Economic Growth, June 2013.

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A suggested solution to the problem set at the exam in Economic Growth, June 12, 2013

(3-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates precise understanding of the concepts and methods needed for analyzing the factors that matter for economic growth.

1. Solution to Problem 1 (45 %)

We are informed that if a person attends school for S years, he or she obtains human capital $h = h(S)$, $h' > 0$. A person "born" at time v (v arbitrary) chooses S to maximize

$$HW_v = \int_{v+S}^{\infty} \hat{w}_t h(S) e^{-(r+m)(t-v)} dt, \qquad (*)$$

where $\hat{w}_t$ is the market-determined real wage per year per unit of human capital at time t, r is a constant real interest rate, and m is a parameter such that the probability of surviving at least until age $\tau > 0$ is $e^{-m\tau}$. It is assumed that owing to technical progress,

$$\hat{w}_t = \hat{w}_0 e^{gt}, \qquad (**)$$

where g is a constant satisfying $0 < g < r + m$.

a) The individual chooses S so as to maximize human wealth (da.: humanformue), that is, the present value of expected future labour earnings; $\hat{w}_t$ multiplied by $h(S)$ is the annual real wage to a person educated S years; m is the mortality rate (here assumed age independent), and $r + m$ is the effective discount rate, given a perfect credit and life annuity market. Behind the supposition that the individual chooses schooling length solely with a view to maximize human wealth lies the implicit assumption that the Separation Theorem holds. This is the theorem saying that under certain conditions, maximizing

[1] The solution below contains *more* details and more precision than can be expected at a three hours exam. The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

lifetime utility can be separated into two decision problems, one of choosing schooling length to maximize human wealth and one of choosing a consumption-saving plan to maximize lifetime utility given the maximized human wealth. A condition needed for this theorem to hold is that there is no direct utility from "going to school" or "being a learned person".

The reason that the horizon in (*) is infinite is that the assumed survival probability, $e^{-m\tau}$, for any age τ is positive. Of course, this is an abstraction. Yet the implied quantitative error may be tolerable because $e^{-m\tau}$, although always positive, is extremely small when τ is large.

b) Substituting (**) into (*), we get

$$\begin{aligned} HW_\nu &= h(S)\int_{\nu+S}^{\infty} \hat{w}_0 e^{gt} e^{-(r+m)(t-\nu)}dt = \hat{w}_0 h(S)\int_{\nu+S}^{\infty} e^{g\nu} e^{[g-(r+m)](t-\nu)}dt \\ &= \hat{w}_0 e^{g\nu} h(S)\left(\left.\frac{e^{[g-(r+m)](t-\nu)}}{g-(r+m)}\right|_{\nu+S}^{\infty}\right) = \hat{w}_0 e^{g\nu} h(S)\frac{e^{[g-(r+m)]S}}{r+m-g} \equiv HW(\nu,S) \quad (1.1) \end{aligned}$$

An interior solution to the problem $\max_S HW(\nu,S)$ satisfies the first-order condition:

$$\begin{aligned} \frac{\partial HW(\nu,S)}{\partial S} &= \frac{\hat{w}_0 e^{g\nu}}{r+m-g}\left[h'(S)e^{[g-(r+m)]S} - h(S)e^{[g-(r+m)]S}(r+m-g)\right] \\ &= HW(\nu,S)\left[\frac{h'(S)}{h(S)} - (r+m-g)\right] = 0, \quad (1.2) \end{aligned}$$

from which follows

$$\frac{h'(S)}{h(S)} = r+m-g \equiv \tilde{r}. \quad (1.3)$$

c) The left-hand side of (1.3) is the proportionate marginal return to schooling (the proportionate return to staying one more year under education). In the optimal plan this equals the effective discount rate appearing on the right-hand side of (1.3), namely the interest rate adjusted for (a) the approximate probability of dying within a year from "now", $1-e^{-m} \approx m$) and (b) wage growth due to technical progress. The trade-off faced by the individual is the following: increasing S by one year results in a higher level of human capital (future earning power), but postpones the time when earning an income begins. The effective interest cost is diminished by g, reflecting the fact that the real wage per unit of human capital will grow by the rate g from the current year to the next year.

