Economic Growth. Exercises

18.05.2011. Christian Groth

Problem Set VIII

VIII.1 Consider the Jones (1995) R&D-based growth model for a closed economy.

Like the original Romer (1990) knowledge-spillover model, the model includes durable physical capital goods. Indeed, the specialized (non-durable) intermediate goods in Acemoglu's §13.2-3 are replaced by specialized capital goods both in the Jones and the Romer model. The number of different varieties of capital goods is called A_t and is treated as an index of the general level of technical knowledge.

There is a Romer-style microeconomic story about the behavior of competitive basic-goods firms demanding specialized capital goods supplied under conditions of monopolistic competition; in addition, the individual R&D firms ignore the knowledge-spillovers. We skip the details and go directly to the implied aggregate level. With Y denoting output of basic goods (not GDP) and K denoting the cumulative non-consumed output of basic goods (otherwise the notation is standard), the aggregate model is:

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\begin{aligned}
Y_t &= K_t^{\alpha}(A_t L_{Yt})^{1-\alpha}, \quad 0 < \alpha < 1, && (1)\\
\dot{K}_t &= Y_t - c_t L_t - \delta K_t, \quad \delta \geq 0, && (2)\\
\dot{A}_t &= \eta A_t^{\varphi} L_{At}, \quad \eta > 0, \varphi < 1, && (3)\\
L_{Yt} + L_{At} &= L_t, && (4)\\
L_t &= L_0 e^{nt}, \ n > 0. && (5)
\end{aligned}
$$

For simplicity we have ignored the R&D duplication externality in Jones (1995).

There is a representative Ramsey household with pure rate of time preference, ρ, and a CRRA instantaneous utility function with parameter $\theta > 0$. To ensure boundedness of the utility integral we assume $\rho - n > (1-\theta)n/(1-\varphi)$.

a) Find an expression for the growth rate of "knowledge", A, under the assumption that this growth rate is positive and constant. *Hint:* start from an expression for g_A derived from (3) and consider the growth rate of g_A.

As the model has *two* state variables, K_t and A_t, it will necessarily exhibit transitional dynamics. The dynamic system will consist of *four* coupled differential equations and is thus relatively complicated.[1] Hence, we shall here concentrate on the balanced growth path (BGP) defined as a path along which Y, K, $C \equiv cL$, and A are positive and g_Y, g_K, g_C and g_A are constant.

b) Along a BGP with $g_A > 0$ and $g_K > -\delta$, find g_Y and g_y, where $y \equiv Y/L$. *Hint:* a possible approach is to divide by $Y = Y^\alpha Y^{1-\alpha}$ on both sides of (1) and then use one of the basic growth theorems.

c) Find the growth rate of c under balanced growth.

It can be shown that the equilibrium real interest rate at time t equals $\alpha^2 Y_t/K_t - \delta$. This information is useful for some of the next questions.

d) Suppose $s_A \equiv L_A/L$ in balanced growth is increased by an R&D subsidy.

  d1) Will this increase affect the long-run per capita growth rate? Comment.

  d2) Will the increase in s_A affect *levels* under balanced growth? Comment. *Hint:* Find an expression for y in terms of $\tilde{k} \equiv K/(AL_Y)$, s_A, and A under balanced growth. Then find an expression for A in terms of L_A under balanced growth. Check that $\tilde{k}$ is independent of s_A; use here that the output-capital ratio in balanced growth can be found from the Keynes-Ramsey rule of the representative household.

e) Is the level of the y path a monotonic function of s_A? Why or why not?

f) The laissez-faire market economy can be shown to generate too little R&D compared to the social planner's solution? What factors might explain this feature?

g) Check whether there is a scale effect on levels in the model. Comment. *Hint:* From Jones (1995, p. 769) we have that s_A under balanced growth is independent of L. Show by use of the Keynes-Ramsey rule that also $\tilde{k}$ under balanced growth is independent of L. Then the stated question can be answered on the basis of the result in d2).

[1] Yet the presence of saddle point stability can be established, cf. Arnold (Rev. Econ. Dynamics, 2006, 143-152).

h) Do you view the presence of scale effects on levels in an endogenous growth model as a strength or weakness of the model? Why?

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