Economic Growth Exercise solutions

18.05.2011. Christian Groth

Suggested solution to Problem VIII.1[1]

For convenience we repeat the equations of the model:

$$\begin{aligned}
Y_t &= K_t^{\alpha}(A_t L_{Yt})^{1-\alpha}, \quad 0<\alpha<1, && (1)\\
\dot{K}_t &= Y_t - c_t L_t - \delta K_t, \quad \delta \geq 0, && (2)\\
\dot{A}_t &= \eta A_t^{\varphi} L_{At}, \quad \eta>0, \varphi<1, && (3)\\
L_{Yt}+L_{At} &= L_t, && (4)\\
L_t &= L_0 e^{nt},\ n>0. && (5)
\end{aligned}$$

a) Dividing through by A_t in (3) gives

$$\frac{\dot{A}}{A} \equiv g_A = \eta A^{\varphi-1} L_A. \tag{6}$$

Presupposing $g_A > 0$, log-differentiating w.r.t. t gives

$$\frac{\dot{g}_A}{g_A} = (\varphi-1)g_A + g_{L_A}. \tag{7}$$

Constancy of g_A implies $\dot{g}_A = 0$ so that (7) gives

$$g_A = \frac{g_{L_A}}{1-\varphi}, \tag{8}$$

where g_{L_A} must be constant. We can then rule out that $g_{L_A} > n$, since $L_A \leq L$ for all $t \geq 0$ by definition. But whether $g_{L_A} = n$ or $0 < g_{L_A} < n$ we cannot tell without further information.

b) As suggested by the hint, note that (1) implies

$$1 = \left(\frac{K}{Y}\right)^{\alpha} \left(\frac{AL_Y}{Y}\right)^{1-\alpha} \tag{9}$$

[1] At several places in this exercise the analytical method is similar to the one applied in LN 8, Section 1.1.

In view of the capital accumulation equation (2) and the assumption $g_K > -\delta$ (positive saving and investment), we have under balanced growth $g_Y = g_K$. Then (9) implies AL_Y/Y constant so that

$$g_Y = g_A + g_{L_Y} = \frac{g_{L_A}}{1-\varphi} + g_{L_Y} = \frac{g_{L_A} + (1-\varphi)g_{L_Y}}{1-\varphi}, \tag{10}$$

from (8). Thus, with both g_Y, g_A, and g_{L_A} constant, also g_{L_Y} must be constant. Then, in view of $L_Y \leq L$ for all $t \geq 0$, $g_{L_Y} \leq n$. We conclude $g_{L_A} = g_{L_Y} = n$, since $g_{L_A} < n$ would lead to the contradiction that $g_{L_Y} > n$.

Thereby, (10) gives $g_Y = n/(1-\varphi) + n$ and so

$$g_y = g_Y - n = \frac{n}{1-\varphi} = g_A. \tag{11}$$

Remark. If we had been asked to completely solve the model (with Ramsey households), including finding the transitional dynamics, the approach would be to first derive the complete system of differential equations like we do in the standard Ramsey model with exogenous technical progress or in the Arrow model of learning by investing. Then one finds that the dynamics are described by a *four*-dimensional dynamic system (in contrast to the standard Ramsey model which has two-dimensional dynamics). Characterizing the solution to that four-dimensional system is possible, but outside the confines of this course.

c) Defining $C \equiv cL$, under balanced growth $g_C = g_Y$ and so

$$g_c = g_C - n = g_Y - n = \frac{n}{1-\varphi} \equiv g_c^*.$$

d) We consider an R&D subsidy which increases $s_A \equiv L_A/L$. Since the model is saddle-point stable, the economy converges to a balanced growth path (BGP) in the long run with growth rate g_y given by (11).

d1) No, a higher s_A will not affect g_y in the long run, since (11) shows that g_y only depends on n and φ, not on s_A. A higher s_A will temporarily increase the growth rate of A and tends to temporarily increase also the growth rate of y. But the fact that $\varphi < 1$ ("diminishing returns to knowledge" in the growth engine) makes it impossible to maintain the higher growth rate in A forever. This is like in a Solow model where an increase in the saving rate raises the growth rate only temporarily due to the falling marginal productivity of capital.