The intuition behind the first-order condition (1.3) is perhaps easier to grasp if we put g on the left-hand-side and multiply by $\hat{w}_t$ in the numerator as well as the denominator. Then the condition reads:

$$\frac{\hat{w}_t h'(S) + \hat{w}_t h(S) g}{\hat{w}_t h(S)} = r+m.$$

On the the left-hand side we now have the actual net rate of return obtained by investing one more year in education. In the numerator we have the direct increase in wage income by increasing S by one unit plus the gain arising from the fact that human capital, $h(S)$, is worth more in earnings capacity one year later due to technical progress. In the denominator we have the educational investment made by letting the obtained human capital, $h(S)$, "stay" one more year in school instead of working. In an optimal plan the actual net rate of return on the marginal investment equals the required rate of return, $r+m$. This is what could be obtained by the alternative strategy, which is to leave school after S years and then invest the first years's labor income in a life annuity paying the net rate of return, $r+m$, per year (until death). That is, the first-order condition can be seen as a no-arbitrage equation. (As is quite usual, our interpretation treats marginal changes as if they were discrete. Thereby our interpretation is, of course, only approximative.)

d) From now we assume

$$h(S) = S^{\eta}, \qquad \eta > 0. \tag{***}$$

Then $h' = \eta S^{\eta-1}$ so that (1.3) gives

$$\frac{h'(S)}{h(S)} = \frac{\eta}{S} = \tilde{r}.$$

Solving for S gives

$$S = \frac{\eta}{\tilde{r}} \equiv S^*. \tag{1.4}$$

We are told that the second-order condition to ensure that the first-order condition gives an optimum is that the elasticity of h' w.r.t. S is smaller than the elasticity of h w.r.t. S at least at $S = S^*$. The latter elasticity is

$$\frac{S}{h}h' = \frac{S}{S^{\eta}}\eta S^{\eta-1} = \eta,$$

and the former is

$$h'' = \eta(\eta-1)S^{\eta-2} \text{ so that } \frac{S}{h'}h'' = \frac{S}{\eta S^{\eta-1}}\eta(\eta-1)S^{\eta-2} = \eta - 1.$$

Hence, the second-order condition is satisfied.

e) With $\eta = 0.6$, $r = 0.06$, $m = 0.008$, and $g = 0.018$ we get

$$S^* = \frac{0.6}{0.06 + 0.008 - 0.018} = 12 \text{ years}.$$

Comment: We may think of these 12 years as the typical length of primary and secondary school together. Alternatively and perhaps closer to the model setup, we may think of

S as the educational length *after* primary school (or later). One might argue that m is set a bit low. Indeed, with the assumed survival probability, $e^{-m\tau}$, life expectancy is $1/m = 125$ years if $m = 0.008$.

f) An increase in life expectancy decreases m. Hence, an increase in life expectancy increases S. The intuition is that the longer the expected period where you can enjoy the fruits of education the higher is the incentive to invest more in education.

We are now told that the economy considered is a small open economy where the representative firm has the production function

$$Y_t = F(K_t, A_t h L_t).$$

There is perfect competition in all markets and r is given from the world market.

g) Suppressing for the moment the explicit dating of the variables, the firm solves the problem:

$$\max_{K^d, L^d} \Pi = F(K^d, AhL^d) - (r+\delta)K^d - wL^d.$$

First-order conditions are

$$\begin{aligned} F_1(K^d, AhL^d) - (r+\delta) &= 0, && \text{(FOC1)} \\ F_2(K^d, AhL^d)Ah - w &= 0. && \text{(FOC2)} \end{aligned}$$

In view of CRS, we have $F(K, AhL) = AhLF(\tilde{k}, 1) \equiv AhLf(\tilde{k})$, where $\tilde{k} \equiv K/(AhL)$. Consequently, by (FOC1),

$$f'(\tilde{k}^d) = F_1(K^d, AhL^d) = r + \delta.$$

A solution for the desired capital intensity, $\tilde{k}^d$, will be unique since $f'' < 0$. Let us denote it $\tilde{k}^*$. So

$$\tilde{k}^* = f'^{-1}(r+\delta).$$

Since $r+\delta$ is constant over time, so is $\tilde{k}^*$.

As $F_2(K, AhL) = f(\tilde{k}) - f'(\tilde{k})\tilde{k}$, (FOC2) together with $\tilde{k} = \tilde{k}^*$ gives

$$w_t = (f(\tilde{k}^*) - f'(\tilde{k}^*)\tilde{k}^*)A_t h_t = (f(\tilde{k}^*) - f'(\tilde{k}^*)\tilde{k}^*)A_t h(S^*), \tag{1.5}$$

for a typical member of the labor force. Here (***) with S given by (1.4) can be inserted.

