Economic Growth, August 2013.

Christian Groth

A suggested solution to the problem set at the re-exam in Economic Growth, August 14, 2013

(3-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates precise understanding of the concepts and methods needed for analyzing the factors that matter for economic growth.

1. Solution to Problem 1 (40 %)

Consider a simple closed economy where two goods get produced: corn and new ideas. Time is continuous. The labor force (in this problem the same as population), L_t, grows at an exogenous constant rate $n \geq 0$, i.e., $L_t = L_0 e^{nt}$. People can work as farmers, L_{Yt}, or researchers, L_{At}, where $L_{Yt} + L_{At} = L_t$ and $L_{At} = s_A L_t$, where, for simplicity, s_A is assumed constant, $0 < s_A < 1$. The production functions are:

$$Y_t = A_t^\sigma J^\alpha L_{Yt}^{1-\alpha}, \qquad \sigma > 0, 0 < \alpha < 1, \tag{*}$$

$$\dot{A}_t = \mu A_t^\varphi L_{At}, \qquad \mu > 0, \varphi < 1, \tag{**}$$

where Y_t is corn output, J is the amount of land (fixed), and A_t the level of technical knowledge. Let the growth rate of a variable $x > 0$ at a given point in time be denoted g_x (not necessarily a constant). Let $y \equiv Y/L$.

a) From (*) follows

$$y_t \equiv \frac{Y_t}{L_t} = A_t^\sigma J^\alpha L_{Yt}^{-\alpha} = A_t^\sigma J^\alpha \left[(1 - s_A) L_t\right]^{-\alpha},$$

so that

$$g_y = \sigma g_A - \alpha g_L = \sigma g_A - \alpha n. \tag{1.1}$$

[1] The solution below contains *more* details and more precision than can be expected at a three hours exam. The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

b) From (**) follows

$$g_A \equiv \frac{\dot{A}_t}{A_t} = \mu A_t^{\varphi-1} s_A L_t > 0,$$

since $s_A > 0$.

c)

$$\frac{\dot{g}_A}{g_A} = (\varphi - 1)g_A + n = n - (1-\varphi)g_A,$$

so that

$$\dot{g}_A = [n - (1-\varphi)g_A]\, g_A \gtreqless 0 \text{ for } g_A \lesseqgtr \frac{n}{1-\varphi},$$

respectively. Hence

$$g_A \longrightarrow \frac{n}{1-\varphi} \equiv g_A^* \text{ for } t \longrightarrow \infty.$$

d) By (1.1),

$$g_y = \sigma g_A - \alpha n \longrightarrow_{t\to\infty} \sigma \frac{n}{1-\varphi} - \alpha n = \left(\frac{\sigma}{1-\varphi} - \alpha\right) n \equiv g_y^*. \tag{1.2}$$

e) When $n > 0$, (1.2) implies

$$g_y^* \gtreqless 0 \text{ for } \frac{\sigma}{1-\varphi} \gtreqless \alpha, \tag{1.3}$$

respectively.

f) From (1.3) follows that when $n > 0$, we have $g_y^* > 0$ if and only if $\sigma/(1-\varphi) > \alpha$. In this case (1.2) implies

$$\frac{\partial g_y^*}{\partial n} = \frac{\sigma}{1-\varphi} - \alpha > 0.$$

The economic intuition behind this is that per capita growth in this model is driven by knowledge creation and knowledge is a nonrival good. This implies a *scale effect* (advantage of scale measured by population size). In view of $\varphi < 1$, it is "only" a *weak* scale effect, also called a scale effect *on levels*. It implies that higher population growth results in higher per capita growth in the long run. This is in contrast to what is known as a *strong* scale effect (corresponding to $\varphi = 1$) whereby a larger population as such (without population growth) would be enough to generate a higher per capita growth rate in the long run.

g) The alternative case, $g_y^* < 0$, is also possible in the model, namely when $\sigma/(1-\varphi) < \alpha$. Here α is the output elasticity w.r.t. land which is a non-producible production factor. So, for a given level of knowledge agriculture faces decreasing returns w.r.t. producible factors and population growth will result in falling per capita income (as in Malthus' theory). To circumvent this inherent tendency to Malthusian misery, either a high output elasticity w.r.t. knowledge (high σ) or a high growth rate of knowledge (high $1/(1-\varphi)$), or both, is needed. The case $\sigma/(1-\varphi) < \alpha$ is a situation where these conditions are *not* satisfied.

h) No! Constant marginal productivity of labor in agriculture corresponds to the case $\alpha = 0$. The key formulas above are still valid, but now we necessarily have $\sigma/(1-\varphi) > \alpha = 0$. This corresponds to the situation described under f) and $g_y^* < 0$ is not possible. Intuitively, in spite of $n > 0$, Malthusian misery is avoided because in agriculture there are no diminishing returns w.r.t. producible inputs. And because of technical progress there is indeed positive per capita growth.

