y = \alpha g_k$. At the same time, however, the Balanced Growth Equivalence Theorem of Chapter 4 says that along a BGP satisfying (A3), $g_Y = g_K$, which implies $g_y = g_k$. As $\alpha \in 0, 1)$, we have thus reached a contradiction unless $g_y = g_k = 0$.

So, as expected, without technological progress there can not exist sustained per capita growth. To put it differently, along a BGP we necessarily have $g_Y = g_K = g_C = n$, where $C \equiv cN$.

10.5 A look ahead

Given the prospect of non-increasing population in the world economy already within a century from now (United Nations, 2013), the prospect of sustained per capita growth in the world economy in the very long run may seem bleak according to the model. Let us take a closer look at the issue.

10.5.1 The case $n = 0$

Suppose $n = 0$ in the above model and return to the assumption (A1). As g_{L_A} can no longer be a positive constant, g_A and g_y can no longer be positive constants. Hence balanced growth with $g_y > 0$ is impossible. Does this imply that there need be economic stagnation in the sense of $g_y = 0$? No, what is ruled out is that $y_t = y_0 e^{g_y t}$ is impossible for any constant $g_y > 0$. So it is *exponential* growth that is impossible.

Still paths along which $y_t \rightarrow \infty$ and $c_t \rightarrow \infty$ for $t \rightarrow \infty$ are technically feasible. Along such paths, g_y and g_c will be positive forever, but with $\lim_{t\rightarrow\infty} g_y = 0$ and $\lim_{t\rightarrow\infty} g_c = 0$. To see this, suppose $L_{At} = L_A$, a positive constant less than L, where L is the constant labor force which is proportional to the constant population. Suppose further, for simplicity, that $\bar{h}$ is can be considered exogenous. Then, from (10.4) and (10.17) follows

$$\dot{A}_t = \gamma A_t^{\varphi} \bar{h} L_A \equiv \xi A_t^{\varphi}, \qquad \xi \equiv \gamma \bar{h} L_A.$$

This Bernoulli differential equation has the solution[4]

$$A_t = \left[A_0^{1-\varphi} + (1-\varphi)\xi \cdot t\right]^{\frac{1}{1-\varphi}} \equiv \left[A_0^{1-\varphi} + (1-\varphi)\gamma \bar{h} L_A \cdot t\right]^{\frac{1}{1-\varphi}} \rightarrow \infty \text{ for } t \rightarrow \infty. \tag{10.22}$$

[4]See Section 7.2 of Chapter 7.

The stock of knowledge thus follows what is known as a *quasi-arithmetic growth* path − a form of less-than-exponential growth. The special case $\varphi = 0$ leads to simple *arithmetic growth*: $A_t = A_0 + (1-\varphi)\eta\bar{h}L_A \cdot t$. In case $0 < \varphi < 1$, A_t features more-than-arithmetic growth and in case $\varphi < 0$, A_t features less-than-arithmetic growth. It can be shown that with a social welfare function of the standard Ramsey type, cf. Chapter 8, the social planner's solution converges, for $t \to \infty$, toward a path where also K_t, Y_t, y_t, and c_t feature quasi-arithmetic growth.[5]

10.5.2 The case of rising life expectancy

There is another demographic aspect of potential importance for future productivity growth, namely the prospect of increasing schooling length in the wake of an increasing life expectancy.

Over the past 30-40 years average years of schooling have tended to grow arithmetically at a rate of about 0.8 years per decade in the EU as a whole, compared to 0.7 years in the US (Montanino et al. 2004). A central factor behind this development is the rising life expectancy due to improved income, salubrity, nutrition, sanitation, and medicine. Increased life expectancy heightens the returns to education. In the first half of the twentieth century life expectancy in the US improved at a rate of four years per decade. In the second half the rate has been smaller, but still close to two years per decade (Arias, 2004). Oeppen and Vaupel (2002) report that since 1840 female life expectancy in the record-holding country in the world has steadily increased by almost a quarter of a year per year. To what extent such developments may continue is not clear. But at least for a long time to come we may expect growth in life expectancy and thereby also in educational investment because of the lengthening of the recovery period for that investment.

Increasing schooling length introduces heterogeneity w.r.t. individual human capital into the model. In a cross-section of workers at a given point in time the workers' h becomes a decreasing function of age. And increasing life expectancy changes the aggregate growth process for population and labor force. This takes us somewhat outside the above analytical framework with a stationary age structure and no schooling heterogeneity. Yet let us speculate a little.

Suppose the schooling technology can be presented by a power function:[6]

$$h = h(S) = S^{\eta}, \qquad \eta > 0. \tag{10.23}$$

[5] Groth et al. (2010).

[6] There is some empirical support for this hypothesis, cf. Section 9.5 of Chapter 9.

Let every member of cohort $v \geq 0$ spend $S(v)$ years in school, thereby leaving school with human capital $h(v) = S(v)^\eta$. Then the growth rate of h of the cohort just leaving school is

$$\frac{dh(v)/dv}{h(v)} = \frac{\eta S(v)^{\eta-1} S'(v)}{S(v)^\eta} = \eta \frac{S'(v)}{S(v)}.$$

Assume sustained arithmetic growth in schooling length takes place due to a steadily rising life expectancy. Then

$$S(v) = S_0 + \mu v, \qquad S_0 \geq 0, \mu > 0. \tag{10.24}$$

Hence,

$$\eta \frac{S'(v)}{S(v)} = \frac{\eta\mu}{S_0 + \mu v} \rightarrow 0 \text{ for } v \rightarrow \infty.$$

On this background the projection will be that also average human capital, $\bar{h}_t$, will be growing over time but at a rate, $g_{\bar{h}}$, approaching 0 for $t \rightarrow \infty$. This gives no chance that the $g_{\bar{h}}$ in the formula (10.13) can avoid approaching nil. So our model rules out exponential per capita growth in the long run when $n = 0$ and $h(v) = S(v)^\eta$.

