Economic Growth Exercises

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Problem set VII

VII.1 *The production side of the lab-equipment model* Consider a closed economy with a given aggregate labor supply L, constant over time. There are three production sectors:

Firms in Sector 1 produce basic goods under perfect competition and free entry.

Firms in Sector 2 produce specialized intermediate goods under monopolistic competition and barriers to entry.

Firms in Sector 3 perform R&D to develop technical designs ("blueprints") for new specialized intermediate goods.

Basic goods and intermediate goods are nondurable goods. There is no physical capital in the economy. Since there is a time lag between R&D outlay and a successful outcome and this time lag is stochastic, research is risky. It is assumed, however, that all risk is ideosyncratic and that the economy is "large" with "many" firms in all sectors. By holding their financial wealth in the form of balanced portfolios consisting of diversified equity shares in the firms, investors (the households) can thus essentially avoid risk. This allows the research labs to act in a risk-neutral manner.

Also the labor market has perfect competition. All firms are profit maximizers. Time is continuous. Unless needed for clarity, the dating of the time-dependent variables is implicit.

The representative firm in Sector 1 has the production function

$$Y = \frac{1}{1-\beta}\left(\sum_{i=1}^{N} x_i^{1-\beta}\right) H^{\beta}, \qquad 0 < \beta < 1, \tag{1}$$

where Y is output per time unit, x_i is input of intermediate good i ($i = 1, 2, ..., N$), H is labor input ("hours"), and N is the number of different currently available types of intermediate goods. The firms in Sector 1 take this number of "varieties" as given.

The output of basic goods is used partly for consumption, C, partly as input in sector 2, X, and partly for input in Sector 3, Z :

$$Y = C + X + Z. \tag{2}$$

Let the basic good be the numeraire and let p_i denote the price of intermediate good i.

a) Find the demand for intermediate good i conditional on full employment. What is the price elasticity of this demand?

b) Suppose $p_i = p$, $\forall i$. Show that the assumed production function, (1), in this case is in conformity with the classical idea from Adam Smith that "there are gains by specialization" or, with another formulation, "variety is productive". *Hint:* check how a rise in N affects Y for given total input of intermediates.

After having invented the technical design i, the inventor has taken out (free of charge) a perpetual patent on the commercial use of this design. The inventor then entered Sector 2, starting to supply the new intermediate good corresponding to this design, that is, the intermediate good i. Performing this role, the inventor is called *firm* i. Given the technical design i, firm i can effortlessly transform basic goods into intermediate goods of type i simply by pressing a button on a computer, thereby activating a computer code. The following linear transformation rule applies to all $i = 1, \ldots, N$:

it takes $\psi > 0$ units of the basic good to supply 1 unit of intermediate good i. (3)

The market value of firm i in Sector 2 can be written

$$V_{it} = \int_t^\infty \pi_{is} e^{-\int_t^s r_\tau d\tau} ds, \tag{4}$$

where π_{is} is the profit at time s and r_τ is the discount rate at time τ.

c) Interpret the expression for V_{it}. What is the relevant discount rate?

Being a monopolist, firm i is a price setter and thus chooses a time path $(p_{is})_{s=t}^{\infty}$ so as to maximize the market value of the firm.

d) This problem can be reduced to a series of static profit maximization problems. Why? Solve the problem. Comment.

e) Find x_{it}, π_{it}, and Y_t in general equilibrium. Comment.

f) Find an expression for V_{it} in general equilibrium. Comment.

All the R&D firms in Sector 3 face the same simple "research technology". The rate at which successful research outcomes arrive is proportional to the flow input of basic goods into research. Consider R&D firm j. Let z_{jt} be the amount of basic goods per time unit the firm devotes in its endeavor to make an invention. With η_{jt} denoting the instantaneous success arrival rate, we have

$$\eta_{jt} = \eta z_{jt}, \qquad \eta > 0, \tag{5}$$

where η is a given parameter reflecting "research productivity".

g) Give an intuitive proof of the claim that the expected payoff per unit of basic goods devoted to R&D per time unit is $V_t\eta$, where V_t is the market value of an arbitrary firm in Sector 2.

h) Let the aggregate research input be denoted Z_t, i.e., $Z_t \equiv \sum_j z_{jt}$. "In equilibrium with $Z_t > 0$, we must have $V_t\eta = 1$." True or false? Why?

i) Find the equilibrium rate of return on financial wealth at time t; comment on your result. *Hint:* consider the rate of return on shares in an arbitrary firm in Sector 2.

