Written exam for the M. Sc. in Economics. Summer 2014

Economic Growth

Master's Course

June 4, 2014

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total including this page.

The weighting of the problems is: Problem 1: 30%, Problem 2: 55%, and Problem 3: 15%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a closed market economy with perfect competition in all markets. Firm no. i, $i = 1, 2, ..., N$, where N is "large", has the production function

$$Y_{it} = F(K_{it}, T_t L_{it}),$$

where F is a neoclassical production function with CRS and satisfying the Inada conditions, L_{it} is labor input, and K_{it} is capital input. It is assumed that the technology level T_t satisfies

$$T_t = K_t^{\lambda}, \qquad 0 < \lambda \leq 1,$$

where $K_t = \sum_i K_{it}$. Time, t, is continuous. There is no uncertainty. The size of the labor force is $L_t = L_0 e^{nt}$, where n is a constant ≥ 0. Aggregate output is Y_t per time unit and is used for consumption, $C_t \equiv c_t L_t$, and investment in physical capital so that

$$\dot{K}_t \equiv \frac{dK_t}{dt} = Y_t - C_t - \delta K_t, \qquad \delta > 0, \qquad K_0 > 0 \text{ given.}$$

a) In general equilibrium, determine the real interest rate, r, and the aggregate production function at time t. Comment.

From now on we consider a special case of this framework, namely a case where $\lambda = 1$ and $n = 0$. Moreover, there is a fixed number, L, of Ramsey-style households with instantaneous CRRA utility with parameter $\theta > 0$ and a constant rate of time preference, ρ. Each household supplies inelastically one unit of labor per time unit. Until further notice there are no taxes and subsidies.

b) Determine the equilibrium real interest rate, r, and the aggregate production function under these circumstances. Comment.

c) Determine the equilibrium growth rate of c_t and name it g_c. (You do not have to *derive* the Keynes-Ramsey rule but may use your general knowledge about this rule to answer the question.)

From now on assume (A1) $F_1(1, L) - \delta > \rho$ and (A2) $\rho > (1 - \theta) g_c$.

d) What could be the motivation for these two assumptions?

e) Determine the growth rate of $k \equiv K/L$ and $y \equiv Y/L$. A detailed derivation involving the transversality condition need not be given. Instead you may refer to a general property of AK and reduced-form AK models in a Ramsey framework.

We introduce a government which contemplates to implement a subsidy to households' saving such that the after-subsidy rate of return on private saving is $(1+\sigma)r$. Let the subsidy be financed by a lump-sum tax on all households.

f) Determine σ so as to establish in a decentralized way the allocation of a social planner with the same criterion function as that of the representative household. *Hint:* Set up the social planner's problem, derive the first-order conditions and the TVC. Determine the implied growth rate of c_t. Next, use your general knowledge about reduced-form AK models to determine the growth rates of k_t and y_t (a brief verbal account is enough). Finally, use that for σ to be optimal, σ should ensure that the net rate of return on saving faced by the consumer equals the net rate of return to capital investment implied by the aggregate production technology.

Problem 2 In the simple increasing variety model (Model I) the representative firm in the basic-goods sector has the production function

$$Y_t = A\left(\sum_{i=1}^{N_t} {x_{it}}^{1-\beta}\right) L^\beta, \qquad A > 0,\ 0 < \beta < 1, \tag{1}$$

where Y_t is output in the sector, x_{it} is input of intermediate good i $(i = 1, 2, ..., N_t)$, N_t is the number of different types of intermediate goods available at time t, and L is labor input (here equalized to the constant labor supply in the model). In the specialized *intermediate-goods* sector firms face marginal costs $\psi > 0$ and in the R&D sector there is a constant research productivity $\eta > 0$. A defining characteristic of Model I is the assumption that once the technical design for intermediate good i has been invented in the R&D sector, the inventor can take out (free of charge) an effective perpetual patent on the commercial use of this design.

We shall consider some of the results forthcoming when this assumption is replaced by the assumption that the duration of monopoly power over the commercial use of an invention is *limited* and *uncertain.* More precisely we shall assume that cessation of monopoly power can be described as a Poisson process with an exogenous Poisson arrival rate $\lambda > 0$, the same for all monopolies. This refers to what is in our syllabus called Model II.

