Problem set for Homework 1

You are encouraged to solve the assignment together with fellow students (max. four students per group). *If you do not know the other students, but would like to have one or two co-workers, you may send me an e-mail about it. Then I might be able to match you with somebody else in the same situation.* You may write your paper in English or Scandinavian as you prefer. You will need a computer to produce some of the required graphs and numerical results. As to the text and mathematical formulas you may use either your computer or readable handwriting. The maximum number of pages is 10.

Time table for the first homework/workshop assignment

Tuesday 15th of March at 8.00 a.m.: The problem set is announced.

Wednesday 16th of March 8.15-10.00 lecture as usual.

Monday 21st of March 8.15-10.00: Each group should bring a laptop to the exercise class because it is needed to solve some of the questions, which are about model simulation. The exercise class will take the form of a workshop where Niklas Brønager acts as a group consultant.

Wednesday 23rd of March 8.15-10.00 no lecture, but I will be available as a consultant, answering questions by e-mail or personal application within this time interval.

Thursday 24th of March in the time interval 10.00-12.30 you should hand in your paper (an attachment in an e-mail is not acceptable). This deadline is a must. The place to hand in your paper is given below.

On the **front page** of your paper you write the following:

1) Full name of all the authors.

2) The first six digits of their cpr. numbers.

3) Date.

4) Name of the course and the lecturer.

The place where you hand in your paper is:

Thursday 24th of March 10.00-12.30: Økonomisk Institut, Studiekontoret (Mia Kildetoft), CSS Building 25, ground floor, office 25.0.09. Postal address: Studiekontoret, Økonomisk Institut, Øster Farimagsgade 5, Building 25, 1353 København K.

I evaluate your papers ("accepted" or "not accepted") and return them, with comments, as soon as possible.

Good luck!

Christian Groth

The weights of the problems are:

Problem 1: 40 %, Problem 2: 40 %, Problem 3: 20 %.

Problem 1 Consider a Ramsey model, in continuous time, of a closed competitive market economy with public consumption, lump-sum taxes (positive or negative), and capital income taxation. For simplicity the government budget is assumed to be always balanced. The model leads to the following differential equations

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \tilde{c}_t - \tilde{\gamma} - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \tag{*}$$

$$\dot{\tilde{c}}_t = \frac{1}{\theta}\left[(1-\tau_r)(f'(\tilde{k}_t) - \delta) - \rho - \theta g\right]\tilde{c}_t, \tag{**}$$

and the condition

$$\lim_{t\to\infty} \tilde{k}_t e^{-\int_0^t \left[(1-\tau_r)(f'(\tilde{k}_s)-\delta)-g-n\right]ds} = 0. \tag{***}$$

Notation is: $\tilde{k}_t = K_t/(A_t L_t)$ and $\tilde{c}_t = C_t/(A_t L_t) = c_t/A_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively, and L_t is population = labor supply, all at time t. Further, A_t is a measure of the technology level and $\tilde{\gamma} \equiv G_t/(A_t L_t)$, where G_t is government consumption (which is assumed not to affect marginal utility of private consumption). Finally, f is a production function on intensive form, derived from an aggregate neoclassical CRS production function.

The remaining symbols stand for parameters and all these are non-negative. It is assumed that

$$\lim_{\tilde{k}\to 0} f'(\tilde{k}) - \delta > \frac{\rho + \theta g}{1-\tau_r} > n + g > \lim_{\tilde{k}\to\infty} f'(\tilde{k}) - \delta.$$

The government controls $\tilde{\gamma}$ (> 0) together with $\tau_r \in [0, 1)$ and the lump-sum taxes. Until further notice $\tilde{\gamma}$ and τ_r are kept constant over time, and $\tilde{\gamma}$ is not so large that no steady state exists. The lump-sum taxes are continuously adjusted so that the government budget remains balanced; the lump-sum taxes may even be negative, in which case they represent lump-sum transfers.

a) Briefly interpret (*), (**), and (***), including the parameters.

b) Draw a phase diagram and illustrate, for a given $\tilde{k}_0 > 0$, the path which the economy follows. Comment.

c) Is it possible for a steady state to exist without assuming f satisfies the Inada conditions? Why or why not?

d) How does the long-run level of per capita consumption depend on θ? Explain the intuition.

e) How does the long-run real interest rate depend on g? Explain the intuition.

f) Suppose the economy has been in steady state until time t_0. Then, suddenly τ_r is changed to a new constant level $\tau_r' > \tau_r$, while lump-sum taxes are decreased to the extent needed to maintain a balanced budget. Illustrate by a phase diagram what happens in the short and long run. Give an economic interpretation of your result.

g) Does the direction of movement of $\tilde{k}$ depend on whether $\theta \lesseqgtr 1$? Comment.

