Economic Growth   Lecture Note 15

14.04.2010.   Christian Groth

A simple model with horizontal innovations

This is an account of the logic of the innovation-driven growth model in B & S, Chapter 6.1. It is a model where productivity growth occurs through purposeful R&D investment by firms in search for monopoly profits. The basic ideas come from Paul Romer (1987, 1990).

It is a model where technical knowledge and its intentional creation is the center of attention. Recall our definition of *technical knowledge* as a list of instructions about how different inputs can be combined to produce a certain output. For example it could be a principle of chemical engineering. Such a list or principle can be copied on the blackboard, in books, in journals, on floppy disks etc. and can, by its nature, be available and used over and over again at arbitrarily many places at the same time. Thus, technical knowledge is a *non-rival* good.[1] At least temporarily, however, new technical knowledge may be *temporarily excludable* by patents or secrecy so that the innovator can maintain a monopoly on the commercial use of new technical knowledge for some time.

The present model focuses on *horizontal* innovations. By this we mean inventions of *new* goods, that is, new lists of instructions (new "technical designs" in the language of Romer) about how to combine different inputs to obtain new goods. In the model the new goods are input goods, but a more general framework would include new consumption goods as well. The rising number of varieties of goods contributes to productivity via increased division of labor and specialization in society. Thus the model belongs to the class of models called "increasing-variety models".

1 Overview of the economy

We consider a closed market economy with L utility maximizing households. Each household supplies inelastically one unit of labor per time unit. There are two production

[1] Even though a particular *medium* on which a copy of a list of inctructions is placed is a rival good, it can usually be reproduced at very low cost in comparison to the cost of making additions to the stock of technical knowledge.

sectors, the *manufacturing sector* (or what Romer calls the "basic-goods" sector) and the *innovative sector*. We call these two sectors Sector 1 and Sector 2, respectively. Sometimes it is convenient to interpret Sector 2 as consisting of two sub-sectors, "activity 2.a" and "activity 2.b" below.

There is no physical capital in the economy, only non-durable intermediate goods. Households' financial wealth consists of shares in monopoly firms in Sector 2. This sector supplies specialized intermediate goods under conditions of *monopolistic competition*. These goods are input in Sector 1, where the firms operate under *perfect competition*. Also the labor market has perfect competition. All firms are profit maximizers. Generally, variables are dated implicitly.

1.1 The production structure

In Sector 1, the *manufacturing sector*, firms combine labor and N different intermediate goods to produce a homogeneous output good. Firm i ($i = 1, 2, ..., M$) in the sector has the production function

$$Y_i = A\left(\sum_{j=1}^{N} {x_{ij}}^{\alpha}\right) {L_i}^{1-\alpha}, \qquad A > 0,\ 0 < \alpha < 1. \tag{1}$$

Here Y_i, L_i, and x_{ij} denote output of the firm, labor input, and input of intermediate good j, respectively, where $j = 1, 2, ..., N$. Aggregate output per time unit in Sector 1 is $Y \equiv \sum_i Y_i$.

This aggregate output of "basic goods" is used partly for consumption, C, partly for investment in R&D, R, and partly for replacing the intermediate goods used up in the production of Y. Hence, we have

$$Y = C + R + X, \tag{2}$$

where $X \equiv \sum_j \sum_i x_{ij}$. This latter identity reflects the assumption that from a purely technological point of view there is a one-to-one relationship between basic goods and intermediate goods, as explained in the next paragraph.[2]

In the *innovative sector*, Sector 2, there are two kinds of activities, Activity 2.a and Activity 2.b. The first activity is to supply already invented intermediate goods. Once the

[2] Apart from naming the aggregate investment in research R, and writing x_{ij} instead of X_{ij}, our notation is basically as in B & S.

technical design, j, has been invented, the inventor can effortlessly transform any number of basic goods into the same number of intermediate goods of type j simply by pressing a button on a computer, thereby activating a computer code. That is,

it takes x units of the basic good to supply x units of intermediate good j. (3)

