Economic Growth Short Note 1.

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1 Skill-biased technical change in the sense of Hicks: An example

Let output be produced through a differentiable three-factor production function $\tilde{F}$:

$$Y = \tilde{F}(K, L_1, L_2, t), \qquad \partial \tilde{F}/\partial t > 0,$$

where K is capital input, L_1 is input of unskilled labor, and L_2 is input of skilled labor. Suppose technological change is such that the production function can be rewritten

$$\tilde{F}(K, L_1, L_2, t) = F(K, H(L_1, L_2, t)), \tag{1}$$

where the function $H(L_1, L_2, t)$ represents a "human capital" aggregate. Let the function H have CRS-neoclassical properties w.r.t. (L_1, L_2) and let $\partial H/\partial t > 0$.

In equilibrium under perfect competition in the labor markets the relative wage, the "skill premium", will be

$$\frac{w_2}{w_1} = \frac{\partial Y/\partial L_2}{\partial Y/\partial L_1} = \frac{F_H \partial H/\partial L_2}{F_H \partial H/\partial L_1} = \frac{H_2(L_1, L_2, t)}{H_1(L_1, L_2, t)} = \frac{H_2(1, L_2/L_1, t)}{H_1(1, L_2/L_1, t)}, \tag{2}$$

where we have used Euler's theorem[1] (saying that if H is homogeneous of degree one in its first two arguments, then the partial derivatives of H are homogeneous of degree zero w.r.t. these arguments).

Hicks' definitions are now: If for all $L_2/L_1 > 0$,

$$\frac{d\left(\frac{H_2(1,L_2/L_1,t)}{H_1(1,L_2/L_1,t)}\right)}{dt}\Bigg|_{\frac{L_2}{L_1}\text{ constant}} \gtreqless 0, \text{ then technical change is}$$

$$\left\{ \begin{array}{c} \text{skill-biased in the sense of Hicks,} \\ \text{skill-neutral in the sense of Hicks.} \\ \text{blue collar-biased in the sense of Hicks,} \end{array} \right. \tag{3}$$

respectively. Combining with (2), we see that if the skill-premium has an upward trend for fixed relative supplies of skilled and unskilled labor, a possible explanation is that technological change is skill-biased in the sense of Hicks.

[1] Acemoglu, p. 29.

In the US the skill premium (measured by the wage ratio for college grads vis-a-vis high school grads) has had an upward trend since 1950 (see Jones and Romer, 2010).[2] If in the same period the relative supply of skilled labor had been roughly constant, a suggested explanation could be skill-biased technical change. In practice the relative supply of skilled labor has also been rising over the same period (in fact even faster than the skill premium). This suggests that the extend of "skill-biasedness" has been even stronger.[3]

An additional aspect of the story is that skill-biasedness helps *explain* the observed increase in the relative *supply* of skilled labor. If for a constant relative supply of skilled labor the skill premium is increasing, this increase strengthens the incentive to go to college. Thereby the fraction of skilled labor in the labor force tends to increase.

2 Capital-skill complementarity

Another potential source of a rising skill premium is *capital-skill complementarity*. Consider the production function

$$Y = \tilde{F}(K, L_1, L_2, t) = F(K, A_{1t}L_1, A_{2t}L_2) = (K + A_{1t}L_1)^{\alpha}(A_{2t}L_2)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where A_{1t} and A_{2t} are technical coefficients that may be rising over time. In this production function capital and unskilled labor are perfectly substitutable (the partial elasticity of factor substitution is $+\infty$). On the other hand there is *direct complementarity* between capital and skilled labor ($\partial^2 Y/(\partial L_2 \partial K) > 0$).

In equilibrium under perfect competition the skill premium is

$$\frac{w_2}{w_1} = \frac{\partial Y/\partial L_2}{\partial Y/\partial L_2} = \frac{(K + A_{1t}L_1)^{\alpha}(1-\alpha)(A_{2t}L_2)^{-\alpha}A_{2t}}{\alpha(K + A_{1t}L_1)^{\alpha-1}A_{1t}(A_{2t}L_2)^{1-\alpha}} = \frac{1-\alpha}{\alpha}\left(\frac{K + A_{1t}L_1}{A_{2t}L_2}\right)\frac{A_{2t}}{A_{1t}}. \qquad (4)$$

Here, even without technical change (A_{1t} and A_{2t} constant), a rising capital stock will, for fixed L_1 and L_2, raise the skill premium.

Equilibrium under perfect competition also implies

$$\frac{\partial Y}{\partial K} = \alpha(K + A_{1t}L_1)^{\alpha-1}(A_{2t}L_2)^{1-\alpha} = \alpha\left(\frac{K + A_{1t}L_1}{A_{2t}L_2}\right)^{\alpha-1} = r_t + \delta, \qquad (5)$$

[2] On the other hand, over the years 1915 - 1950 the skill premium had a downward trend (Jones and Romer, 2010).

[3] As the H function has CRS-neoclassical properties w.r.t. L_1 and L_2, $H_{22} < 0$ and $H_{12} > 0$, cf. LN 2. Hence, with skill-neutral technical change we should have observed a *declining* skill premium (even more so with blue collar-biased technical change).

where r_t is the real interest rate at time t and δ is the (constant) capital depreciation rate. If in the long run r_t tends to be constant (cf. Kaldor's stylized facts), then also $(K + A_{1t}L_1)/(A_{2t}L_2)$ will tend to be constant. In this case, (4) shows that capital-skill complementarity is *not sufficient* for a rising skill premium. For the skill premium to remain increasing in this case, we need that technical change brings about a rising A_{2t}/A_{1t}. This amounts to skill-biasedness in a strong form.

The above observations are consistent with a story where capital equipment gradually replaces unskilled labor and a rising skill premium induces more and more people to go to college. The rising level of education in the labor force contributes to productivity. This together with continued technical change constitutes the basis for further capital accumulation and productivity increases.

In particular since the early 1980s the skill premium has been sharply increasing in the US (see Acemoglu, p. 498). This is also the period where ICT technologies took off.

3 Literature

Duffy. J., C. Papageorgiou, and F. Perez-Sebastian, 2004, Capital-Skill Complementarity? Evidence from a Panel of Countries, *The Review of Economics and Statistics*, vol. 86(1), 327-344.

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