As a thought experiment, suppose instead that the schooling technology is exponential:

$$h = h(S) = e^{\psi S(v)}, \qquad \psi > 0. \tag{10.25}$$

Then the growth rate of the human capital of the cohort just leaving school is

$$\frac{dh(v)/dv}{h(v)} = \frac{e^{\psi S(v)} \psi s'(v)}{e^{\psi S(v)}} = \psi S'(v) > 0. \tag{10.26}$$

Assume arithmetic growth in life expectancy as well as schooling length, the latter following (10.24). Then $\psi S'(v) = \psi\mu$, a positive constant. My conjecture is that also average human capital, $\bar{h}_t$, will in this case under certain conditions grow at the constant rate, $\psi\mu$, at least approximately (I have not made the required demographic calculus).

Let us try some numbers. Suppose life expectancy in modern times steadily increases by λ years per year and let schooling time and retirement age be constant fractions of life expectancy. Let the schooling time fraction be denoted ω. Then $S'(v) = \mu = \omega\lambda$ and $g_{\bar{h}} = \psi S'(v) = \psi\omega\lambda$. With $\lambda = 0.2$, $\omega = 0.2$, and $\psi = 0.10$, we get $g_{\bar{h}} = 0.004$.[7] Suppose $n = 0.005$ and $\varphi = 0.5$

[7] As reported by Krueger and Lindahl (2001), in cross-section studies ψ is usually estimated to be in the range $(0.05, 0.15)$.

(as suggested by Jones, 1995).[8]

Along a BGP with R&D we then have, by (10.13),

$$g_A = \frac{g_{\bar{h}} + n}{1 - \varphi} = \frac{0.004 + 0.005}{0.5} = 0.018.$$

In case $\sigma = 1 - \alpha$, (10.16) thus yields

$$g_y = g_A + g_{\bar{h}} = 0.018 + 0.004 = 0.022.$$

If instead $n = 0$ (in accordance with the long-run projection) and $L_{At} = L_A \in (0, L)$, we get along a BGP with R&D

$$g_y = \frac{g_{\bar{h}}}{1 - \varphi} + g_{\bar{h}} = \frac{0.004}{1 - \varphi} + 0.004 = 0.012.$$

In spite of $n = 0$, the thought experiment (10.25) thus leads to a non-negligible level of exponential growth. With the compounding effects of exponential growth it is certainly substantial. I call it a "thought experiment" because the empirical foundation of the exponential human capital production function (10.25) is weak if not non-existing.

10.6 Concluding remarks

In a semi-endogenous growth setting we have considered human capital formation and knowledge creating R&D. The latter is ultimately the factor driving productivity growth unless one is willing to make very strong assumptions about the human capital production function. Technical knowledge is capable of performing this role because it is a nonrival good and is "infinitely expansible", as emphasized by Paul Romer (1990) and Danny Quah (1996). Contrary to this, in Lucas (1988) the distinction between technical knowledge and human capital is not emphasized and it is the accumulation of human capital that is driving long-run productivity growth.[9]

[8] $n = 0.005$ per year may seem a low number for the empirical growth rate of research labor (scientists and engineers) in the US and other countries over the last century. On the other hand, for simplicity our model has ignored the likely duplication externality due to overlap in R&D at the economy-wide level. Taking that overlap into account, we should replace $\bar{h}L_A$ in (10.4) by $(\bar{h}L_A)^{1-\pi}$, and n in (10.20) by $(1 - \pi)n$, where $\pi \in (0, 1)$ measures the extent of duplication. Jones (1995) suggests $\pi = 0.5$.

[9] Although distinguishing between human capital and knowledge creation, the approach by Dalgaard and Kreiner (2001) is very different from the one we have followed above and has affinity partly with Lucas and partly with Mankiw et al. (1992).

In the above analysis we have ignored the role of scarce natural resources for limits to growth. We will come back to this issue in chapters 13 and 16.

We have ruled out $\varphi = 1$ because in combination with $n > 0$ it would tend to generate a forever growing productivity growth rate, a feature not in accordance with the actual economic evolution of the industrialized world over the last century. We have ruled out $\varphi > 1$ because in combination even with $n = 0$, it would tend to generate economic "explosion" in a very dramatic and implausible sense: infinite output in finite time! Jones (2005) argues that the empirical evidence speaks for $\varphi < 1$ in modern times.

The above analysis simply tells us what the growth rate *must* be in the long run provided that the system considered converges to balanced growth. On the other hand, specification of the market structure and the household sector, including demography and preferences, will be needed if we want to study the adjustment processes outside balanced growth or determine an equilibrium real interest rate or similar.

It is due to the semi-endogenous growth setting (the $\varphi < 1$ assumption) that one can find the long-run per capita growth rate from knowledge of technology parameters and the rate of population growth alone. How the market structure and the household sector are described, is immaterial for the long-run growth rate. These things will in the long run have "only" *level* effects.

Only if economic policy affects the technology parameters or the population growth rate, will it be able to affect the long-run growth rate. Still, economic policy can *temporarily* affect economic growth and in this way affect the *level* of the long-run growth path.

10.7 References

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