Remark. An alternative approach to determining w_t is to do this without introducing the production function on intensive form, f. As F is homogeneous of degree one, the partial derivatives, F_1 and F_2, are, by Euler's theorem, homogeneous of degree zero. From (FOC1) therefore follows that $F_1(\tilde{k}^d, 1) = r + \delta$. In view of $F_{11} < 0$, this determines $\tilde{k}^d$ uniquely as $\tilde{k}^*$. Then, by (FOC2), w_t is determined as $w_t = F_2(\tilde{k}^*, 1)A_t h(S^*)$.

h) In terms of the real wage per unit of human capital, $\hat{w}_t$, we have

$$w_t = \hat{w}_t h(S^*).$$

Inserting (**) gives

$$w_t = \hat{w}_0 e^{gt} h(S^*) = w_0 e^{gt},$$

showing that the growth rate of w_t is g. Comparing with (1.5) we see that also the technology level, A_t, must grow at the rate g.

i) We have

$$y_t \equiv \frac{Y_t}{L_t} = \frac{F(K_t, A_t h L_t)}{L_t} = \frac{A_t h L_t f(\tilde{k}^*)}{L_t} = A_t h(S^*) f(\tilde{k}^*).$$

As h is an increasing function of S, more education is seen to imply higher per capita income, y_t.

The growth rate of y_t equals the growth rate, g, of A_t and is independent of the level of education.

As a *comment* one could for instance refer to the parallel with the saving rate in the Solow model. In that model, a higher saving rate will have a *temporary* growth effect but no permanent growth effect (in the regular case at least). In the present model, more education (from S_1 to $S_2 > S_1$, say) will have a *temporary* growth effect. But in the long run there will only be a *level effect.* In real life a hypothetical transition from S_1 to S_2 in the whole labor force takes quite some time during which the level effect builds up, thereby giving growth a boost with some durability although not permanent. If we go a little outside the model and assume there is sustained arithmetic growth in life expectancy, we may imagine that also S, and thereby $h(S)$, will be forever growing, although at a diminishing rate. The boost to the growth rate of y will thereby peter out in the very long run.

2. Solution to Problem 2 (40 %)

For convenience, key equations are repeated here:

$$\dot{K}_t = Y_t - C_t - \delta K_t, \qquad \delta \geq 0.$$

The production function of firm $i = 1, 2, \ldots, N$ is

$$Y_{it} = K_{it}^{\alpha}(A_t L_{it})^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{*}$$

where A_t is the economy-wide technology level, $\sum_i K_{it} = K_t$, and $\sum_i L_{it} = L_t$, where $L_t = L_0 e^{nt}$, $n \geq 0$, is the labor force (= employment = population). Each firm is small relative to the economy as a whole and perceives it has no influence on aggregate variables, including A_t.

a) We suppress the time index when not needed for clarity. Consider firm i. Its maximization of profits, $\Pi_i = K_i^{\alpha}(AL_i)^{1-\alpha} - (r+\delta)K_i - wL_i$, leads to the first-order conditions

$$\begin{aligned} \partial\Pi_i/\partial K_i &= \alpha K_i^{\alpha-1}(AL_i)^{1-\alpha} - (r+\delta) = 0, \\ \partial\Pi_i/\partial L_i &= (1-\alpha)K_i^{\alpha}A^{1-\alpha}L_i^{-\alpha} - w = 0. \end{aligned} \tag{2.1}$$

We can write (2.1) as

$$\alpha A^{1-\alpha}k_i^{\alpha-1} = r + \delta, \tag{2.2}$$

where $k_i \equiv K_i/L_i$. From (2.2) follows that the chosen k_i will be the same for all firms, say $\bar{k}$. In equilibrium $\sum_i K_i = K$ and $\sum_i L_i = L$, where K and L are the available amounts of capital and labor, respectively (both pre-determined). Since $\sum_i K_i = \sum_i k_i L_i$ $= \sum_i \bar{k}L_i = \bar{k}L$, the chosen capital intensity, k_i, satisfies

$$k_i = \bar{k} = \frac{K}{L} \equiv k, \qquad i = 1, 2, ..., N. \tag{2.3}$$

Substituting into (2.2) gives $r = \alpha A^{1-\alpha}k^{\alpha-1} - \delta$. Reintroducing explicit dating of the variables, the solution for the equilibrium interest rate at time t is

$$r_t = \alpha A_t^{1-\alpha}k_t^{\alpha-1} - \delta, \tag{2.4}$$

where both A_t and k_t are pre-determined.

