Economic Growth. Lecture Note 9.

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Understanding the equilibrium in the R&D sector, cf. Acemoglu, Ch. 12.5 and 13.1

There is a time lag of random length between outlay on R&D and the arrival of a successful research outcome, an invention. During this period which can in principle be infinitely long, an R&D firm has costs but no revenue. Thus R&D is risky and there is a financing problem. In the simple R&D models considered in this course, all risk is assumed ideosyncratic. At the same time the economy is assumed "large" and so risk is fully diversifiable. Research labs can thus act in a risk-neutral manner in spite of the fact that the investors (the households), who ultimately finance all investment, are risk averse.[1]

The specific "story" in the lab-equipment model in Acemoglu, Section 13.1, is as follows. There is a "large" number of R&D firms and free entry and exit. All R&D firms operate under the same conditions with regard to "research technology" and success potential. The analysis is based on the simplifying assumption of *stochastic independence, no memory, and no overlap* in research. The "no overlap" assumption amounts to assuming that inventions can go in so many directions that the likelihood of different research labs chasing and making the same invention is negligible.

When a successful research outcome (an invention) arrives, the inventor can take out (free of charge) a perpetual patent on the commercial use of the invention. This gives the invention a certain market value, V_t. The inventor can realize this market value either by licensing the right to use the invention commercially or by directly herself entering manufacturing as a (monopolist) producer of the new good made possible by the invention. In the model, the market value of any invention arriving at time t is V_t, the same for all research firms:

$$V_t = \pi \int_t^\infty e^{\int_t^s r_\tau d\tau} ds, \tag{1}$$

where π is the (constant) profit per time unit obtainable by commercial use of the invention.

[1] Recall that $u''(c) < 0$, i.e., the instantaneous utility function is strictly concave.

The "research technology" faced by the R&D firms is characterized by the following assumption: the rate at which successful research outcomes (inventions) arrive is proportional to the flow input of basic goods into research. Consider an arbitrary R&D firm j. Let z_{jt} be the amount of basic goods per time unit the firm devotes in its endeavor to make an invention. With η_{jt} denoting the instantaneous success arrival rate, the research technology is given by

$$\eta_{jt} = \eta z_{jt}, \qquad \eta > 0, \tag{2}$$

where η is a given parameter reflecting "research productivity". The aggregate research input is $Z_t \equiv \sum_j z_{jt}$.

CLAIM 1 The expected payoff per time unit per unit of basic goods invested per time unit is

$$E_t(\text{R\&D payoff}) \ = V_t \eta. \tag{3}$$

Proof[2] Consider an arbitrary R&D firm j. The probability of a successful research outcome in a "small" time interval $(t, t + \Delta t]$ is approximately $\eta_{jt}\Delta t$. We thus have

$$E_t(\text{R\&D payoff})\Delta t \approx V_t \eta_{jt}\Delta t + 0 \cdot (1 - \eta_{jt}\Delta t) = V_t \eta_{jt}\Delta t. \tag{4}$$

With $z_{jt} = 1$, we have $\eta_{jt} = \eta$ in view of (2). Substituting this into (4) and dividing through by $z_{jt}\Delta t$ gives (3). □

We are then ready to explain the following claim in Acemoglu (p. 437):

CLAIM 2 (i) In any equilibrium, whether with $Z_t = 0$ or $Z_t > 0$, we must have

$$V_t \eta \leq 1. \tag{5}$$

(ii) In any equilibrium with $Z_t > 0$, we must have

$$V_t \eta = 1. \tag{6}$$

Proof. (i) Suppose that, contrary to (5), we have $V_t \eta > 1$. Expected R&D payoff is then higher than the R&D cost and so expected pure profit is zero. As the financing conditions are such that R&D firms act in a risk-neutral manner, their flow demand for finance will be infinite. The flow supply of finance, ultimately coming from household saving, is,

[2] Admittedly, this and the following "proofs" are of a rather informal character.

however, finite and thus there is excess demand for funds. This drives share prices down and the required rate of return, r_t, up, thus lowering V_t (cf. (1)) until $V_t\eta = 1$. Thus $V_t\eta > 1$ can be ruled out as an equilibrium and this leaves (5) as the only possible state in an equilibrium.

(ii) Any R&D firm j will choose $z_{jt} > 0$ in $(t, t + \Delta t]$, where Δt is "small", only if expected payoff is at least as large as the costs, i.e., $V_t\eta z_{jt}\Delta t \gtrapprox z_{jt}\Delta t$, which in the limit, for $\Delta t \to 0$, amounts to

$$V_t\eta \geq 1. \tag{7}$$

This is, however, only a necessary condition for $Z_t > 0$ in an equilibrium. Since any equilibrium requires (5) to hold, we are left with (6) as the only possible state in an equilibrium with $Z_t > 0$. □

From where does the finance for aggregate R&D investment ultimately come? Let aggregate financial wealth at time t be denoted A_t. Then

$$A_t = a_t L = VN_t = \frac{1}{\eta}N_t,$$

in equilibrium. In view of $\dot{N}_t = \eta Z_t$, we therefore have

$$\dot{A}_t = V\dot{N}_t = \frac{1}{\eta}\dot{N}_t = \frac{1}{\eta}\eta Z_t = Z_t = S_t.$$

As expected, the aggregate R&D investment is thus ultimately financed through the simultaneous aggregate household saving, S_t.

Will there not be both losers and winners in this investment process? No! Imagine all the saving, $S_t\Delta t$, in the short time interval $(t, t + \Delta t]$ first goes to large mutual funds that (without administrative costs) instantly use the receipts to finance current aggregate R&D. Since $V\dot{N}_t = S_t$, this financial investment is at the aggregate level immediately "paid back" in the form of possession of valuable shares in monopolies supplying newly invented intermediates. From then on, holding these shares gives the normal rate of return in the economy, r^*. Although some of the R&D firms have not been successful in this time interval, others have been more-than-normally successful by making a higher-than-expected number of inventions. The unequal occurrence of failures and successes across the many different R&D firms is neutralized when it comes to the payout to the customers with deposits in the mutual funds.

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