The second activity in the innovative sector is R&D. New "technical designs" (blueprints) for making new specialized intermediate goods are invented. Ignoring indivisibilities, we assume the aggregate number of new technical designs invented in the economy per time unit is

$$\dot{N} \equiv \frac{dN}{dt} = \frac{R}{\eta}, \qquad \eta > 0,\ \eta \text{ constant}, \tag{4}$$

where R is the aggregate R&D investment per time unit measured in terms of basic goods as indicated by (2). All that is required for research is to direct an amount of resources, or more precisely R units of the basic good per time unit, to innovative activity. Then R/η new technical designs are invented per time unit. In this model it takes no special resources, like skilled workers or scientists, to make inventions, only "standard" resources that could otherwise be used to produce consumption goods. Moreover, even in R&D there is no uncertainty.[3]

The parameter η can be interpreted as the required input of basic goods in R&D per invention, a fixed cost per innovation. For simplicity it is assumed that inventions can go in so many directions that the likelihood of different research labs chasing and making the same invention is negligible.

After an invention has been made, the inventor enters Activity 2.a and begins selling the new intermediate good to firms in Sector 1. The inventor retains a perpetual monopoly over the production and sale of the invented intermediate good. We assume this is possible either by concealment of the new technical design or by taking out an infinitely-lived patent, which, for simplicity, is assumed free of charge.

At first sight this whole production setup may seem peculiar. In Sector 2 a part of the output from Sector 1 is used as input both in supplying specialized intermediate goods and in R&D, but no complementary labor input appears in Sector 2. Formulating the three production activities in the economy this way, however, is only a convenient (and quite common) way of saving notation in this type of models.[4] A more realistic full-fledged

[3] This is an odd feature of the model. More advanced models of course include uncertainty.

[4] At the same time it is this multi-faceted use of outout from Sector 1 that motivates the term "basic goods".

description of the production structure would start from specific production functions, with both labor and intermediate goods as inputs, for all three production activities. Then an assumption would be imposed that the production functions (technologies) are essentially the same (apart from the constant η) in the three activities. Setting the model up this way would change none of the conclusions.

1.2 The potential for sustained productivity growth

Already the production function (1) conveys the basic idea of an "increasing-variety model". In equilibrium we get $x_{ij} = x_i$, since all intermediate goods end up having the same price (see below). Thus, (1) can be written

$$Y_i = ANx_i^{\alpha}L_i^{1-\alpha} = A(Nx_i)^{\alpha}(NL_i)^{1-\alpha} \equiv f(Nx_i, NL_i),$$

where Nx_i is the total input of intermediate goods. We see that

$$\frac{\partial Y_i}{\partial N}|_{Nx_i=\text{const.}} = f_2(Nx_i, NL_i)L_i > 0.$$

This says that for a given total input, Nx_i, of intermediate goods, the higher the number of varieties (with which follows a lower x_i of each intermediate), the more productive is this total input. "Variety is productive". There are "gains to division of labor and specialization in society". Thus the number of input varieties, N, can be interpreted as a measure of the level of technical knowledge.

1.3 National income accounting

Before considering agents' behavior, it will probably be clarifying to do a little national income accounting.

The production side The aggregate production in the basic-goods sector is $Y = \sum_{i=1}^{M} Y_i$ and $X \equiv \sum_j \sum_i x_{ij}$ is the amount of basic goods used up as input (in the form of specialized intermediate goods) in the production of Y (along with the labor input). Using the basic good as our unit of account, all the specialized intermediate goods will in equilibrium have the price P (see below). We therefore have

$$\text{value added in Sector 1 } = Y - PX.$$

Thus, in this model the production function in (1) is *not* a function giving value added by firm i as a function of inputs, but giving gross output of the firm as a function of the inputs. This is a feature that distinguishes multi-sector models, where inputs include non-durable intermediate goods, from our usual one-sector models with only capital and labor as explicit inputs.

There are two components of value added in Sector 2:

$$\begin{aligned}\text{value added in Activity 2.a} &= PX - X,\\ \text{value added in Activity 2.b is} &= V\dot{N} - R,\end{aligned}$$

where V is the market value of an innovation. In an equilibrium with $\dot{N} > 0$, this value equals η (see below). Thus GDP, or aggregate value added, is

$$\begin{aligned}GDP &= Y - PX + \eta\dot{N} - R + PX - X\\ &= Y - X, \qquad (5)\end{aligned}$$

in view of (4).

