Economic Growth Lecture Note 19

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Conceptual aspects of the choice of social discount rate

A controversial issue within economists' debate on the climate change problem is the choice of discount rate. This choice matters a lot for the present value of a project which involves costs that begin now and benefits that occur only after many years, say 75-100-200 years from now, as is the case with the measures against global warming.

The value of receiving 1000 inflation-corrected euros a hundred years from now is worth less than 1 euro today when evaluated at a 7 percent discount rate. On the other hand, if a 1 percent discount rate is used, the 1000 inflation-corrected euros a hundred years from now is worth more than 300 euros today.

Unfortunately it is not always recognized that "discount rate" can mean many different things. This has lead to serious confusion, even within the academic debate about policies addressing climate change. This lecture note is an attempt at clarifying some of the conceptual distinctions.

A *discount rate* is an interest rate applied in the construction of a *discount factor*, that is, a factor by which future costs and benefits measured in some unit of account are converted into present equivalents. The higher the discount rate, the lower the associated discount factor.

Think of "period t" as running from date t to date $t+1$, that is, as the time interval $[t, t+1)$ on a continuous time axis with a time unit equal to the period length. Thus time t is the same as the "beginning of period t". Unless otherwise indicated our period length, hence our time unit, will be one year.

One of the reasons that we have to distinguish between different types of discount rates is that there is a variety of applied units of account.

1 The unit of account

1.1 Money as the unit of account

When the unit of account is *money*, we talk about a *nominal discount rate*. More specifically, if the money unit is euro, we talk about an euro discount rate. A payment stream $x_0, x_1, \ldots, x_t, \ldots, x_T$, where x_t ($\gtrless 0$) is the net payment in euro due at the end of period t, has present value

$$PV_0 = \frac{x_0}{1+i_0} + \frac{x_1}{(1+i_0)(1+i_1)} + \cdots + \frac{x_{T-1}}{(1+i_0)(1+i_1)\cdots(1+i_{T-1})} \tag{1}$$

euro. If i_0 is the nominal interest rate in euro on a one-period bond from date 0 to date 1, the *nominal discount factor* from date 1 to date 0 is $1/(1+i_0)$. This discount factor tells how many euro need be invested in the bond at time 0 to obtain 1 euro at time 1. When the interest rate in this way appears as a constituent of a discount factor, it is called a *discount rate*. Like an interest rate it tells how many additional units of account (here euros) is returned after one period of unit length, if one unit of account (one euro) is invested at the beginning of the period.[1]

The *average nominal discount rate* from date T to date 0 is the number $\bar{i}_{0,T}$ satisfying

$$1 + \bar{i}_{0,T} = \left((1+i_0)(1+i_1)\cdots(1+i_{T-1})\right)^{1/T}. \tag{2}$$

The corresponding nominal discount factor is

$$(1+\bar{i}_{0,T})^{-T} = \frac{1}{(1+i_0)(1+i_1)\cdots(1+i_{T-1})}. \tag{3}$$

If i is constant, the average nominal discount rate is of course the same as i and the nominal discount factor is simply $1/(1+i)^T$.

In continuous time with continuous compounding the formulas corresponding to (1), (2), and (3) are

$$PV_0 = \int_0^T x(t)\, e^{-\int_0^t i(\tau)d\tau} dt, \tag{4}$$

$$\bar{i}(0,T) \equiv \frac{\int_0^T i(\tau)d\tau}{T}, \quad \text{and} \tag{5}$$

$$e^{-\bar{i}(0,T)T} = e^{-\int_0^T i(\tau)d\tau}. \tag{6}$$

And as above, if i is constant, the nominal discount factor takes the simple form e^{-iT}.

[1] A discount factor is by definition a non-negative number. Hence, a discount rate is by definition greater than -1.

