Problem set for midterm paper

This is the mandatory homework assignment in Economic growth, Spring 2014. You are encouraged to solve the assignment together with fellow students (max. four students per group). *If you do not know the other students, but would like to have one or two co-workers, you may send me an e-mail about it. Then I might be able to match you with somebody else in the same situation.* You may write your paper in English or Scandinavian as you prefer. As to the text and mathematical formulas in the paper you may use either your computer or readable handwriting. For the present assignment you will need a computer or a pocket calculator to produce some of the numerical results. The maximum accepted number of pages is 8 (10 if hand writing). Be brief and precise in your text. Be careful with your language.

Time table for course activity during the homework/workshop period:

Tuesday March 24 at 16:00. The homework/workshop assignment is posted at the course website.

Thursday March 26 class 8:15-10:00. At least the last half of the exercise class will take the form of a workshop where Niklas Brønager acts as a group consultant. NB! Each group should bring at least one *laptop* **with MATLAB installed**. You can download MATLAB from the Software Library at KUnet (intranet.ku.dk). It is recommended that you also, in advance, go to

http://www.econ.ku.dk/okocg/Computation/Relaxation%20mm%202011/relax.html

and download the following material: 1) the system files for the Relaxation Algorithm, 2) the Instruction Manual, and 3) the MATLAB code for "Ramsey model with a public sector".

Monday March 30. Lecture as usual.

Thursday April 9 class 8:15-10:00. Again a part of the class will be in homework-workshop form.

Deadline for your paper is: Monday April 13 at 10:15. You may either upload your paper, as a pdf-file, in Absalon before deadline (then you automatically have a kind of acknowledgement) or give the paper to me at the lecture that Monday. The *deadline is a must*! The **front page** of your paper should provide the following information:

1) Full names of all the authors. 2) Date. 3) Name of the course and the lecturer.

I evaluate your paper (“accepted” or “not accepted”) and return it, with comments, as soon as possible. The paper must be accepted (grade at least 2) in order to go in for the final written exam.

Good luck! Christian Groth

The weights of the problems are:

Problem 1: 25 %, Problem 2: 40 %, Problem 3: 15 %, Problem 4: 5 %, Problem 5: 15 %.

Problem 1 *Taxation, saving, and economic evolution* Consider a Ramsey model, in continuous time, of a closed competitive market economy with public consumption, lump-sum taxes (positive or negative), and capital income taxation. The government satisfies its intertemporal budget constraint. The model leads to the following differential equations

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \tilde{c}_t - \tilde{\gamma} - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \tag{1}$$

$$\dot{\tilde{c}}_t = \frac{1}{\theta}\left[(1-\tau_r)(f'(\tilde{k}_t) - \delta) - \rho - \theta g\right]\tilde{c}_t, \tag{2}$$

and the condition

$$\lim_{t\to\infty} \tilde{k}_t e^{-\int_0^t \left[(1-\tau_r)(f'(\tilde{k}_s)-\delta)-g-n\right]ds} = 0. \tag{3}$$

Notation is: $\tilde{k}_t \equiv K_t/(A_t L_t)$ and $\tilde{c}_t \equiv C_t/(A_t L_t) \equiv c_t/A_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively, and L_t is population = labor supply, all at time t. Further, A_t is a measure of the technology level and $\tilde{\gamma} \equiv G_t/(A_t L_t)$, where G_t is government consumption. Finally, f is a production function in intensive form, derived from an aggregate neoclassical CRS production function. The remaining symbols stand for parameters and all these are positive.

