

# Lecture Notes in Economic Growth

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# Preface

This is a collection of earlier separate lecture notes in Economic Growth. The notes have been used in recent years in the course Economic Growth within the Master's Program in Economics at the Department of Economics, University of Copenhagen.

Compared with the earlier versions of the lecture notes some chapters have been extended and in some cases divided into several chapters. In addition, discovered typos and similar have been corrected. In some of the chapters a terminal list of references is at present lacking.

The lecture notes are in no way intended as a substitute for the textbook: D. Acemoglu, *Introduction to Modern Economic Growth*, Princeton University Press, 2009. The lecture notes are meant to be read along with the textbook. Some parts of the lecture notes are alternative presentations of stuff also covered by the textbook, while many other parts are complementary in the sense of presenting additional material. Sections marked by an asterisk, \*, are cursory reading.

For constructive criticism I thank Niklas Brønager, class instructor since 2012, and plenty of earlier students. No doubt, obscurities remain. Hence, I very much welcome comments and suggestions of any kind relating to these lecture notes.

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