1.2 Consumption as the unit of account

When the unit of account is a basket of consumption goods or, for simplicity, just a homogeneous consumption good, we talk about a *consumption discount rate* (or a *real* discount rate). Let the consumption good's price in terms of euros be P_t, $t = 0, 1, \dots, T$. A consumption stream $c_0, c_1, \dots, c_t, \dots, c_T$, where c_t is available at the end of period t, has present value

$$PV_0 = \frac{c_0}{1+r_0} + \frac{c_1}{(1+r_0)(1+r_1)} + \cdots + \frac{c_{T-1}}{(1+r_0)(1+r_1)\cdots(1+r_{T-1})}. \tag{7}$$

Instead of the nominal interest rate, the proper discount rate is now the *real* interest rate, r_t, on a one-period bond from date t to date $t+1$. Ignoring indexed bonds, the real interest rate is not directly observable, but can be calculated in the following way from the observable nominal interest rate i_t :

$$1 + r_t = \frac{P_{t-1}(1+i_t)}{P_t} = \frac{1+i_t}{1+\pi_t},$$

where P_t is the price (in terms of money) of a consumption good delivered at the end of period t and $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate from date t to date $t+1$. The *consumption discount factor* (or *real* discount factor) from date t to date $t+1$ is $1/(1+r_t)$. This discount factor tells how many consumption goods' worth need be invested in the bond at time t to obtain one consumption good's worth at time $t+1$. The stream $c_0, c_1, \dots, c_t, \dots, c_T$ could alternatively represent an income stream measured in current consumption units. Then the real interest interest rate, r_t, would still be the relevant real discount rate and (7) would give the present real value of the income stream.

The *average consumption discount rate* and the corresponding consumption discount factor are defined in a way analogous to (2) and (3), respectively, but with i_t replaced by r_t. Similarly for the continuous time versions (4), (5), and (6).

1.3 Utility as the unit of account

Suppose intertemporal preferences can be represented by a sum of period utilities discounted by a constant rate ρ. Then ρ is a *utility discount rate.* Usually we then write

$$U(c_0, c_1, \cdots, c_{T-1}) = u(c_0) + \frac{u(c_1)}{1+\rho} + \cdots + \frac{u(c_{T-1})}{(1+\rho)^{T-1}},$$

where $u(\cdot)$ is the period utility function. The associated *utility discount factor* from date T to date 0 is $1/(1+\rho)^{T-1}$. The utility function $\tilde{U}(c_0, c_1, \cdots, c_{T-1}) \equiv (1+\rho)^{-1}U(c_0, c_1, \cdots, c_{T-1})$

represents the same preferences and here the utility discount factor from date T to date 0 is $1/(1+\rho)^T$, which in form corresponds to (3). In continuous time (with continuous compounding) the "sum" of discounted utility is

$$U_0 = \int_0^T u(c(t))e^{-\rho t}dt,$$

where $e^{-\rho t}$ is the utility discount factor from time t to time 0.[2]

2 Further distinctions

In addition to the applied unit of discount there are other criteria that lead to a partitioning of discount rates. Exploring these criteria in detail would take us too far.[3] But here is a list.

1. *Length of the time horizon* (some authors draw a line between less than vs. more than 40 years).

2. *Certainty vs. risk vs. fundamental uncertainty.*

3. *One vs. several consumption goods.* As we shall see below, the relevant consumption discount rate in a given context depend on several factors, including the growth rate of consumption. When different consumption goods, say an ordinary produced consumption good, c, and services from the eco-system, q, enter the utility function, the relevant consumption discount rate becomes a more intricate. A brief introduction is in Heal (2008).

4. *Private vs. social.* Discounting from an individual household's or firm's point of view, as it occurs in private cost-benefit analysis, is one thing. Discounting from a policy maker's point of view is another. The latter appears in *social cost-benefit analysis* and should take into account externalities and other market failures. Whatever the unit of account, a discount rate applied in social cost-benefit analysis is called a *social discount rate.*

[2]Note that a first-order Taylor approximation of e^x around $x = 0$ gives $e^x \approx e^0 + e^0(x-0) = 1 + x$ for x "small". Replacing x by ρ and taking powers, we see the analogy between $e^{-\rho t}$ and $(1+\rho)^{-t}$. But because of the continuous compounding, $e^{-\rho t} < (1+\rho)^{-t}$ whenever $\rho > 0$ and $t > 0$.

[3]The reader is referred to the list of references.

