Economic Growth. Exercises

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Problem Set V

V.1 A famous paper by Mankiw, Romer, and Weil (1992) carries out a cross-country regression analysis (98 countries, 1960-1985) based on the aggregate production function,

$$Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta}, \qquad 0 < \alpha < \alpha + \beta < 1, \tag{*}$$

where Y is GDP, K aggregate capital input, H aggregate human capital input, A the technology level, and L input of man-hours, $L_t = L_0 e^{nt}$, n constant. The gross investment rates in the two types of capital are a fraction s_K and s_H of GDP, respectively. Assuming that $A_t = A_0 e^{gt}$, $g \geq 0$, is the same for all countries in the sample (apart from a noise term affecting A_0), the authors conclude that $\alpha = \beta = 1/3$ fits the data quite well.

Let h denote average human capital, i.e., $h \equiv H/L$, and suppose all workers at any time t have the same amount of human capital, equal to h_t.

a) Show that (*) can be rewritten on the form $Y_t = F(K_t, X_t L_t)$, where F is homogeneous of degree one. Indicate what X_t must be in terms of h and A and what the implied "quality function" is.

b) When we study individual firms' decisions, this alternative way of writing the production function is more convenient than the form (*). Explain why.

c) Within a Ramsey-style set-up, where s_K and s_H are endogenous and time-dependent, it can be shown that the economy converges to a steady state with $\tilde{y} \equiv Y/(AL) = (\tilde{k}^*)^{\alpha}(\tilde{h}^*)^{\beta}$, where $\tilde{k}^*$ and $\tilde{h}^*$ are the constant steady state values of $\tilde{k} \equiv K/(AL)$ and $\tilde{h} \equiv h/A$. Find the long-run growth rate of $y \equiv Y/L$. Does human capital accumulation drive per capita growth in the long run?

In Section 11.2 in the textbook by Acemoglu the author presents a Ramsey-style one-sector approach to human and physical capital accumulation. The production function is

$$Y_t = F(K_t, h_t L_t), \tag{**}$$

where F is a neoclassical production function with CRS satisfying the Inada conditions. We shall compare the implications of (*) and (**) under the assumption that A_t in (*) is time-independent and equals 1.

d) Does (*) and (**) imply the same or different answers to the last question in c)? Comment.

e) Briefly evaluate the set-up in Section 11.2 in the Acemoglu textbook from a theoretical as well as empirical perspective.

f) If we want a linear quality function, as implicit in (**), to be empirically realistic, there is an alternative approach that might do better. What approach is that?

V.2 Consider a closed economy with human capital formation and two production sectors, manufacturing and R&D. For simplicity we may imagine that the R&D sector is governed by the government. Time is continuous. At the aggregate level we have:

$$\begin{aligned} Y_t &= A_t^{\gamma} K_t^{\alpha} (\bar{h}_t L_{Y,t})^{1-\alpha}, \qquad \gamma > 0, 0 < \alpha < 1, && (1) \\ \dot{K}_t &= Y_t - C_t - \delta K_t, \qquad \delta \geq 0, && (2) \\ \dot{A}_t &= \eta A_t^{\varphi} (\bar{h}_t L_{A,t})^{1-\varepsilon}, \qquad \eta > 0, \varphi < 1, 0 \leq \varepsilon < 1, && (3) \\ L_{Y,t} + L_{A,t} &= L_t = \text{ labor force.} && (4) \end{aligned}$$

Here A_t measures the stock of technical knowledge and $\bar{h}_t$ is average human capital in the labor force at time t (otherwise notation is standard). The case $\varphi > 0$ corresponds to the "standing on the shoulders" hypothesis and the case $\varepsilon > 0$ corresponds to the "stepping on the toes" hypothesis (ε reflects the degree of overlapping in R&D).

From now we ignore the explicit dating of the variables unless needed for clarity. Let the growth rate of a variable $x > 0$ be denoted g_x (not necessarily positive and not necessarily constant over time). Assume that all variables in the model are positive and remain so.

a) Write down a growth accounting relation expressing g_Y in terms of g_A, g_K, $g_{\bar{h}}$, and g_{L_Y}.

b) Express g_A in terms of A, $\bar{h}$, L_A.

c) Presupposing $g_A > 0$, express the growth rate of g_A in terms of g_A, $g_{\bar{h}}$, and g_{L_A}.

Let the time unit be one year. Suppose an individual "born" at time v (v for "vintage") spends the first S years of life in education and then enters the labor market with a human capital level which at time $t \geq v + S$ is $h(S)$, where $h' > 0$. After leaving education the individual works full-time until death (for simplicity). We ignore the role of teachers and schooling equipment. At least to begin with, we assume for simplicity that S is a constant and thus independent of v. Then with a stationary age distribution in society,

$$L_t = (1 - \sigma)N_t, \tag{5}$$

where N_t is the size of the (adult) population at time t and σ is the constant fraction of this population under education (σ will be an increasing function of S). We assume that life expectancy is constant and that the population grows at a constant rate $n \geq 0$:

$$N_t = N_0 e^{nt}. \tag{6}$$

Let a balanced growth path (BGP) in this economy be defined as a path along which g_Y, g_C, g_K, g_A, $g_{\bar{h}}$, g_{L_A}, and g_{L_Y} are constant.

d) Show that

$$g_A = \frac{(1-\varepsilon)(g_{\bar{h}} + n)}{1 - \varphi}.$$

e) From a certain general proposition we can be sure that along a BGP, $g_Y = g_K$. What proposition and why?

f) We can also be sure that $g_{L_Y} = n$ under balanced growth. Why?

g) Defining $y \equiv Y/L$, it follows that under balanced growth,

$$g_y = \frac{\gamma g_A}{1 - \alpha} + g_{\bar{h}}.$$

How?

h) It is possible to express g_y under balanced growth in terms of only one endogenous variable, $g_{\bar{h}}$. Show this.

i) Comment on the role of n in the resulting formula for g_y.

V.3 This problem presupposes that you have solved Problem V.2, in particular question h).

a) Consider two connected statements: "The model in Problem V.2 assumes diminishing marginal productivity of knowledge in knowledge creation;" and "hence, sustained exponential per capita growth requires $n > 0$ or $g_{\bar{h}} > 0$." Evaluate these statements.

b) Given the prospect of non-increasing population in the world economy in the long run, what is the prospect of sustained exponential per capita growth in the world economy according to the model?

Suppose $h(S) = S^{\mu}$, $\mu > 0$.

c) Demographic data exists saying that life expectancy tends to grow arithmetically, in fact, almost by a quarter of a year per year. Assuming this to continue, and going a little outside the model, what is the prospect of sustained exponential per capita growth in the world economy? Discuss.

d) Although hardly realistic, suppose $h(S)$ is exponential as in the Mincer equation. Then again answer c).

V.4 *Short questions*

a) In the theory of human capital and economic growth we encounter different hypotheses about the schooling technology. List some examples. Briefly comment.

b) "Arrow's learning-by-investing model predicts that the share of capital income in national income is constant in the long run if and only if the aggregate production function is Cobb-Douglas." True or false? Why?

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