We assume that capital is essential and that the instantaneous utility function is

$$u(c, G) = \frac{c^{1-\theta}}{1-\theta} + v(G), \qquad \theta > 0. \tag{4}$$

We further assume that

$$\lim_{\tilde{k}\to 0} f'(\tilde{k}) - \delta > \frac{\rho + \theta g}{1-\tau_r} > n + g > \lim_{\tilde{k}\to\infty} f'(\tilde{k}) - \delta. \tag{5}$$

The government controls $\tilde{\gamma}$ (> 0) together with $\tau_r \in [0, 1)$ and the lump-sum taxes. Until further notice τ_r is kept constant over time; $\tilde{\gamma}$ is kept constant throughout and is of moderate size so as to not prevent existence of a steady state.

a) Briefly interpret (1) - (5), including the parameters.

b) Draw a phase diagram and illustrate, for a given $\tilde{k}_0 > 0$, the path which the economy follows. Comment.

c) Is it possible for a non-trivial steady state to exist without assuming f satisfies the Inada conditions? Why or why not?

d) How does the long-run level of per capita consumption depend on θ? Explain the intuition.

e) How does the long-run real interest rate depend on g? Explain the intuition.

Problem 2 *Aspects of transitional dynamics* This problem is about the same model as Problem 1, but answering the questions requires numerical simulation on a computer. Since consumption in the model is a forward-looking variable, our (nonlinear) two-dimensional dynamic system has only one pre-determined variable. The solution path depends on the exogenous initial value of this variable and on the transversality condition which in this model amounts to a requirement of convergence to a unique steady state which is a saddle point. The solution path in the phase diagram therefore coincides with a section of a saddle path. We need a computer algorithm capable of finding this (generally nonlinear) saddle path. One such algorithm is known as the Relaxation Algorithm which you can run in MATLAB, see my personal website:

http://web.econ.ku.dk/okocg/Computation/Relaxation%20mm%202011/relax.html[1]

Let the function f be Cobb-Douglas with output elasticity w.r.t. capital equal to $\alpha = 1/3$. Let $\theta = 1$, $\rho = 0.02$, $g = 0.015$, $n = 0.005$, and $\delta = 0.05$, given that the time unit is one year. As to the policy parameters, let $\tilde{\gamma} = 0.1$ and $\tau_r = 0.2$. Your paper should include figures with the plots mentioned below.

a) Let $\tilde{k}_0$ equal $0.6{\cdot}\tilde{k}^*$, where $\tilde{k}^*$ is the steady-state value of $\tilde{k}$. Simulate the model solution and plot the time profile for $\tilde{k}_t$ and $\tilde{c}_t$ for $t \geq 0$.

The Ramsey model package in MATLAB also calculates the half-life of the distance of $\tilde{k}$ from its steady-state value. Let us see how this feature can be utilized.

[1] You are welcome to use other methods if you master them yourself. In the exercise class there will be help if you use the Relaxation Algorithm in MATLAB.

b) Let the time path of a variable x be $x_t = x_0 e^{-\beta t}$, where β is a positive constant. Show analytically that (i) the instantaneous speed of convergence (SOC_t) in this case equals β and that (ii) the half-life of the distance of x from its long-run value is

$$h = \frac{\ln 2}{\beta} \text{ years.} \tag{h}$$

Hint: the instantaneous speed of convergence at time t (SOC_t) of a monotonically converging variable is defined as the (proportionate) rate of decline at time t of the distance to the steady-state value:

$$\text{SOC}_t = -\frac{d(|x_t - x^*|)/dt}{|x_t - x^*|}.$$

Generally in growth models which converge to a steady state, SOC_t for a given variable is not exactly constant during the adjustment, but depends on the size and the sign of $x - x^*$ at time t (that is why the qualifier "instantaneous" is added).[2] SOC_t converges for $t \to \infty$ to the *asymptotic SOC,* which can therefore be regarded as an approximation of SOC_t.[3] The Ramsey model package in MATLAB calculates both the half-life and the asymptotic SOC for $\tilde{k}$.

c) For the simulation in a), report the half-life of the adjustment of $\tilde{k}$ and the asymptotic SOC.

d) From knowledge of the half-life one can on the basis of equation (h) calculate the "average SOC" for $\tilde{k}$ during the first half-life. Do that. *Hint:* interpret the difference $\tilde{k}_t - \tilde{k}^*$ as x_t and apply equation (h).[4]

e) Compare your result in d) with both the average SOC and the asymptotic SOC as reported by MATLAB.

f) If we know that a model converges to a steady state, why bother about measures of speed of convergence?

g) Let the value of $\tilde{k}_0$ equal $0.5{\cdot}\tilde{k}^*$. Simulate the model and plot the time profile for $\tilde{k}_t$ and $\tilde{c}_t$ in the cases $\theta = 1$ and $\theta = 10$, respectively (the other parameters unchanged).