The implied aggregate production function is

$$\begin{aligned} Y &= \sum_i Y_i \equiv \sum_i y_i L_i = \sum_i k_i^{\alpha}A^{1-\alpha}L_i = k^{\alpha}A^{1-\alpha}\sum_i L_i = k^{\alpha}A^{1-\alpha}L \\ &= k^{\alpha}L^{\alpha}A^{1-\alpha}L^{1-\alpha} = K^{\alpha}(AL)^{1-\alpha}. \end{aligned}$$

So

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}. \tag{2.5}$$

We are now told that A_t evolves according to

$$A_t = K_t^{\lambda}, \qquad 0 < \lambda \leq 1, \tag{**}$$

where λ is a constant.

b) According to the general hypothesis of *learning-by-investing* the economy-wide technology level in (*) is an increasing function of society's previous experience, proxied by cumulative aggregate net investment:

$$A_t = \left(\int_{-\infty}^{t} I_s^n ds \right)^{\lambda} = K_t^{\lambda}, \quad 0 < \lambda \leq 1, \tag{2.6}$$

where I_s^n is aggregate net investment and λ is the "learning parameter".

The idea is that investment – the production of capital goods – as an unintended *by-product* results in *experience* or what we may alternatively call on-the-job *learning*. This adds to the knowledge about how to produce the capital goods in a cost-efficient way and how to design them so that in combination with labor they are more productive and better satisfy the needs of the users. The learning is assumed to benefit essentially all firms in the economy. There are knowledge spillovers across firms and these spillovers are reasonably fast relative to the time horizon relevant for growth theory.

c) To calculate the social marginal productivity of capital we first insert (**) into (2.5) to get

$$Y_t = K_t^{\alpha}(K_t^{\lambda} L_t)^{1-\alpha} = K_t^{\alpha+\lambda(1-\alpha)} L_t^{1-\alpha}.$$

So, with SP indicating "from a social planner's perspective", we have

$$\frac{\partial Y_t}{\partial K_t}_{|SP} = [\alpha + \lambda(1-\alpha)]\, K_t^{\alpha+\lambda(1-\alpha)-1} L_t^{1-\alpha} = [\alpha + \lambda(1-\alpha)]\, K_t^{\lambda(1-\alpha)} k_t^{\alpha-1}. \tag{2.7}$$

When calculating the private marginal productivity of capital, we keep A_t constant in (2.5) so as to get

$$\frac{\partial Y_t}{\partial K_t}_{|A_t \text{ fixed}} = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} = \alpha A_t^{1-\alpha} k_t^{\alpha-1} = \alpha K_t^{\lambda(1-\alpha)} k_t^{\alpha-1}, \tag{2.8}$$

by (**). We see that

$$\frac{\partial Y_t}{\partial K_t}_{|A_t \text{ fixed}} = \frac{\alpha}{\alpha + \lambda(1-\alpha)} \frac{\partial Y_t}{\partial K_t}_{|SP}. \tag{2.9}$$

Thereby, whenever $\lambda > 0$, the social marginal productivity of capital is greater than the private marginal productivity of capital.

d) The learning-by-investing in this economy is a positive externality. It therefore "invites" government intervention in the market mechanism.

e) The policy proposal is to offer an investment subsidy $s \in (0, 1)$ to the firms so that their capital costs are reduced to $(1-s)(r+\delta)$ per unit of capital per time unit. Given

s, (2.1) is replaced by

$$\partial \Pi_i / \partial K_i = \alpha K_i^{\alpha-1} (AL_i)^{1-\alpha} - (1-s)(r+\delta) = 0,$$

so that in equilibrium we have

$$\alpha A_t^{1-\alpha} k_t^{\alpha-1} = (1-s)(r+\delta),$$

where again $k_t \equiv K_t/L_t$ is pre-determined from the supply side. Thus, the equilibrium interest rate satisfies

$$r_t = \frac{\alpha A_t^{1-\alpha} k_t^{\alpha-1}}{1-s} - \delta = \frac{\frac{\partial Y_t}{\partial K_t}_{|A_t \text{ fixed}}}{1-s} - \delta, \tag{2.10}$$

by (2.8).

