Economic Growth. Lecture Note 7.

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Heterogeneity and the dynamics of wealth distribution (a supplement to B & S § 2.6.7)

In Section 2.6.7 of their book B & S briefly consider questions about applicability of the Ramsey model in case households are heterogeneous. B & S give an account of the following Caselli and Ventura (2000) results:

1. If households have the same preferences, but differ wrt. labor productivity and initial financial wealth, it is possible to construct a meaningful "representative household" by averaging across the households. The Ramsey model with this representative household will generate a path of consumption and financial wealth which correctly describes the path of average consumption and average financial wealth in an economy populated with this kind of heterogeneous households.

2. If the households' preferences differ in a very limited sense (see p. 121), it is still possible to construct a meaningful representative household. Otherwise, it is not.

A second issue in relation to result 1 is: how will the distribution of financial wealth in the population evolve over time? This is a complicated issue which unfortunately has no clear-cut answer. Since the arguments in B & S are very compact, I will here attempt a slightly more detailed explanation. To understand the context and the notation you have to first read pp. 118-120 in B & S.

Below, equation numbers refer to the text in B & S.

Dynamics of wealth distribution

Whereas c_j/c stays constant over time (as indicated by (2.46) and (2.50)), this is not so for the relative financial wealth, a_j/a, of household j. Indeed, the relative financial wealth of household j changes according to

$$\begin{aligned}
\frac{d}{dt}\left(\frac{a_{jt}}{a_t}\right) &= \frac{a\dot{a}_j - a_j\dot{a}}{a^2} = \frac{a\dot{a}_j - \frac{a_j}{a}\dot{a}}{a^2} \\
&= \frac{(r-n)a_j + w\pi_j - c_j - \frac{a_j}{a}\left((r-n)a + w - c\right)}{a} \quad \text{(from (2.45) and (2.48))} \\
&= \frac{w\pi_j - c_j - \frac{a_j}{a}w + \frac{a_j}{a}\mu(a+\tilde{w})}{a} \\
&= \frac{w\pi_j - \mu(a_j + \pi_j\tilde{w}) - \frac{a_j}{a}w + \frac{a_j}{a}\mu(a+\tilde{w})}{a} \quad \text{(from (2.47) and (2.49))} \\
&= \frac{\pi_j(w-\mu\tilde{w}) - \mu a_j - \frac{a_j}{a}(w-\mu\tilde{w}) + \mu a_j}{a} = \frac{\pi_j(w-\mu\tilde{w}) - \frac{a_j}{a}(w-\mu\tilde{w})}{a}.
\end{aligned}$$

With explicit dating of the time-dependent variables we thus have

$$\frac{d}{dt}\left(\frac{a_{jt}}{a_t}\right) = \frac{(w_t - \mu_t\tilde{w}_t)(\pi_j - \frac{a_{jt}}{a_t})}{a_t}. \tag{2.51}$$

Note that $\tilde{w}_t \equiv (1/J)\sum_{j=1}^{J}\tilde{w}_{jt}$ is average human wealth in that $\tilde{w}_{jt}$ is per capita human wealth of household j :

$$\tilde{w}_{jt} = \int_t^\infty w_\tau \pi_j e^{-\int_t^\tau (r_s - n)ds} d\tau. \tag{1}$$

Moreover, μ_t is the marginal propensity to consume out of wealth in that

$$c_{jt} = \mu_t(a_{tj} + \tilde{w}_{jt}).$$

The general formula for μ_t, also valid outside steady state, is (see B & S, p. 94, or my Chapter 7, p. 284):

$$\mu_t = \frac{1}{\int_t^\infty e^{\int_t^\tau (\frac{1-\theta}{\theta}r_s - \frac{\rho}{\theta} + n)ds} d\tau}.$$