To help memory about Model II, here is some information: In equilibrium aggregate output in the basic-goods sector is $Y_t = \left[1 + ((1-\beta)^{-(1-\beta)/\beta} - 1)\frac{N_t^{(c)}}{N_t}\right] Y_t^{(m)}$, where $Y_t^{(m)} \equiv AN_t(x^{(m)})^{1-\beta}L^\beta$ is the equilibrium output of basic goods in Model I and $N_t^{(c)}$

denotes the number of intermediate-goods varieties that at time t are competitively supplied while the remainder, $N_t - N_t^{(c)}$, are still supplied under monopolistic conditions; moreover, $x^{(m)} \equiv (A(1-\beta)^2/\psi)^{1/\beta} L$.

a) Explain, both formally and intuitively, why in equilibrium in Model II the aggregate output in the basic-goods sector is larger than in Model I.

b) In both Model I and Model II it can be shown that in an equilibrium with active R&D, the market value, V_t, of a monopoly satisfies the equation $V_t\eta = 1$. Give an intuitive explanation of this result.

c) In Model II find the real interest rate in equilibrium with active R&D. *Hint:* Combine the result in b) with the no-arbitrage condition $(\pi^{(m)} + \dot{V}_t^{(+)} - \lambda V_t)/V_t = r_t$, where $\pi^{(m)} = [\beta/(1-\beta)]\,\psi x^{(m)}$ is profit per time unit of a monopoly firm and $\dot{V}_t^{(+)}$ is the increase per time unit in the market value of the monopoly firm at time t, conditional on its monopoly position remaining in place also in the next moment.

d) Comment on your result and compare with the real interest rate in an equilibrium with active R&D in Model I. Give a brief intuitive explanation of the difference.

e) As in Model I, the households in Model II are Ramsey households with parameters θ and ρ. Find the growth rate, g_c^{II}, of per capita consumption in Model II. Comment. (You do not have to *derive* the Keynes-Ramsey rule but may use your general knowledge about this rule to answer the question.)

f) State a parameter condition ensuring that your result in e) gives a positive rate. Further, state a parameter condition ensuring boundedness of the utility integral in Model II.

For Model II it can be shown that:

(i) $s_t \equiv N_t^{(c)}/N_t$ approaches $s^* = \lambda/(g_N^* + \lambda)$ over time, where g_N^* is the long-run growth rate of N_t; and

(ii) $g_N^* = g_c^{II} \equiv g^*$.

g) How does the size of A, η, and λ, respectively, affect the long-run growth rate, g^*? Comment.

h) Model II illustrates a classical dilemma in antitrust policy and patent legislation. Explain.

i) Now consider a government in the economy described by Model II. Suppose the social welfare function coincides with the intertemporal utility function of the representative Ramsey household. Let the government choose to pay a subsidy at constant rate, σ, to purchases of any monopolistically supplied intermediate good, i, such that the net price of this intermediate good is $(1-\sigma)p_i$, where $p_i = \psi/(1-\beta)$ is the price set by the monopolist supplier. Suppose you, as an economic advisor, are asked by the government to suggest an appropriate value for σ. What value of σ would you suggest? Why?

j) Suppose the government contemplates whether to implement, in addition, a subsidy, s, to reduce research cost, $1/\eta$, per expected invention. Would your recommendation be that it should do this or not? Give your argument.

k) As to the financing, the government plans to finance its subsidy policy by taxing consumption. Will it be possible to do this in a non-distortionary way, given that an optimal subsidy policy has been chosen and that a balanced budget is maintained forever? *Hint:* Under this condition the economy behaves as a reduced-form AK economy.

Problem 3 *Short questions*

a) In poor countries capital per worker, measured as K/L, tends to be much lower than in rich countries. Can we, while accepting the neoclassical assumption of diminishing marginal productivity of capital, explain why the capital flows from rich to poor countries are not much larger than they are? Why or why not?

b) In our syllabus there are models where externalities in R&D play a role. Give two examples, one being an inter-temporal externality, the other an intra-temporal externality.

c) In the lab-equipment model the aggregate number of new technical designs (inventions) per time unit is

$$\dot{N}_t \equiv \frac{dN_t}{dt} = \eta Z_t, \qquad \eta > 0,$$

where Z_t is the aggregate R&D investment per time unit, measured in basic goods. Also this model could be extended with either a negative or a positive intra-temporal externality in R&D. Suggest how. Give an interpretation of each of the two cases.

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