Chapter 8

Natural resources and economic growth

VIII.1 *Natural resources and endogenous growth* There is an aggregate production function for manufacturing goods,

$$Y_t = A_t^{\varepsilon} K_t^{\alpha} L_t^{\beta} R_t^{\gamma} J^{\lambda}, \qquad \varepsilon, \alpha, \beta, \gamma, \lambda > 0,\ \alpha + \beta + \gamma + \lambda = 1, \tag{8.1}$$

where K_t, L_t, R_t, and J are inputs of capital, labor, a non-renewable resource, and land (a renewable resource), respectively, per time unit at time t. The amount of land is considered an exogenous constant. Total factor productivity is A_t^{ε} where the variable A_t is assumed proportional to the stock of technical knowledge accumulated through R&D investment. Due to this proportionality we simply identify A_t with the stock of knowledge at time t.

Aggregate output is used for consumption, C_t, investment, I_{Kt}, in physical capital, and investment, I_{At}, in R&D,

$$C_t + I_t \equiv C_t + I_{Kt} + I_{At} = Y_t.$$

Accumulation of capital occurs according to

$$\dot{K} = I_K - \delta K = Y - C - I_A - \delta K, \qquad \delta \geq 0, \qquad K_0 > 0 \text{ given}, \tag{8.2}$$

where δ is the rate of depreciation of capital. Accumulation of knowledge occurs through R&D investment, I_A,

$$\dot{A} = I_A - \delta A, \qquad \delta \geq 0 \qquad A_0 > 0 \text{ given}. \tag{8.3}$$

We allow depreciation of knowledge in order to take into account the possibility that as technology advances, old knowledge becomes obsolete. To simplify the dynamics, we assume that the rate of depreciation of knowledge is the same as that of capital, δ.

Extraction of the non-renewable resource is described by

$$\dot{S}_t = -R_t \equiv -u_t S_t, \qquad S_0 > 0 \text{ given}, \tag{8.4}$$

where S_t is the stock of the non-renewable resource (e.g., oil reserves) and u_t is the depletion rate. Extraction involves no costs apart from the depletion of the reserves. Since we must have $S_t \geq 0$ for all t, there is a finite upper bound on cumulative resource extraction:

$$\int_0^\infty R_t dt \leq S_0. \tag{8.5}$$

Finally, population (= labor force) grows according to

$$L_t = L_0 e^{nt}, \qquad n \geq 0, \qquad L_0 > 0 \text{ given}.$$

Uncertainty is ignored.

a) With respect to the way knowledge creation is modeled, the model has affinity with one of the model types encountered earlier in this course. What is the name of this model type.

We shall in this exercise concentrate on *(technically) efficient paths*, i.e., paths such that consumption can not be increased in some time interval without being decreased in another time interval.

b) Find the value of the knowledge-capital ratio, A/K, at which the net marginal productivities of A and K are the same. Denote your result $\bar{x}$.

c) Suppose $A_0/K_0 > \bar{x}$ (Case 1). An efficient economy with $I_t > 0$ for all $t \geq 0$ will then for a while invest only in capital, i.e., there will be a phase where $I = I_K$ and $I_A = 0$. Why?

d) At some finite time $\bar{t} > 0$ it will hold that $A_{\bar{t}}/K_{\bar{t}} = \bar{x}$. Why?

e) For all $t > \bar{t}$ it will hold that $A_t/K_t = \bar{x}$. Why? Find the value of $a \equiv I_A/I$ for $t > \bar{t}$ and sketch the time profile of A/K in the $(t, A/K)$ plane for $t \geq 0$. *Hint:* Let $x_t \equiv A_t/K_t$ and consider the equation $\dot{x}_t/x_t = 0$.

f) Suppose instead that $A_0/K_0 > \bar{x}$ (Case 2). Sketch the time profile of A/K in the $(t, A/K)$ plane in this case.