In steady state,[1] $r_t = \rho + \theta x \equiv r^*$, so that μ_t becomes a constant, μ^*, given by

$$\begin{aligned}
\mu^* &= \frac{1}{\int_t^\infty e^{(\frac{1-\theta}{\theta}r^* - \frac{\rho}{\theta} + n)(\tau - t)} d\tau} \\
&= \frac{\rho - (1-\theta)r^*}{\theta} - n \\
&= \rho - n - (1-\theta)x. \qquad (2)
\end{aligned}$$

Thus, in steady state

$$c_{jt} = \mu^*(a_{tj} + \tilde{w}_{jt}) = \left[\rho - n - (1-\theta)x\right](a_{tj} + \tilde{w}_{jt}).$$

[1] When talking about "steady state", we mean steady state of the *aggregate* Ramsey model.

Claim 1 In steady state $w_t = \mu^* \tilde{w}_t$ (that is, the real wage is proportional to average human wealth).

Proof In steady state the equilibrium real wage grows at the rate of technical progress, x, a constant exogenous rate. Thus, $w_\tau = w_t e^{x(\tau - t)}$. Moreover, in steady state human wealth of the individual household, $\tilde{w}_{jt}$, is proportional to the real wage w_t because (1) gives

$$\begin{aligned}
\tilde{w}_{jt} &= \int_t^\infty w_t e^{x(\tau-t)} \pi_j e^{-(r^*-n)(\tau-t)} d\tau \\
&= w_t \pi_j \int_t^\infty e^{-(r^*-n-x)(\tau-t)} d\tau \\
&= w_t \pi_j \frac{1}{r^* - n - x} = w_t \pi_j \frac{1}{\rho - n - (1-\theta)x} \\
&= w_t \pi_j \frac{1}{\mu^*},
\end{aligned}$$

by (2). Thus, the average human wealth satisfies

$$\tilde{w}_t \equiv \frac{\sum_{j=1}^J \tilde{w}_{jt}}{J} = \frac{\sum_{j=1}^J w_t \pi_j \frac{1}{\mu^*}}{J} = \frac{w_t \sum_{j=1}^J \pi_j}{J} \frac{1}{\mu^*} = w_t \frac{1}{\mu^*},$$

in view of $(1/J) \sum_{j=1}^J \pi_j = 1$. Reordering gives $w_t = \mu^* \tilde{w}_t$. $\square$

Substituting Claim 1 into (2.51) gives, in steady state,

$$\frac{d}{dt}\left(\frac{a_{jt}}{a_t}\right) = \frac{(w_t - \mu^* \tilde{w}_t)(\pi_j - \frac{a_{jt}}{a_t})}{a_t} = 0,$$

implying that in steady state the relative financial wealth position of household j is constant.

Recall that average productivity, $\frac{1}{J}\sum_j \pi_j$, equals one. Hence, for household j the ratio $\pi_j/1$ or, what is the same, the size of π_j, indicates the relative productivity position of household j. Now, suppose the economy is outside the steady state. There are two cases to look at.

Case 1: $w_t > \mu_t \tilde{w}_t$. Consider household j. Suppose that its relative asset position is not as good as its relative productivity position. From (2.51),

$$\frac{a_{jt}}{a_t} < \frac{\pi_j}{1} \Rightarrow \frac{d}{dt}\left(\frac{a_{jt}}{a_t}\right) > 0, \text{ i.e., } \frac{a_{jt}}{a_t} \uparrow .$$

That is, when the relative asset position of household j is not as good as its relative productivity position, the relative asset position gradually moves up towards the comparatively high productivity position (as long as $w_t > \mu_t \tilde{w}_t$ holds). The intuition is that household j, which is comparatively "stronger" in labor productivity than in assets, in view of $w_t > \mu_t \tilde{w}_t$, saves more relative to its assets (a_{jt}), than the average household (with $\pi = 1$) does relative to *its* assets (a_t). It does so because it has comparatively high labor income.

