So our parameter restrictions are

$$(\frac{1}{\alpha}-1)\left(\alpha^2 A\right)^{\frac{1}{1-\alpha}}\frac{L}{\eta} \;>\; p+\rho, \qquad \text{and} \tag{A1}$$

$$\rho \;>\; (1-\theta)\frac{1}{\theta}\left[(\frac{1}{\alpha}-1)\left(\alpha^2 A\right)^{\frac{1}{1-\alpha}}\frac{L}{\eta}-p-\rho\right] \tag{A2}$$

6 Transitional dynamics

Given that cessations of individual monopolies follow the assumed independent Poisson processes with "arrival rate" p, the aggregate number of transitions per time unit from monopoly to competitive status follow a Poisson process with "arrival rate" $pN^m(t)$. The expected number of transitions per time unit from monopoly to competitive status is then

$$E_t\dot{N}^c(t) = pN^m(t).$$

Assuming $N^m(t)$ is "large", the difference between actual and expected transitions per time unit will be negligible (by the law of large numbers), and we simply write

$$\dot{N}^c(t) = pN^m(t) = p(N(t) - N^c(t)). \tag{22}$$

Defining $u \equiv N^c/N$ and $g_x \equiv \dot{x}/x$ for any positively-valued variable x, we get

$$\begin{aligned} g_u &= g_{N^c} - g_N = p\frac{N-N^c}{N^c} - g_N = p(u^{-1}-1) - g_N \\ &= pu^{-1} - (p+g_N), \end{aligned} \tag{23}$$

by (22). In steady state ($\dot{u} = 0$), we thus have

$$u = \frac{p}{p+g_N} = \frac{p}{p+g_N^*} \equiv u^*, \tag{24}$$

where g_N^* is the constant growth rate of N required for u to be constant.

The general movement i N is given by (4), which together with (2) and (11) implies that $\dot{N}$

$$
\begin{aligned}
&= \frac{1}{\eta}R = \frac{1}{\eta}(Y - X - C) = \frac{1}{\eta}\left[Y - (N^m X^m + N^c X^c) - C\right] \\
&= \frac{1}{\eta}\left[Y^m\left(1 + (\alpha^{\frac{-\alpha}{1-\alpha}} - 1)\frac{N^c}{N}\right) - (N - N^c)X^m - N^c X^c - C\right] \quad \text{(by (11) and (9))} \\
&= \frac{1}{\eta}\left[\left(\bar{A}\left(1 + (\alpha^{\frac{-\alpha}{1-\alpha}} - 1)\frac{N^c}{N}\right) - X^m - (X^c - X^m)\frac{N^c}{N}\right)N - C\right] \quad \left(\text{by } \frac{Y^m}{N} \equiv \alpha^{\frac{2\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L \equiv \bar{A}\right) \\
&= \frac{1}{\eta}\left[\bar{A} - X^m + (\alpha^{\frac{-\alpha}{1-\alpha}} - 1)\bar{A}u - (X^c - X^m)u - \tilde{c}L\right]N \quad \left(\text{by } u \equiv \frac{N^c}{N} \text{ and } \tilde{c} \equiv \frac{C}{NL} \equiv \frac{c}{N}\right) \\
&= \frac{1}{\eta}\left[(1 - \alpha^2)\bar{A} + \left(\alpha^{\frac{-\alpha}{1-\alpha}} - 1 - (\alpha^{\frac{1-2\alpha}{1-\alpha}} - \alpha^2)\right)\bar{A}u - \tilde{c}L\right]N \quad \text{(by (6) and (7))} \\
&= \frac{1}{\eta}\left[(1 - \alpha^2)\bar{A} + \left(\alpha^{\frac{-\alpha}{1-\alpha}} - (1 + \alpha)\right)(1 - \alpha)\bar{A}u - \tilde{c}L\right]N.
\end{aligned}
$$

Hence, the growth rate of N is

$$g_N = \frac{1}{\eta}\left(B_1 + B_2 u - \tilde{c}L\right), \tag{25}$$

where $B_1 \equiv (1 - \alpha^2)\bar{A} > 0$ and $B_2 \equiv \left(\alpha^{\frac{-\alpha}{1-\alpha}} - (1 + \alpha)\right)(1 - \alpha)\bar{A} > 0$.

We can now construct the implied dynamic system in the endogenous variables u and $\tilde{c}$. From (23) follows $\dot{u} = p - (p + g_N)u$, which combined with (25) yields

$$\dot{u} = p - \left(p + \frac{1}{\eta}\left(B_1 + B_2 u - \tilde{c}L\right)\right)u. \tag{26}$$

Similarly, from $\tilde{c} \equiv c/N$ follows $\dot{\tilde{c}}/\tilde{c} = g_c - g_N = g_c^* - g_N$, by (21). So,

$$\dot{\tilde{c}} = \left(g_c^* - \frac{1}{\eta}\left(B_1 + B_2 u - \tilde{c}L\right)\right)\tilde{c}. \tag{27}$$

The differential equations (26) and (27) constitute a dynamic system with two endogenous variables u and $\tilde{c}$, the first of which is a predetermined variable, while the second is a jump variable.

