Interpretation of the first-order condition (7.9)

We may rewrite (7.9) as $r = \rho - \dot{\lambda}/\lambda$. Here we have on the left-hand-side, the actual *real* rate of return on saving, as given from the market. And the right-hand-side can be interpreted as the *required* real rate of return. To see this, suppose that at time t you make a deposit of v_t utility units in a bank that offers you a rate of interest in terms of utility (with continuous compounding) equal to your required utility rate of return, ρ. By definition, a utility rate of return at time t is the (proportionate) rate at which the utility value of the deposit expands per time unit. Thus, the bank's offer is

$$\frac{\dot{v}_t}{v_t} = \rho.$$

My claim is that the corresponding required *real* rate of return is $\rho - \dot{\lambda}_t/\lambda_t$. By definition, a real rate of return is the (proportionate) rate at which the real value of a deposit expands per time unit, when no withdrawal from the deposit is made. The real value of the deposit is its value in terms of the consumption good, which is also the unit in which we measure real financial wealth. To calculate the real value of the deposit at time t, we divide the number of utility units, v_t, by λ_t, since λ_t is the (shadow) price, measured in utility units, of one unit of real financial wealth (you may interpret the financial wealth as an accumulated amount of consumption goods). Thus, a financial wealth consisting of $v_t/\lambda_t \equiv m_t$ real goods is worth v_t utility units (each real unit of the financial wealth is worth λ_t utility units). Hence, the real rate of return at time t on the deposit is

$$\frac{\dot{m}_t}{m_t} = \frac{\dot{v}_t}{v_t} - \frac{\dot{\lambda}_t}{\lambda_t} = \rho - \frac{\dot{\lambda}_t}{\lambda_t},$$

as was to be explained.

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