Written exam for the M. Sc. in Economics, summer 2011

Economic Growth

Master's Course

June 15, 2011

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is: Problem 1 45%, Problem 2 45%, and Problem 3 10%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a closed economy with profit maximizing firms, operating under perfect competition. The size of the labor force (= employment = population) is L_t. Aggregate output (GDP) at time t is Y_t per time unit. Output is used for consumption and investment in physical capital, K_t, so that $\dot{K}_t = Y_t - C_t - \delta K_t$, where C_t is consumption and δ is the rate of physical decay of capital, $\delta \geq 0$. The initial value $K_0 > 0$ is given. There is a perfect market for loans with a short-term real interest rate r_t. Time is continuous and there is no uncertainty.

The production function of firm i is

$$Y_{it} = K_{it}^{\alpha}(A_t L_{it})^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where A_t is the economy-wide technology level, $\sum_i K_{it} = K_t$, and $\sum_i L_{it} = L_t$. Suppose each firm is small relative to the economy as a whole and perceives it has no influence on aggregate variables, including A_t.

a) In general equilibrium, determine r and the aggregate production function at time t.

b) For a given A_t, find the TFP level (total factor productivity) at time t.

For any variable $x > 0$, let g_x denote its growth rate, $\dot{x}/x$.

c) Following the basic idea in growth accounting, express g_Y analytically in terms of the "contributions" from growth in K, L, and a residual, respectively.

d) Find the TFP growth rate, the gross income share of capital (aggregate gross income to capital owners divided by GDP = GNP), and the labor income share, respectively.

From now, suppose A_t evolves according to

$$A_t = e^{\varepsilon t} K_t^{\lambda}, \qquad \varepsilon > 0, \quad 0 < \lambda < 1, \tag{*}$$

where ε and λ are given constants.

e) Briefly interpret (*).

f) Given (*), express g_Y analytically in terms of the "contributions" from growth in K, L, and a residual, respectively.

g) As a thought experiment, suppose we have empirical data for this economy. Will applying standard growth accounting on the basis of these data lead to over- or underestimation of the "contribution" to output growth from growth in capital? Why?

Let $L_t = L_0 e^{nt}$, where n is a positive constant.

h) Determine the growth rate of $y \equiv Y/L$ under balanced growth, assuming saving is positive. *Hint:* use a certain general balanced growth property.

i) Briefly explain what constitute the ultimate sources of per capita growth according to the model. Compare with what the growth accounting in c) suggested.

Problem 2 The bulk of empirical evidence suggests that market economies do too little R&D investment compared to the optimal level as defined from the perspective of a social planner respecting the preferences of an assumed representative infinitely-lived household.

a) Is the "lab-equipment" version of the expanding input variety model consistent with this evidence? Briefly discuss.

b) What kind of subsidy and taxation scheme is capable of implementing the social planner's allocation in the "lab-equipment" model?

c) Our syllabus describes two other versions of the expanding input variety model. The aggregate invention production functions in these two versions are two alternative cases within the common form

$$\dot{N}_t = \eta N_t^{\varphi} L_{Rt}, \qquad \eta > 0, \varphi \leq 1,$$

where N_t is the number of existing different varieties of intermediate goods (indivisibilities are ignored) and L_{Rt} the input of research labor at time t (time is continuous). Briefly interpret.

d) Are these versions consistent with the mentioned evidence? Why or why not?

e) Are there features in these versions that may call for additional policy measures compared with b)? Briefly discuss.

f) The patent-R&D ratio is defined as the number of new patents per year divided by aggregate R&D expenditures. With w_t denoting the real wage, write down an expression for the patent-R&D ratio according to the model versions mentioned under c).

g) What prediction concerning the time path of the patent-R&D ratio can we derive from the two alternative model versions mentioned under c), assuming balanced growth?

h) Since the late fifties, in the US a systematic decline in the empirical patent-R&D ratio has taken place. Briefly relate to your result in g).

Problem 3 In the theory of human capital and economic growth we encounter the concept of a schooling technology. List some possible specifications of a schooling technology and briefly discuss.

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