g) To fix ideas, return to Case 1. For $t > \bar{t}$, the evolution of the economy can be described in terms of a single measure of "hybrid capital" $\tilde{K} \equiv K + A$. Show that this addition makes sense and show why aggregate output in manufacturing can for $t > \bar{t}$ be written:

$$Y = B\tilde{K}^{\tilde{\alpha}}L^{\beta}R^{\gamma}J^{\lambda}, \tag{8.6}$$

where B is a positive constant and $\tilde{\alpha} + \beta + \gamma + \lambda > 1$. Find $\tilde{\alpha}$.

h) Show that the accumulation of $\tilde{K}$ for $t > \bar{t}$ is given by

$$\dot{\tilde{K}}_t = \dot{A}_t + \dot{K}_t = Y_t - c_t L_t - \delta\tilde{K}_t, \tag{8.7}$$

where c_t is per capita consumption.

Let g_z denote the growth rate, $\dot{z}/z$, of any smooth time-dependent variable z. Define a balanced growth path (BGP) as a path along which g_K, g_Y, g_C and g_S are constant.

i) Along a BGP the depletion rate, u, is a constant as well and satisfies

$$\begin{aligned} u &\equiv R/S = -\dot{S}/S \equiv -g_S > 0, \qquad \text{and} \\ g_R &= g_S = -u. \end{aligned}$$

Why is this true? Show that along a BGP,

$$(1 - \tilde{\alpha})g_c + \gamma u = (\tilde{\alpha} + \beta - 1)n. \tag{*}$$

Hint: In (8.6) log-differentiate w.r.t. t on both sides (as in growth accounting) and apply the Balanced Growth Equivalence Theorem.

j) Show that a BGP has $g_c > 0$ if and only if

$$(\tilde{\alpha} + \beta - 1)n > 0 \qquad \text{or} \qquad \tilde{\alpha} > 1.$$

k) Based on Nordhaus (1992), $\alpha \approx 0.2, \beta \approx 0.6, \gamma \approx 0.1$, and $\lambda \approx 0.1$ seem reasonable. What is then the greatest lower bound for ε if:

(i) semi-endogenous growth should be technically feasible?

(ii) fully endogenous growth should be technically feasible?

ℓ) "Along a BGP, policies that decrease (increase) the depletion rate u (and only such policies) will increase (decrease) the per capita growth rate (here we presuppose $\tilde{\alpha} < 1$, the plausible case)". True or false? Why?

VIII.2 *Adding preferences and a social planner* We consider the same framework and use the same notation as in Problem VIII.1 (it is an advantage if you have already solved that problem). We "close" the model by adding Ramsey households with CRRA preferences (θ, ρ), where $\theta > 0$ and $\rho \geq n \geq 0$, and a utilitarian social planner. The social planner chooses a path $(c_t, R_t)_{t=0}^{\infty}$ so at to maximize

$$U_0 = \int_0^\infty \frac{c_t{}^{1-\theta}}{1-\theta} L_t e^{-\rho t} dt, \tag{8.8}$$

subject to the constraints given by technology ((8.6), (8.7) and (8.4)) and initial conditions. It can be shown that the first-order conditions lead to the equation

$$(1-\theta)g_c + u = \rho - n. \tag{**}$$

a) The equations (*) (from Exercise VIII.1) and (**) constitute a linear equation system with two unknowns, g_c and u. Find g_c and u, assuming that the determinant $D = 1 - \tilde{\alpha} - \gamma + \theta\gamma > 0$.

Suppose a journal article on this basis states the following:

(i) If there is impatience ($\rho > 0$), then even when a non-negative g_c is technically feasible, a negative g_c can be optimal.

(ii) Population growth is *good* for economic growth. In its absence, when $\rho > 0$, we get $g_c < 0$ along an optimal BGP; if $\rho = 0$, $g_c = 0$ when $n = 0$.

b) For each of these claims, check its validity.