Since there is no capital income taxation, the equilibrium interest rate constitutes the intertemporal rate of transformation faced by the consumer. Efficiency requires that this rate equals the *net* social marginal productivity of capital, which is $\partial Y_t / \partial K_{t|SP} - \delta$. Substituting (2.9) into (2.10) the requirement thus is that s satisfies

$$\frac{\alpha}{(1-s)\left[\alpha + \lambda(1-\alpha)\right]} \frac{\partial Y_t}{\partial K_t}_{|SP} - \delta = \frac{\partial Y_t}{\partial K_t}_{|SP} - \delta.$$

Both δ and $\partial Y_t / \partial K_{t|SP}$ cancels out and we get

$$1 - s = \frac{\alpha}{\alpha + \lambda(1-\alpha)},$$

or

$$s = \frac{\lambda(1-\alpha)}{\alpha + \lambda(1-\alpha)}.$$

f) With $\alpha = 1/3$ and $\lambda = \frac{1}{2}$, we get

$$s = \frac{\frac{1}{2} \cdot 2/3}{1/3 + \frac{1}{2} \cdot 2/3} = \frac{1}{2}.$$

With $\alpha = 1/3$ and $\lambda = 1$, we get

$$s = \frac{2/3}{1/3 + 2/3} = \frac{2}{3}.$$

So in the latter case the efficient subsidy is larger. The intuition behind this fact is that a higher value of the learning parameter reflects a larger positive externality which requires a larger subsidy to be fully internalized.

From now on we consider the case $0 < \lambda < 1$ and $n > 0$.

g) For any production function $Y = F(K, AL)$ which is homogeneous of degree one, we have

$$\frac{Y}{K} = F(1, \frac{AL}{K}). \tag{2.11}$$

Under balanced growth with positive gross saving, Y/K is constant over time. Then (2.11) implies that AL/K is constant over time so that

$$g_K = g_A + g_L. \tag{2.12}$$

In the present case $A = K^{\lambda}$ and $g_L = n$. Hence, (2.12) implies

$$g_K = \lambda g_K + n,$$

from which follows

$$g_K = \frac{n}{1-\lambda}.$$

Thereby

$$g_y = g_Y - n = g_K - n = \frac{n}{1-\lambda} - n = \frac{\lambda n}{1-\lambda}.$$

h) With $\lambda < 1$, the model generates semi-endogenous growth (positive per capita growth is generated by an internal mechanism in the model but to sustain the growth rate, growth in some exogenous factor, here the labor force, is needed). The subsidy rate, s, has no permanent growth effect, "only" a *level effect* and, if raised to a higher constant level, a *temporary* growth effect.

A higher n, however, implies a higher g_y in balanced growth and thereby a *permanent* growth effect.

Comment: Although there are diminishing marginal returns to capital at the aggregate level, there are increasing returns to scale w.r.t. capital *and* labor taken together. In this case growth in the labor force not only counterbalances the falling marginal productivity of aggregate capital (this counter-balancing role reflects the complementarity between K and L), but also upholds sustained productivity growth − the more so the higher is n.

3. Solution to Problem 3 (15 %)

a) The answer to this question is a matter of opinion. But a reasonable argument should be given.

A "scale effect" is present if some variable representing the *size* of the economy affects either the productivity level in the economy or the long-run productivity growth rate. Usually, by size is meant population size.

Personally, I tend to view the presence of "weak" scale effects (i.e., scale effects on *levels,* not on growth) in an endogenous growth model as a strength of the model. My case for this view is that I consider technical knowledge to be the main driving force in the evolution of productivity − and technical knowledge is a nonrival good, thereby suggesting an advantage of scale.

Let the aggregate production function be $Y = F(K, AL)$, where A is the level of technical knowledge and F is a CRS production function with positive marginal productivities. Consider (average) labor productivity: $y = Y/L = F(K/L, A)$. Because of the *nonrivalry* of technical knowledge, labor productivity depends on the total stock of knowledge, not on this stock per worker (in contrast, labor productivity depends on capital per worker, not on the total stock of capital). Hence, for the following reasons a scale effect on the productivity level should be expected:

- Everything else equal, a larger population breeds more new ideas per time unit.
- The per capita cost of creating increases in knowledge through R&D is smaller the larger is the population.
- As illustrated by the model of Problem 2, to the extent that learning by investing is operative, it is (via knowledge spillovers) total investment that matters rather than per capita investment; and, everything else equal, total investment is larger in a larger economy.