2. Solution to Problem 2 (45 %)

We consider a closed economy with two production sectors, manufacturing and R&D. Time is continuous. At the aggregate level we have:

$$\begin{aligned} Y_t &= T_t K_t^\alpha (\bar{h}_t L_{Yt})^{1-\alpha}, \quad 0 < \alpha < 1, && (2.1)\\ \dot{K}_t &= Y_t - c_t N_t - \delta K_t, \quad \delta \geq 0, && (2.2)\\ T_t &= A_t^\sigma, \quad \sigma > 0, && (2.3)\\ \dot{A}_t &= \eta A_t^\varphi \bar{h}_t L_{At}, \quad \eta > 0, \varphi < 1, && (2.4)\\ L_{Yt} + L_{At} &= L_t, && (2.5) \end{aligned}$$

where Y_t is manufacturing output, K_t is capital input, $\bar{h}_t$ is average human capital in the labor force, c_t is per capita consumption, and N_t is population at time t; otherwise notation corresponds to that in Problem 1.

We assume that all variables in the model are positive and remain so.

a) By (2.1),

$$g_Y = g_T + \alpha g_K + (1-\alpha)(g_{\bar{h}} + g_{L_Y}). \tag{2.6}$$

The TFP growth rate is defined as

$$g_{TFP} \equiv g_Y - (\alpha g_K + (1-\alpha)(g_{\bar{h}} + g_{L_Y})) = g_T, \tag{2.7}$$

here.

b) In (2.4) the case $0 < \varphi < 1$ represents the "standing on the shoulders" case where knowledge creation becomes easier the more knowledge there is already. In contrast, the case $\varphi < 0$ represents the "fishing out" case, also called the "easiest inventions are made first" case, where it becomes more and more difficult to create the next advance in technical knowledge.

c) From (2.4) follows

$$g_A \equiv \frac{\dot{A}_t}{A_t} = \mu A_t^{\varphi-1}\bar{h}_t L_{At} > 0,$$

so that

$$\frac{\dot{g}_A}{g_A} = (\varphi - 1)g_A + g_{\bar{h}} + g_{L_A}. \tag{2.8}$$

Additional information: After schooling in S years, individual human capital is $h(S)$, $h' > 0$. No role for teachers and schooling equipment. Life expectancy constant over time and S the same for all individuals independently of time of birth. After leaving school, individuals works full-time until death. The population grows at a constant rate $n > 0$:

$$N_t = N_0 e^{nt}. \tag{2.9}$$

As age distribution is assumed stationary,

$$L_t = (1 - \beta)N_t, \tag{2.10}$$

where β is the constant fraction of this population under education (β is an increasing function of S). It is further assumed that:

(i) the economy has balanced growth ($g_Y, g_C, g_K, g_A, g_{\bar{h}}, g_{L_A}$, and g_{L_Y} are constant); and

(ii) $Y - cN > 0$ for all t.

d) Yes, as g_{L_A} and g_{L_Y} are constant along a BGP, we can be sure that $g_{L_Y} = n$ along the BGP. Indeed, suppose that instead $g_{L_Y} > n$ and still constant (as it must be along a BGP). Then after some time we would have L_t $(= L_{Yt} + L_{At})$ growing at a rate above n, which is a contradiction. And if we imagine that $g_{L_Y} < n$, then, in order for the sum, $L_{Yt} + L_{At}$, to grow at the rate n, we would need $g_{L_A} > n$ and still constant, which again leads to a contradiction.

e) The proposition says that given the accumulation equation (2.2) (where N is proportional to L) and given gross saving is positive, we have along a BGP that $g_Y = g_K$.

f) It follows from the additional information above that $\bar{h} = h(S)$ so that $g_{\bar{h}} = 0$. Then, by (2.6) and (2.3),

$$g_y = g_Y - n = g_T + \alpha g_K + (1-\alpha)n - n = g_T + \alpha(g_K - n) = g_T + \alpha g_k. \tag{2.11}$$

g) As $g_Y = g_K$, we have $g_y = g_k$. Then, by (2.11),

$$g_y = \frac{\sigma g_A}{1-\alpha}. \tag{2.12}$$

h) The result (2.11) decomposes productivity growth into a "contribution" from technical change and a contribution from "capital deepening" (growth in k). The result (2.12), however, goes deeper and displays that technical change in generated by knowledge growth and that knowledge growth is the *only* source of productivity growth.

In view of (2.7) and (2.3), we have $g_{TFP} = g_T = \sigma g_A$. So we can re-write (2.11):

$$g_y = g_{TFP} + \alpha g_k. \tag{2.13}$$

This equation corresponds to the statement: "The TFP growth rate measures the contribution to productivity growth from growth in technology"; one could continue the statement by saying that the remainder contribution comes from capital deepening. The statement is somewhat misleading since it understates the contribution from knowledge growth. Indeed, (2.12) displays that knowledge growth is the *only* source of productivity growth and the reason is that also capital deepening is due to knowledge growth.