d2) We have

$$y \equiv \frac{Y}{L} = \frac{Y}{L_Y}\frac{L_Y}{L} = \frac{Y}{L_Y}(1-s_A) = \tilde{k}^{\alpha}A(1-s_A), \tag{12}$$

where $\tilde{k} \equiv K/(AL_Y)$. We consider s_A as fixed by policy. Under balanced growth one can infer stocks from flows. Indeed, from (6) and (8) follows

$$\eta A^{\varphi-1}L_A = \frac{n}{1-\varphi},$$

implying

$$A_t = \left(\frac{n}{\eta(1-\varphi)}\right)^{\frac{1}{\varphi-1}} L_{At}^{\frac{1}{1-\varphi}} = \left(\frac{n}{\eta(1-\varphi)}\right)^{\frac{1}{\varphi-1}} (s_A L_t)^{\frac{1}{1-\varphi}}.$$

Substituting into (12) gives

$$y_t = (\tilde{k}^*)^{\alpha}\left(\frac{n}{\eta(1-\varphi)}\right)^{\frac{1}{\varphi-1}} (s_A L_0 e^{nt})^{\frac{1}{1-\varphi}}(1-s_A) \tag{13}$$

in balanced growth where $\tilde{k}$ takes some constant value, say $\tilde{k}^*$. If $\tilde{k}^*$ is independent of s_A, (13) unambiguously shows that the path for y_t depends on s_A.

We now show that $\tilde{k}^*$ *is* indeed independent of s_A. From the aggregate production function we have

$$\frac{Y}{K} = K^{\alpha-1}(AL_Y)^{1-\alpha} = \tilde{k}^{\alpha-1} = (\tilde{k}^*)^{\alpha-1}$$

along the BGP. With r denoting the real interest rate, using the household's Keynes-Ramsey rule we have, along the BGP,

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r-\rho) = \frac{1}{\theta}\left(\alpha^2\frac{Y}{K} - \delta - \rho\right) = \frac{1}{\theta}\left(\alpha^2(\tilde{k}^*)^{\alpha-1} - \delta - \rho\right) = g_c^* = \frac{n}{1-\varphi}. \tag{14}$$

This equation determines $\tilde{k}^*$ independently of s_A as was to be shown. It thus follows from (13) that the path for y_t depends on s_A and so policy has long-run *level* effects.

e) The answer is no, the effect on the level of the y path of an increase in s_A is of ambiguous sign. To show this, consider (13). We see that the term

$$z \equiv s_A{}^{\frac{1}{1-\varphi}}(1-s_A)$$

is of key importance for the answer to the role of s_A. Note that

$$\begin{aligned}\frac{\partial z}{\partial s_A} &= (1-s_A)\frac{1}{1-\varphi}s_A^{\frac{1}{1-\varphi}-1} - s_A^{\frac{1}{1-\varphi}} \\ &= \frac{s_A^{\frac{1}{1-\varphi}-1}}{1-\varphi}\left[1-s_A-(1-\varphi)s_A\right] \\ &= \frac{s_A^{\frac{1}{1-\varphi}-1}}{1-\varphi}\left[1-(2-\varphi)s_A\right] \gtreqqless 0 \text{ for } s_A \lesseqqgtr \frac{1}{2-\varphi}.\end{aligned}$$

Thus, if s_A is not "too large", an increase in s_A will have a positive level effect on y via the productivity-enhancing effect of more knowledge creation. But if s_A is already quite large, L_Y will be small, which implies that $\partial Y/\partial L_Y$ is large. This large marginal product constitutes the opportunity cost of increasing s_A and dominates the benefit of a higher s_A, when $s_A > 1/(2-\varphi)$.

f) That inventors can by secrecy or patents obtain monopoly on the commercial application of their inventions helps to support the economic incentive to do R&D. The negative aspect is, however, the monopoly pricing associated with the monopoly power: prices of specialized capital goods will be set above the marginal cost of supplying them. This implies too little use of these expensive specialized inputs in the economy. There is thus a static inefficiency resulting in a productivity level that is lower than it would be without monopoly pricing. This inefficiency also implies that the expected size of the market for future innovations is smaller than otherwise and underinvestment in R&D tends to result.