There is, however, a counteracting factor. As environmental economics has emphasized, a tendency to positive scale effects on levels *may* be more or less counteracted by *congestion* and aggravated *environmental problems* ultimately caused by increased population and a population density above some threshold.

As to the empirical aspects of the issue, we should first of all remember that in view of cross-border diffusion of ideas and technology, a positive scale effect (whether weak or strong) should not be seen as a prediction about individual countries, but rather as pertaining to larger regions, nowadays probably the total industrialized part of the world. So cross-country regression analysis is not the right framework for testing for scale effects, whether on levels or the growth rate. The relevant scale variable is not the size of the country, but the size of a larger region to which the country belongs, perhaps the whole world; and multivariate time series analysis seems the most relevant approach.

Considering the *very*-long run *history* of population and per capita income of different regions of the world, there clearly exists evidence in favour of scale effects (Kremer, 1993).

Also more recent econometric studies supporting the hypothesis of positive scale effects on levels are available. Finally, considering the economic growth in China and India in the last three decades, we must acknowledge that this impressive performance at least does not speak *against* the existence of positive scale effects on levels.

b) The phrase "variety is productive" is associated with a certain class of R&D-based endogenous growth models, namely those where the productivity-driving force comes from "horizontal innovations" involving new specialized inputs. The assumption is that the "expanding input variety" allows gains from increased specialization and division of labor.

This idea is for instance implicit in the production function of the manufacturing sector in the model known as the "lab-equipment" model:

$$Y_t = A \left(\sum_{i=1}^{N_t} {x_{it}}^{1-\beta} \right) L_t^{\beta}, \qquad A > 0,\ 0 < \beta < 1, \tag{3.1}$$

where Y_t, L_t, and x_{it} denote output of the firm, labor input, and input of the specialized intermediate good i, respectively, where $i = 1, 2, ..., N_t$. Because the intermediate goods enter the production function in a symmetric way and at the same time have the same price (coming from the same markup on the same unit costs), in equilibrium the same amount of each will be demanded, i.e., $x_{it} = x_t$, $i = 1, 2, ..., N_t$. So (3.1) gives

$$Y_t = AN_t x_t^{1-\beta} L_t^{\beta} = A(N_t x_t)^{1-\beta}(N_t L_t)^{\beta},$$

where $N_t x_t$ measures the total input of intermediate goods. Keeping this constant (i.e., letting x_t decrease along with increases in N_t), we see that nevertheless output increases along with increases in N_t. Indeed,

$$\frac{\partial Y_t}{\partial N_t}_{|N_t x_t \text{ fixed}} > 0,$$

reflecting that "variety is productive". The mathematical background is that as (3.1) is specified, the marginal productivity of each intermediate good is higher when its quantity is lower. So splitting a given total quantity, $N_t x_t$, up into more specialized intermediate goods (higher N_t) gives higher output.

c) The R&D-based growth model by Charles Jones is an "expanding input variety" model where "knowledge-spillovers" play a key role. In the notation by Acemoglu, the expected output of new technical designs in R&D firm j is

$$E_t \dot{N}_{jt} = \tilde{\eta} L_{jt}, \qquad \tilde{\eta} > 0.$$

The individual R&D firms are "small" and take the research productivity, $\tilde{\eta}$, as given. Nevertheless this productivity is determined according to

$$\tilde{\eta} = \eta N_t^{\varphi}, \qquad \eta > 0, \varphi < 1, \tag{3.2}$$

where N_t is the total number of input varieties in the economy at time t. Because the uncertainty is ideosyncratic and the economy is assumed "large", at the aggregate level the actual number of new technical designs invented per time unit coincide with the expected number, i.e., $\dot{N}_t = E_t \dot{N}_t$ (by the "law of large numbers").

Yet we do not necessarily have $\dot{N}_t = \sum_j \dot{N}_{jt} = \tilde{\eta} L_{Rt}$, where $L_{Rt} = \sum_j L_{jt}$. Instead, according to Jones, we have

$$\dot{N}_t = \tilde{\eta} L_{Rt}^{\lambda}, \qquad 0 < \lambda \leq 1,$$

where $\tilde{\eta}$ is still given by (3.2). The point is that $\lambda < 1$ is possible. The interpretation is that some "overlap" ("stepping on toes") in R&D is likely. The "degree of overlapping" in research is then measured by $1 - \lambda$.