A way of reconciling the interpretations of (2.13) and (2.12) is by saying that (2.13) displays the two factors behind the *current* increase in y, while (2.12) takes into account that the capital deepening is itself, in a long-run perspective, driven by knowledge growth.

i) By (2.8) together with $g_{\bar{h}} = 0$ and $g_{L_A} = n$ follows that along a BGP (where g_A is constant)

$$0 = (\varphi - 1)g_A + n,$$

whereby we find

$$g_A = \frac{n}{1-\varphi}. \tag{2.14}$$

Hence, by (2.12),

$$g_y = \frac{\sigma n}{(1-\varphi)(1-\alpha)} \equiv g_y^*. \tag{2.15}$$

Here we have taken into account that in our endogenous growth model also knowledge growth is endogenous. It is determined by resource allocation to R&D. Along a BGP the growth rate of knowledge is determined as in (2.14), implying that g_y is ultimately determined as in (2.15).

In (2.15) the role of n, σ and φ is analogue to that in Problem 1. However, the role of the elasticity parameter, α, w.r.t. the non-human rival production factor in manufacturing is here completely different from the role of α in the agricultural economy in Problem 1. In the present model a higher α implies higher g_y^* when $n > 0$, while in the agricultural economy model it implied lower g_y^*, cf. (1.2). The reason is that when the non-human production factor is capital, it can be accumulated and its accumulation contributes to growth. When the non-human production factor is land, however, it is not producible, and its scarcity implies a drag on growth.

3. Solution to Problem 3 (15 %)

a) Formulas like $g_y = \lambda n/(1-\lambda)$ come from semi-endogenous knowledge-driven growth models like Arrow's or Jones'. Here population growth contributes to per capita growth as explained under f) in Problem 1. In view of cross-border diffusion of ideas and technology, this proposition should not be seen as a prediction about individual countries, however. It should rather be seen as pertaining to larger regions, nowadays probably the total industrialized part of the world. So the single country is not the relevant unit of observation and cross-country regression analysis thereby not the right framework for testing such a link from n to g_y.

b) True!

We have

$$Y_{jt} = F(K_{jt}, A_{jt}L_{jt}) = f(\tilde{k}_{jt})A_{jt}L_{jt}, \qquad j = 1, 2, ..., N,$$

where f is the production function on intensive form and $\tilde{k}_{jt} \equiv K_{jt}/(A_{jt}L_{jt})$ is the effective capital intensity. Let the common capital depreciation rate be denoted δ. From firms profit maximization under perfect competition then follows

$$f'(\tilde{k}_{jt}) = r + \delta,$$

which shows that the chosen effective capital intensity is the same for all countries and constant over time. Let it be denoted $\tilde{k}^*$. The equilibrium real wage in country j at time t will be

$$w_{jt} = \frac{\partial Y_{jt}}{\partial L_{jt}} = \frac{\partial \left(f(\tilde{k})A_{jt}L_{jt} \right)}{\partial L_{jt}} = f(\tilde{k})A_{jt} = f(\tilde{k})A_{j0}e^{gt}.$$

Hence, in countries with a relatively high technology level, A_{j0}, the wage level will also be relatively high for the same quality of labor (according to the present framework). There will therefore be a strong economic incentive for workers to migrate.

c) There are two main versions of the expanding input variety model, namely the lab-equipment version and the knowledge-spillover version. In the lab-equipment version the invention production function is specified as

$$\dot{N}_t = \eta Z_t, \qquad \eta > 0, \ \eta \text{ constant}, \tag{3.1}$$

where knowledge (proportional to the number of existing varieties of intermediate goods) is by Acemoglu denoted N_t while Z_t is the aggregate R&D investment per time unit. This investment is simply a flow of basic goods allocated to R&D.

In the knowledge-spillover version the input to R&D is research labor and the invention production function is typically specified as

$$\dot{N}_t = \eta N_t^{\varphi} L_{Rt}, \qquad \eta > 0, \varphi \leq 1, \tag{3.2}$$

where L_{Rt} is the input of research labor at time t.

The lab-equipment version features no positive externality from knowledge creation, yet there is underinvestment in R&D. This is because the monopoly position of innovators implies that the invented specialized intermediate goods are priced above marginal costs. Consequently, "too little" of these goods is demanded, that is, the market for each variety is "too small". A sufficient policy remedy turns out to be a subsidy (of appropriate size) to purchases of intermediate goods.

In the knowledge-spillover version market failure derives not only from monopoly pricing but also from the positive intertemporal externality of R&D via the economy-wide productivity factor N_t^{φ}. This calls for a research subsidy in addition to the subsidy to purchases of intermediate goods.

There are two famous sub-versions of (3.2). There is the knife-edge case $\varphi = 1$ which, together with $n = 0$ (where n is the growth rate of the labor force $L = L_N + L_Y$), gives the Romer version. This version features "fully endogenous" growth in the sense that positive per capita growth is generated by an internal mechanism in the model and the "growth engine" is strong enough to sustain a positive per capita growth rate forever without the support by growth in an exogenous factor.

The alternative sub-case of (3.2) is the case $\varphi < 1$ which, together with $n \geq 0$, gives the Jones version. With $n > 0$, growth is here "semi-endogenous" growth in the sense that positive per capita growth is generated by an internal mechanism in the model, but the "growth engine" is *not* strong enough to sustain a positive per capita growth rate forever without the support by growth in an exogenous factor. In both the Jones version and the models of Problem 1 and Problem 2 this exogenous factor is the population size.

—