5. *Micro vs. macro.* Social cost-benefit analysis may be carried out at a *microeconomic* level, in which a lot of circumstances are exogenous (like in partial equilibrium analysis). Or it may be carried out at a *macroeconomic* level. At this level more circumstances are endogenous, including possibly the natural environment and economic growth. And in macroeconomic cost-benefit analysis also intergenerational ethical issues and risks are of importance.

3 The debate on discounting in social cost-benefit analysis

There has been a lot of disagreement among economists about how to discount in social cost-benefit analysis, in particular when the whole economy and a long time horizon are involved. Some views are:

(i) The discount rate applied in social cost-benefit analysis should equal the observed market rate of return.

Critics of this view point out that there are market failures[4] and that people who benefit are not the same as those who bear the costs. Where different - and as yet unborn - generations are involved, difficult ethical questions are involved.

Moreover, there is the *isolation paradox* (Sen 1961). Suppose each old has an altruistic concern for *all* members of the next generation. Then a transfer from any member of the old generation to the heir entails an externality that benefits all other members of the old generation. A nation-wide coordination (political agreement) that internalizes these externalities would raise intergenerational transfers (bequests etc.), which corresponds to a lowering of ρ. More generally, members of the present generations may be willing to join in a collective contract of more saving and investment by all, though unwilling to save more in isolation. Other reasons for a relatively low social discount rate have been proposed. One is based on the *super-responsibility argument:* the government has responsibility over a longer time horizon than those currently alive. Another is based on the *dual-role argument*: the members of the currently alive generations may in their political or public role be more concerned about the welfare of the future generations than they are in their private economic decisions.

[4] Heal (2008) asks: "Is it appropriate to assume no market failure in evaluating a consumption discount rate for a model of climate change?".

(ii) Even when considering the climate change problem and caring seriously about future generations, the market rate of return is the relevant discount rate. This is because funds used today to pay the cost of mitigating climate change could be set aside and invested in other things and thereby accumulate at this rate of return for the benefit of the future generations.

Critics of this view make the point that the future damages could easily be underestimated. And, if nothing is done now, the risk of the damage being irreparable at any cost becomes higher. Further, there is no guarantee that the necessary funds *are* set aside for investment. Anyway, applying the market rate of return as discount rate for damages occurring in 100-200 years implies that these damages become almost imperceptible.

These disagreements notwithstanding it may be useful and clarifying to consider a specified social cost-benefit problem at the macroeconomic level and with a long time horizon.

4 The standard optimal growth problem

The perspective is that of an "all-knowing and all-powerful" social planner facing an intertemporal allocation problem in a closed economy. From the outset it is worth emphasizing:

- The analysis will show that the only *independent* discount rate in the problem is the *utility* discount rate that enters the objective function. The consumption discount rate is part of neither the objective function, nor the constraints. Instead it is a by-product of the *solution* to the problem and is thus endogenous.
- There is no such thing as a "correct" social discount rate in any objective and unconditional sense. Ethical judgments inevitably enter.

4.1 The setting

We place our social planner in the simplest neoclassical technology set-up with exogenous technical change. Although time is continuous, for simplicity we date the variables by sub-indices, thus writing Y_t etc. The aggregate production function is neoclassical and has CRS:

$$Y_t = F(K_t, T_t L_t) \equiv T_t L_t f(\tilde{k}_t), \tag{8}$$

where Y_t is output, K_t physical capital input, and L_t labor input (equal to the labor force which for simplicity equals population). The argument in the production function on intensive form is defined by $\tilde{k}_t \equiv K_t/(T_t L_t)$. The factor T_t represents the economy-wide level of technology and grows according to

$$T_t = T_0 e^{gt}, \tag{9}$$

where $T_0 > 0$ and $g \geq 0$ are given constants. Population grows at the constant rate $n \geq 0$. Output is used for consumption and investment so that

$$\dot{K}_t = Y_t - c_t L_t - \delta K_t, \tag{10}$$

where c_t is per capita consumption and $\delta \geq 0$ a constant capital depreciation rate.