In addition, if $\varphi > 0$, the knowledge-spillover is positive, which amounts to a positive externality associated with firms' R&D. The absence of a remuneration from the market to this contribution to future productivity reduces the economic incentive to do R&D.

g) We are informed that s_A under balanced growth can be shown to be independent of L. By (14) we see that $\tilde{k}^*$ is independent of L. Hence, (13) implies

$$\frac{\partial y_t}{\partial L_0} > 0.$$

So the answer is: yes, there is a scale effect on levels in the model.

h) We may interpret the question the following way: Are there good theoretical and/or empirical reasons to believe in the existence of (positive) scale effects on levels?

From the point of view of economic *theory*, we should recognize that offsetting forces are in play. On the one hand, there is the *Malthusian* view emphasizing the limited natural resources. For a given level of technology, if there are CRS w.r.t. capital, labor, *and* land (or other natural resources), there are diminishing returns to capital and labor taken together. In this perspective, an increased scale (increased population) results in lower rather than higher per capita output, that is, a negative scale effect should be expected.

On the other hand, there is the *anti-Mathusian* view that repeated improvements in technology tend to overcome, or rather *more* than overcome, this force, if appropriate socio-economic conditions are present. Here the theory of endogenous technical change comes in by telling us that a large population may be good for technical progress. A larger population breeds more ideas, the more so the better educated it is; a larger population also promotes division of labor. This helps in the creation of new technologies or, from the perspective of an open economy, it helps in the local adoption of already existing technologies. It also helps in the less spectacular way by furthering day-by-day productivity increases due to learning by doing and learning by watching. The non-rival character of technical knowledge is important feature behind all this. It implies that output per capita depends on the *total* stock of ideas, not on the stock per person. This implies - everything else equal - an advantage of scale.

In the models considered so far in this course, natural resources and the environment have been more or less ignored. This can only be defended as a first approach intended to clarify certain *mechanisms* − in abstraction from numerous things. The models are primarily focused on aspects of an industrialized economy. Environmental economics has pointed out that a tendency to positive scale effects on levels *may* be more or less counteracted by congestion and aggravated environmental problems ultimately caused by increased population and a population density above some threshold.

What can we say from an *empirical* point of view? First of all we should remember that in view of cross-border diffusion of ideas and technology, a positive scale effect (whether weak or strong) should not be seen as a prediction about individual countries, but rather as pertaining to larger regions, perhaps the total industrialized part of the world. Now, considering the *very*-long run *history* of population and per capita income of different regions of the world, there does indeed exist evidence in favour of scale effects (Kremer, QJE, 1993). Whether advantages of scale are present also in a contemporary context is more debated. Recent econometric studies supporting the hypothesis of positive scale effects on levels include Antweiler and Trefler (AER, 2002) and Alcalá and Ciccone (QJE, 2004). Finally, considering the economic growth in China and India in the last four decades, we must acknowledge that this impressive growth at least does not speak *against* the existence of positive scale effects.

Acemoglu seems to find positive scale effects on levels plausible at the theoretical level (pp. 113-114). At the same time, however, he seems somewhat skeptical as to the existence of empirical support (p. 448).

My personal view on the matter is that although we should, of course, recognize that offsetting forces and a lot of uncertainty are in play, it seems likely that at least up to a certain point there are positive scale effects on levels. If this holds true, it supports the view that international economic integration is generally a good idea. The concern about congestion and environmental problems, in particular global warming, should, however, preclude recommending governments and the United Nations to try to *promote* population growth.

Moreover, one should always remember the distinction between the global and the local level. When discussing economic policy from the perspective of a single country at a particular point in time, we need to incorporate all aspects of relevance in the given local context. For a developing country with limited infrastructure and weak educational system, family-planning programs may certainly be of relevance from both a social and a productivity perspective.

—