Continued from p. 15:

LEMMA A1 Let the function $a(t)$ be continuous and the function $f(t)$ differentiable. Then

$$\int_{t_0}^{t_1} (f'(t) - a(t)f(t))e^{-\int_{t_0}^{t} a(s)ds} dt = f(t_1)e^{-\int_{t_0}^{t_1} a(s)ds} - f(t_0).$$

Proof. Integration by parts from time t_0 to time t_1 yields

$$\int_{t_0}^{t_1} f'(t)e^{-\int_{t_0}^{t} a(s)ds} dt = f(t)e^{-\int_{t_0}^{t} a(s)ds} \Big|_{t_0}^{t_1} + \int_{t_0}^{t_1} f(t)a(t)e^{-\int_{t_0}^{t} a(s)ds} dt.$$

Hence,

$$\begin{aligned} & \int_{t_0}^{t_1} (f'(t) - a(t)f(t))e^{-\int_{t_0}^{t} a(s)ds} dt \\ = \quad & f(t_1)e^{-\int_{t_0}^{t_1} a(s)ds} - f(t_0) + \int_{t_0}^{t_1} f(t)a(t)e^{-\int_{t_0}^{t} a(s)ds} dt - \int_{t_0}^{t_1} a(t)f(t)e^{-\int_{t_0}^{t} a(s)ds} dt, \end{aligned}$$

where the last two terms cancel out. □