Problem 2 This problem is about the same model as Problem 1, but answering the questions requires numerical simulation on a computer. Since consumption in the model is a forward-looking variable, our (nonlinear) two-dimensional dynamic system has only one pre-determined variable. The solution path depends on the exogenous initial value of this variable and on the transversality condition which in this model amounts to a requirement of convergence to a unique steady state which is a saddle point. The solution path in the phase diagram therefore coincides with a section of a saddle path. We need a computer algorithm capable of finding this (generally nonlinear) saddle path. One such algorithm is the so-called Relaxation Procedure which you can run in Matlab, see:

http://www.econ.ku.dk/okocg/Computation/Relaxation/relax.html[1]

[1] You are welcome to use other methods if you master them yourself. In the exercise class there will be help if you use the Relaxation Procedure in Matlab.

Let $f(\cdot)$ be Cobb-Douglas with output elasticity w.r.t. capital equal to $\alpha = 1/3$. Let $\theta = 1$, $\rho = 0.02$, $g = 0.015$, $n = 0.005$, and $\delta = 0.05$, given that one year is the time unit. As to the policy parameters, let $\tilde{\gamma} = 0.1$ and $\tau_r = 0.2$. Your paper should include figures with the plots mentioned below.

a) Let $\tilde{k}_0$ equal $0.6 \cdot \tilde{k}^*$, where $\tilde{k}^*$ is the steady-state value of $\tilde{k}$. Simulate the model solution and plot the time profile for $\tilde{k}_t$ and $\tilde{c}_t$ for $t \geq 0$.

The Ramsey model package in Matlab also calculates the half-life of the distance of $\tilde{k}$ from its steady-state value. Let us see how this feature can be utilized.

b) Let the time path of a variable x be $x_t = x_0 e^{-\beta t}$, where β is a positive constant. Show that the instantaneous speed of convergence (SOC_t) equals β and that the half-life of the distance of x_t (for a fixed t) from its long-run value is

$$h = \frac{\ln 2}{\beta}. \tag{h}$$

Hint: the instantaneous speed of convergence at time t (SOC_t) of a converging variable is defined as the (proportionate) rate of decline at time t of the distance to the steady-state value:

$$\text{SOC}_t = -\frac{d(x_t - x^*)/dt}{x_t - x^*}. \tag{SOC}$$

Generally in growth models which converge to a steady state, SOC_t for a given variable is not exactly constant during the adjustment, but depends more or less on the size and sign of the distance to the steady state (that is why the qualifier "instantaneous" is added). SOC_t itself converges for $t \to \infty$ to the so-called *asymptotic SOC*.[2] The Ramsey model package in Matlab calculates both the half-life and the asymptotic SOC for $\tilde{k}$.

c) For the simulation in a), report the half-life of the adjustment of $\tilde{k}$ and the asymptotic SOC.

d) From knowledge of the half-life one can calculate a kind of "average SOC" for $\tilde{k}$ during the first half-life.[3] Do that and compare your result with the asymptotic SOC. *Hint:* interpret the difference $\tilde{k}_t - \tilde{k}^*$ as x_t and apply (h).

[2] Mathematically, in the Ramsey model the asymptotic SOC equals the absolute value of the negative eigenvalue of the Jacobian matrix based on the right-hand sides of (*) and (**) and evaluated in the steady state. This needs not distract us here, however.

[3] If the instantaneous rate of decline of a variable x at time t is $m(t)$, then the average rate of decline in the time interval $[0, T]$ is $\mu = (\int_0^T m(t)dt)/T$ so that $x_T = x_0 e^{-\mu T}$.

e) If we know that a model converges to a steady state, why bother about measures of speed of adjustment?

f) Let the value of $\tilde{k}_0$ equal $0.5{\cdot}\tilde{k}^*$. Simulate the model and plot the time profile for $\tilde{k}_t$ and $\tilde{c}_t$ in the cases $\theta = 1$ and $\theta = 10$, respectively (the other parameters unchanged).

g) Report the percentage deviation of $\tilde{c}_0$ from $\tilde{c}^*$ in the two cases. Give an intuitive explanation of the sign of the difference between these two percentage deviations.

Problem 3 *Short questions*

a) Several spending items which in national income accounting are classified as "public consumption" are from an economic point of view better described as public investment. List some examples.

b) "If there are constant returns with respect to physical capital, labor, and land taken together, then, considering technical knowledge as a fourth production factor, there tends to be increasing returns w.r.t. to all four production factors taken together." True or false? Explain why.

c) Does the model in Problem 1 imply balanced growth in the long run? Why or why not?

d) Consider a Ramsey model with exogenous Harrod-neutral technical progress and a neoclassical CRS production function which is not Cobb-Douglas. Can the predictions of the model be consistent with Kaldor's "stylized facts"? Give a reason for your answer.

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