Suppose the objective function, the social welfare function, is time separable with (i) an instantaneous utility function $u(c)$, where $u' > 0$ and $u'' < 0$, (ii) a pure rate of time preference, ρ, (iii) an effective utility discount rate equal to $\rho - n$, and (iv) an infinite time horizon. The social planner's optimization problem is to choose a plan $(c_t)_{t=0}^{\infty}$ so as to optimize

$$\begin{aligned}
W_0 &= \int_0^{\infty} u(c_t) e^{-(\rho-n)t} dt \qquad \text{s.t.} && (11)\\
c_t &\geq 0, && (12)\\
\dot{\tilde{k}}_t &= f(\tilde{k}_t) - \frac{c_t}{T_t} - (\delta + g + n)\tilde{k}_t, && (13)\\
\tilde{k}_t &\geq 0 \qquad \text{for all } t \geq 0. && (14)
\end{aligned}$$

Comments

1. If there are feasible paths along which the improper integral W_0 goes to $+\infty$, a maximum of W_0 does not exist. By "optimizing" we then mean finding an "overtaking optimal" solution or a "catching-up optimal" solution, assuming one of either exists (cf. Sydsæter et al. 2008).

2. By taking population growth, n, into account in the *effective* utility discount rate, the welfare function (11) respects the principle of *discounted classical utilitarianism* (a principle also used throughout in B & S). A positive *pure* rate of time preference, ρ, implies discounting the utility of future people just because they belong to the future. Sometimes this discounting of the future is defended by claiming it is a typical characteristic of an individual's preferences. A differing view is that this is not a valid argument for

long-horizon evaluations because these involve different persons and even as yet unborn generations. For example Stern (2007) argues that the only ethically defensible reason for choosing a positive ρ is that there is always a small risk of extinction of the human race due to for example a devastating meteorite or nuclear war. This issue aside, in (11) the *effective* utility discount rate is $\rho - n$. This is based on the view that if $n > 0$, the future should have more weight than otherwise because more people will be available.[5]

4.2 Characterizing the solution

To characterize the optimal solution to the problem we use the Maximum Principle. The current-value Hamiltonian is

$$H(\tilde{k}, c, \lambda, t) = u(c) + \lambda \left[f(\tilde{k}) - \frac{c}{T} - (\delta + g + n)\tilde{k} \right],$$

where λ is the adjoint variable associated with the dynamic constraint (13). An interior optimal path $(\tilde{k}_t, c_t)_{t=0}^{\infty}$ will satisfy that there exists a continuous function $\lambda = \lambda(t)$ such that, for all $t \geq 0$,

$$\frac{\partial H}{\partial c} = 0, \text{ i.e., } u'(c) = \frac{\lambda}{T}, \quad \text{and} \tag{15}$$

$$\frac{\partial H}{\partial \tilde{k}} = \lambda(f'(\tilde{k}) - \delta - g - n) = (\rho - n)\lambda - \dot{\lambda} \tag{16}$$

hold along the path and the transversality condition,

$$\lim_{t \to \infty} \tilde{k}_t \lambda_t e^{-(\rho - n)t} = 0, \tag{17}$$

is satisfied.

Take logs on both sides of (15) and differentiate wrt. t to get

$$\frac{u''(c_t)}{u'(c_t)} \dot{c}_t = \frac{\dot{\lambda}_t}{\lambda_t} - g = \rho - (f'(\tilde{k}_t) - \delta),$$

where the last equality comes from (16). Reordering gives

$$\rho - \frac{c_t u''(c_t)}{u'(c_t)} \frac{\dot{c}_t}{c_t} = f'(\tilde{k}_t) - \delta. \tag{18}$$

This can of course also be written on the standard Keynes-Ramsey rule form, where $\dot{c}_t / c_t$ is isolated on the left-hand side. But from the perspective of discount rates, and therefore

[5] In contrast, the principle of discounted *average* utilitarianism ic characterized by the population growth rate *not* affecting the effective utility discount rate.

rates of return, the form (18) is more useful. A feasible path satisfying both (18) and the transversality condition (17) with $\lambda_t = T_t u'(c_t)$ will be an optimal path.[6]

The optimality condition (18) expresses the general principle that in the optimal plan the marginal rate of substitution between consumption in two successive time intervals equals the marginal rate of transformation as given by technology, i.e., $MRS = MRT$. Accordingly, in the optimal plan the marginal unit of per capita output is equally valuable whether used for current consumption or investment. When used for investment it gives a marginal rate of return equal to the net marginal productivity of capital, i.e., the right-hand side of (18). The latter depends on the endogenous state variable $\tilde{k}$ and is, in a dynamic perspective, just as endogenous and time-dependent as the left-hand side of (18).