In steady state of the system, by (27), $\frac{1}{\eta}\left(B_1 + B_2 u - \tilde{c}L\right) = g_c^*$. By (A1) and (25) we then have $g_N^* = g_c^* > 0$, so that $u^* = p/(p+g_c^*)$, by (24). Finally, $\tilde{c}^* = (B_1 + B_2 u^* - \eta g_c^*)/L$. As shown in Appendix B, where also the phase diagram is sketched, the system is saddle-point stable.

7 Conclusion

We conclude that the economy converges towards a steady state featuring balanced growth with Y, C, N, and N^c growing at the same constant rate, g_c^*, given in (21). As to the total supply, X, of intermediate goods we have $X = N^m X^m + N^c X^c = (N - N^c)X^m + N^c X^c$ $= [(1 - u^*)X^m + u^* X^m] N$. Hence, along the balanced growth path X is proportional to N and therefore grows at the same rate as N. The same is true for $R = \eta \dot{N} = \eta g_N^* N$.

So Model II generates fully endogenous growth. As (21) indicates, the common long-run growth rate, g_c^*, of the key variables is smaller than in Model I with permanent monopolies, which in turn is smaller than the social planner's growth rate, cf. Lecture Note 16. The reason that the growth rate ends up not only lower than the social planner's, but also lower than in a corresponding economy with permanent monopolies, is that the erosion of monopoly power implies less "protection" of the invention, hence less profitability of R&D. Whereas erosion of monopoly power leads to a static efficiency gain as described in Section 3, it also implies a dynamic efficiency *loss*.

Government intervention is motivated. Two policy instruments are needed. To diminish the monopolist price distortion and encourage demand of monopolized intermediates, a subsidy on purchases of monopolized intermediate goods is required. To encourage R&D in a situation with imperfect protection of inventions, a subsidy to R&D investment is also needed. By comparing with the social planner's solution, it is possible to find exact formulas for the subsidy rates such that the social planner's allocation can be replicated in a decentralized way.

8 Appendix

A. Leibniz' formula

Leibniz' formula is:

$$\begin{aligned} F(t) &= \int_{a(t)}^{b(t)} f(z,t)dz => \\ F'(t) &= f(b(t),t)b'(t) - f(a(t),t)a'(t) + \int_{a(t)}^{b(t)} \frac{\partial f(z,t)}{\partial t} dz. \end{aligned}$$

In our case above we have $b(t) = \infty$ and $a(t) = t$, so that $b'(t) = 0$ and $a'(t) = 1$.[3]

B. Stability analysis

[3]For details, see any Math textbook, e.g., Sydsæter vol. II, or B & S, p. 625.

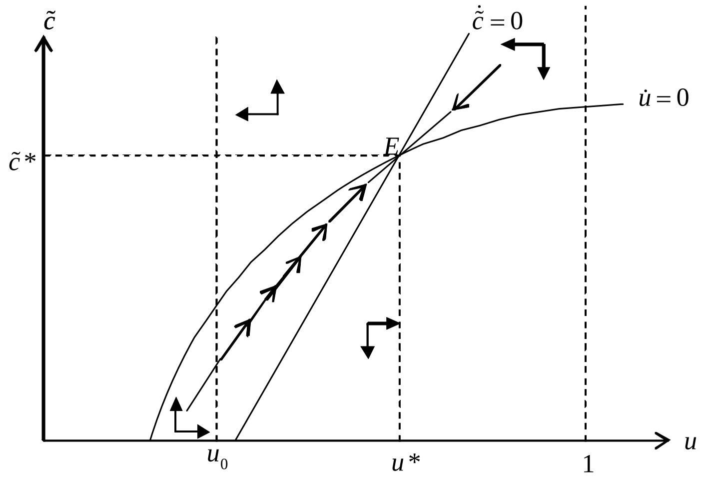

Figure 1:

The Jacobian matrix, evaluated in steady state, is

$$J^* = \begin{bmatrix} \partial \dot{u}/\partial u & \partial \dot{u}/\partial \tilde{c} \\ \partial \dot{\tilde{c}}/\partial u & \partial \dot{\tilde{c}}/\partial \tilde{c} \end{bmatrix}_{|(u^*,\tilde{c}^*)} = \begin{bmatrix} -(p + g_N + B_2 u^*/\eta) & Lu^*/\eta \\ -B_2\tilde{c}^*/\eta & L\tilde{c}^*/\eta \end{bmatrix}.$$

The determinant of this matrix is

$$\det J^* = -(p + g_N + B_2 u^*/\eta)L\tilde{c}^*/\eta + Lu^*/\eta B_2 \tilde{c}^*/\eta = -(p + g_N)L\tilde{c}^*/\eta < 0.$$

Hence, the eigenvalues are of opposite sign and the steady state is saddle-point stable. The phase diagram is sketched in Fig. 1.