[2] An exception is the Solow model with Cobb-Douglas production function, see Exercise II.2. Here the SOC_t for the capital-output ratio is constant and equal to the asymptotic SOC for the capital-output ratio.

[3] Mathematically, in the Ramsey model the asymptotic SOC equals the absolute value of the negative eigenvalue of the Jacobian matrix based on the right-hand sides of (1) and (2) and evaluated in the steady state. This needs not distract us here, however.

[4] If the instantaneous rate of decline of a variable x at time t is $m(t)$, then the *average rate of decline* in the time interval $[0, T]$ is $\mu = (\int_0^T m(t)dt)/T$ so that $x_T = x_0 e^{-\mu T}$.

h) Report the percentage deviation of $\tilde{c}_0$ from $\tilde{c}^*$ in the two cases. Give an intuitive explanation of the sign of the difference between these two percentage deviations.

Problem 3 *Is the end of the Malthusian epoch inevitable?* Suppose a pre-industrial economy can be described by:

$$\begin{aligned} Y_t &= A_t^{\sigma} L_t^{\alpha} Z^{1-\alpha}, \qquad \sigma > 0, 0 < \alpha < 1, \\ \dot{A}_t &= \lambda A_t^{\varepsilon} L_t, \qquad \lambda > 0, \varepsilon > 0, \qquad A_0 > 0 \text{ given}, \\ L_t &= \frac{Y_t}{\bar{y}}, \qquad \bar{y} > 0, \end{aligned}$$

where Y is aggregate output, A the level of technical knowledge, L the labor force (= population), Z the amount of land (fixed), and $\bar{y}$ subsistence minimum. Both Z and $\bar{y}$ are considered as constant parameters. Time is continuous and it is understood that a kind of Malthusian population mechanism is operative behind the scene.

a) The data in Kremer (1993, p. 682) indicates that the global population growth rate has been more or less proportional to the size of population until the1960s. Derive a value of the paramater ε consistent with this observation. *Hint:* a power function $z_t = bx_t^{\alpha}$, where a and b are constants ($b \neq 0$), has growth rate $g_z = ag_x$.

b) Does your answer help to assess which of the two versions of the "population-breeds-ideas model" in Acemoglu, p. 113-114, is the more plausible one? Why or why not?

Problem 4 *Technical attributes of different goods* Economic goods may be distinguished according to whether they are *rival* or *nonrival* and whether they are *excludable* or *nonexcludable*.

a) Define these concepts.

b) Draw a 2×2 table with the rival-nonrival distinction in the head of the table, thus defining the two columns, and the excludable-nonexcludable distinction defining the two rows. Enter some examples related to economic growth theory in the different cells.

Continued next page.

Problem 5 *Short questions*

a) "If there are constant returns to scale with respect to physical capital, labor, and land taken together, then, considering technical knowledge as a fourth production factor, there will be increasing returns w.r.t. to all four production factors taken together." True or false? Explain why.

b) In cross-country growth regressions the estimated coefficient to country size (as measured by the log of population) is generally found to be close to zero and not significant (see for instance Barro and Sala-i-Martin, Economic Growth, 2nd ed., pp. 536-537). Some people interpret this finding as a rejection of the hypothesis that a larger population means larger capacity to create ideas. There is a serious problem, however, associated with using cross-country regression analysis as a test of this hypothesis. What can this problem be?

c) An important aspect of growth analysis is to pose good questions in the sense of questions that are both interesting and manageable. If we set aside some time for discussion in one of the remaining lectures, what question would you suggest should be discussed? Please, state your question in English.

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