More specifically, (18) says that the social planner will sacrifice per capita consumption today for more per capita consumption tomorrow, that is, go for $\dot{c} > 0$, only up to the point where this postponement is compensated by a marginal rate of return sufficiently above ρ. Naturally, the required compensation is higher, the faster marginal utility declines when consumption increases (i.e., the larger is $-u''/u'$). Indeed, higher c in the future than today implies a lower marginal utility of consumption in the future than of consumption today. So the marginal unit of investment today is only warranted if the marginal rate of return is sufficiently above ρ, and this is what (18) indicates.

4.3 The social consumption discount rate

We may also interpret the optimality condition (18) as saying that the *actual* marginal rate of return on investing at time t (the right-hand side) must equal the *required* marginal rate of return at time t (the left-hand side). Denoting the required marginal rate of return r_t^{SP} and letting the optimal path be marked by an asterisk, we thus have from (18)

$$r_t^{SP} = \rho + \theta(c_t^*)\frac{\dot{c}_t^*}{c_t^*}, \tag{19}$$

where $\theta(c) \equiv -cu''(c)/u'(c) > 0$ (the absolute elasticity of marginal utility of consumption). For given $\theta(c_t^*)$, a higher growth rate implies a higher required marginal rate of return on *further* saving. Similarly, for a given growth rate $\dot{c}_t^*/c_t^* > 0$, the required marginal rate of return on further saving is higher, the larger is $\theta(c_t^*)$. This is because $\theta(c_t^*)$ reflects aversion towards consumption inequality over time.

[6] Generally, this will be unique.

Still these remarks are essentially only various ways of interpreting the optimality condition (18). The right-hand side of (19) is not something given in advance, but an endogenous variable equal to the endogenous obtainable rate of return on capital investment represented by equation (18)'s right-hand side.

If instantaneous utility belongs to the CRRA family, $\theta(c)$ is a constant, say $\theta > 0$:

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta}, & \text{when } \theta > 0, \theta \neq 1, \\ \ln c, & \text{when } \theta = 1. \end{cases} \tag{20}$$

If in addition the optimal solution is close to a balanced growth path, we may replace $\dot{c}_t^*/c_t^*$ in (19) by the exogenous rate of technical progress, g. Then (19) reduces to a *constant* required consumption rate of return given by

$$r^{SP} = \rho + \theta g. \tag{21}$$

In this form we recognize a popular suggestion for an appropriate consumption discount rate in social cost-benefit analysis. This discount rate may be useful at a *micro*economic level, that is, for evaluating *small* investment projects in the neighborhood of the already established "big macro plan". Carrying out the microeconomic project would imply only negligible changes at aggregate level.

For long-horizon projects at the macro level the discount rate given by (21) is, however, less useful. First, it was not used in the derivation of the solution to the social planner's problem, but rather a by-product. Second, if new long-horizon projects under consideration affect g, one has to reformulate the social planners problem and again only use the utility discount rate as a starting point.

5 Concluding remarks

I personally agree with the view that when dealing with the climate change problem, the choice of a social consumption discount rate cannot be based on observed market rates of return or growth rates of consumption (as unfortunately for example Nordhaus, 2007, does). This is because the climate change problem is a truly large-scale and in fact global economic problem with implications for economic development in centuries. Future growth rates and rates of return depend on policies chosen now within the set of options, ranging from "business-as-usual" laissez-faire policy to for example the series of concrete "act now" measures suggested in the Stern Review (Stern 2006). In such a context a consumption discount rate is *endogenous,* that is, a part of the *solution* to the problem, not something we have in advance. The only discount rate that can be chosen in advance in this context is the utility discount rate.

In his brief analysis of the economics of the climate change Arrow (2007) finds the fundamental conclusion of the Stern Review justified even if one, unlike the Stern Review, heavily discounts the utility of future generations. Uncertainty and risk aversion plays a key role in the argument. In many areas of life high insurance premia are willingly paid to reduce risks. In Arrow's view this should also be so in relation to the greenhouse gas problem.

6 References

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