Consider the opposite case: the relative productivity position of household j is worse than the relative asset position. We have

$$\frac{a_{jt}}{a_t} > \frac{\pi_j}{1} \Rightarrow \frac{d}{dt}(\frac{a_{jt}}{a_t}) < 0, \text{ i.e., } \frac{a_{jt}}{a_t} \downarrow .$$

This says that when the relative asset position of household j is better than its relative productivity position, the relative asset position of household j gradually moves down towards the comparatively low productivity position (as long as $w_t > \mu_t \tilde{w}_t$ holds).

Case 2: $w_t < \mu_t \tilde{w}_t$. Consider again household j. Suppose its relative asset position is not as good as its relative productivity position. We have

$$\frac{a_{jt}}{a_t} < \frac{\pi_j}{1} \Rightarrow \frac{d}{dt}(\frac{a_{jt}}{a_{tv}}) < 0, \text{ i.e., } \frac{a_{jt}}{a_t} \downarrow .$$

That is, when the relative asset position of household j is not as good as its relative productivity position, the relative asset position gradually moves further away from the comparatively high productivity position (as long as $w_t < \mu_t \tilde{w}_t$ holds). The intuition is that household j, which is "comparatively stronger" in labor productivity than in assets, in view of $w_t < \mu_t \tilde{w}_t$ saves less relative to its assets (a_{jt}), than the average household (with $\pi = 1$) does relative to its assets (a_t). So a_{jt}/a_t decreases over time.

Consider the opposite case: the relative asset position of household j is better than its relative productivity position. We have

$$\frac{a_{jt}}{a_t} > \frac{\pi_j}{1} \Rightarrow \frac{d}{dt}(\frac{a_{jt}}{a_t}) > 0, \text{ i.e., } \frac{a_{jt}}{a_t} \uparrow .$$

In this case, as long as $w_t < \mu_t \tilde{w}_t$ holds, the relative asset position of household j gradually moves further above the comparatively low relative productivity position of this household.

Let us sum up. In case 1, the relative asset positions *converge* towards the relative productivity positions. In case 2, the relative asset positions *diverge* away from the relative productivity positions.

Labor income distribution is typically less skew than asset distribution. Hence, the distribution of productivity is probably less skew than the distribution of assets. Therefore, the majority of the population (or households) is likely to have $a_{jt}/a_t < \pi_j$. Thus, a simple case that corresponds *roughly* to the real world situation is the case where all households have approximately the same productivity, i.e., $\pi_j = 1$ for all j, but a small fraction of them own most of the assets (due to inheritance). So for the majority of the population

$$\frac{a_{jt}}{a_t} < \pi_j \approx 1.$$

From this we conclude, first, that in case 1 above there is a tendency for the distribution of assets to become *less* skew over time and in case 2 *more* skew. Second, in case 1 there is a tendency for most people in the population to experience an *improvement* in their relative asset position and in case 2 a *worsening*.

Unfortunately, however, *when* and *whether* case 1 or case 2 obtains is not simple to say. The answer is not linked unambiguously to whether $\hat{k}_t < \hat{k}^*$ or $\hat{k}_t > \hat{k}^*$. And during the adjustment process for $\hat{k}_t$, the economy may shift from case 1 to case 2 or the opposite. Hence, the model does not have a clear prediction about how the relative asset position for most households moves during the transition towards the steady state.[2]

Outlook

If households have the same preferences, but differ wrt. labor productivity and initial financial wealth, it is possible to construct a meaningful "representative household" by averaging across the households. Still, there is no simple way to establish from the model whether the inequality in possession of financial assets tends to diminish or increase over time. And even though construction of a representative household is possible, there is no obvious way to decide what weights a social planner should give to the utility of the different households in a social welfare function.

[2] A calibration exercise is performed in Glachant, J., and C. Vellutini (2002), Quantifying the relationship between wealth distribution and aggregate growth in the Ramsey model, *Economics Letters 74*, 237-241.