Under the simplifying assumption of independence, no memory, and no overlap in research, the expected aggregate number of inventions per time unit at time t is ηZ_t.

j) Ignoring indivisibilities and appealing to the law of large numbers, express $\dot{N}_t$ ($\equiv dN_t/dt$) in terms of Z_t.

Problem VII.2 below considers, from a national income perspective, what is going on in this economy. Problem VII.3 introduces a household sector into the model and considers the implied economic growth.

VII.2 *National income accounting in the lab-equipment model* This problem presupposes that you have already solved Problem VII.1. We consider the same model and use the same notation. We assume that general equilibrium obtains in the economy.

a) A correct answer to e) of Problem VII.1 implies that, the total quantity, Q_t, of intermediate goods produced per time unit at time t can be written $Q_t = xN_t$. Why?

b) Referring to (2), we have $X_t = \psi x N_t$. Why?

c) By defining $A(L)$ appropriately, we now have the following relationship (which is useful in many contexts):

$$Y = A(L)N_t = C_t + \psi x N_t + \eta^{-1}\dot{N}_t. \tag{6}$$

Show this.

d) Show that

$$GDP_t = Y_t - \psi x N_t.$$

Hint: add up the value added in the three sectors and use your conclusions to h) and j) of Problem VII.1.

e) We also have

$$GDP_t = C_t + S_t,$$

and

$$GDP_t = w_t L + \pi N_t,$$

where S_t is aggregate saving, w_t is the real wage, and π is profit per firm in Sector 2. Explain these equations.

VII.3 *R&D-driven fully endogenous growth* This problem presupposes that you have already solved Problem VII.1. Using the same notation, we "close" the model by specifying the household sector.

Suppose there are L infinitely-lived households (L "large"), all alike. Each household consists of one person and each person supplies inelastically one unit of labor per time unit. Given $\theta > 0$ and $\rho > 0$, each household chooses a plan $(c_t)_{t=0}^{\infty}$ to maximize

$$\begin{aligned}
U_0 &= \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \quad \text{s.t.} \\
c_t &\geq 0, \\
\dot{a}_t &= r a_t + w_t - c_t, \qquad a_0 \text{ given}, \\
\lim_{t\to\infty} a_t e^{-rt} &\geq 0,
\end{aligned} \tag{7}$$

where a_t is financial wealth.

a) Comment on the absence of a time subscript on r; express a_t in terms of V and N_t.

b) Find the growth rate of c_t; comment on your result. *Hint:* a result from i) of Problem VII.1 is useful here.

We assume the economy is in equilibrium and that the parameter values are such that there is positive consumption growth.

c) Write down the required parameter restriction.

d) Write down the parameter restriction needed to ensure bounded utility.

e) Find the growth rate of N_t, Y_t, and w_t; comment on your result. *Hint:* there are two features of the model that indicates it is a kind of reduced-form AK model; this allows you to give a quick answer.

f) How does the growth rate of c depend on η and L, respectively? Comment on the intuition.

How the growth rate of c depends on β (the elasticity of Sector-1 output w.r.t. labor) is ambiguous. In Acemoglu, p. 436, the parameter link $\psi \equiv 1-\beta$ is introduced (misleadingly called a "normalization").

g) With $\psi \equiv 1 - \beta$, the dependency of the growth rate of c on β appears to have a unique sign. Show this.

h) Discuss whether the conclusion in g) is economically meaningful or not.

i) The resource allocation in the economy is not Pareto optimal. Why not?