The income side There are two kinds of income in the economy, wage income and profits. With w denoting the real wage per unit of labor and π the profit per time unit earned by each monopolist firm in Sector 2, and immediately paid out to the share owners, the income side of GDP is

$$GDP = wL + \pi N.$$

Owing to perfect competition and CRS, firms in Sector 1 do not earn profits. Aggregate income is used for consumption and saving:

$$wL + \pi N = C + S.$$

The use side As indicated by (2), aggregate output of basic goods is used partly for consumption, partly for investment in R&D, and partly to replace intermediate goods used up as raw material in the production of basic goods. Accordingly, final output is $GDP = Y - X$, as in (5). Aggregate saving is

$$S = wL + \pi N - C = GDP - C = R,$$

by (5) and (2). Not surprisingly, aggregate saving in a closed economy equals aggregate investment, which is here investment in R&D represented by the R&D expense R.

2 The competitive producers of basic goods

Firm i in the basic-goods sector maximizes profit under perfect competition:

$$
\begin{aligned}
\max_{L_i,(x_{ij})_{j=1}^N} \Pi_i &= Y_i - \sum_{j=1}^N P_j x_{ij} - wL_i \text{ s.t.} \\
Y_i &= A L_i{}^{1-\alpha} \sum_{j=1}^N (x_{ij})^\alpha .
\end{aligned}
$$

The first-order conditions are:

$$
\begin{aligned}
\partial \Pi_i / \partial L_i &= \partial Y_i / \partial L_i - w = (1-\alpha) A L_i^{-\alpha} \sum_{j=1}^N (x_{ij})^\alpha - w = 0, && (6) \\
\partial \Pi_i / \partial x_{ij} &= \partial Y_i / \partial x_{ij} - P_j = \alpha A L_i{}^{1-\alpha} x_{ij}^{\alpha-1} - P_j = 0, \quad j = 1, 2, ..., N. && (7)
\end{aligned}
$$

Here (7) gives the demand function

$$
x_{ij} = L_i (\alpha A)^{\frac{1}{1-\alpha}} P_j^{-\frac{1}{1-\alpha}}, \quad j = 1, 2, ..., N. \tag{8}
$$

So aggregate demand for intermediate goods j is

$$
X_j^d = \sum_i x_{ij} = L\,(\alpha A / P_j)^{\frac{1}{1-\alpha}} \equiv X_j(P_j), \tag{9}
$$

since $\sum_i L_i$ in equilibrium equals aggregate labor supply, L.

3 The monopolist supplier of intermediate good j

In principle the decision problem of monopolist j is the following. Subject to the demand function (9), a price and quantity path $(P_j(\tau), X_j(\tau))_{\tau=t}^\infty$ should be chosen so as to maximize the value of the firm (the present value of the future cash flows):

$$
V_j(t) = \int_t^\infty \pi_j(\tau) e^{-\int_t^\tau r(s)ds} d\tau.
$$

The cash flow ("profit") at time τ is

$$
\pi_j(\tau) = (P_j(\tau) - 1) X_j(\tau),
$$

and the discount rate is $r(s)$, the real interest rate at time s. Since, by assumption, there is no uncertainty, there is only one interest rate, the risk-free rate $r(s)$. The cost (in terms of basic goods) per unit of X_j is 1, in view of (3). Since there is no interdependence over time in the intertemporal optimization problem, it can be reduced to a series of static problems, one for each t:

$$\max_{P_j} \pi_j = (P_j - 1)X_j \text{ s.t.} \tag{10}$$

$$X_j = L(\alpha A)^{\frac{1}{1-\alpha}} P_j^{-\frac{1}{1-\alpha}}. \tag{11}$$

Here we can substitute the constraint (11) into (10), take the derivative wrt. P_j, and then equalize the result with zero.

Alternatively, we may use the rule that the profit maximizing price of a monopolist is the price at which marginal revenue equals marginal cost ($MR = MC$). This is the more intuitive route we will take. We have

$$TR \text{ (= total revenue)} = P_j X_j(P_j) = P_j(X_j)X_j,$$

where, from (11),

$$P_j(X_j) = (X_j/L)^{-(1-\alpha)}\alpha A,$$

which is the maximum price at which the amount X_j can be sold. We find

$$\begin{aligned} MR &= \frac{dTR}{dX_j} = P_j + X_j\frac{dP_j}{dX_j} = P_j(1 + \frac{X_j}{P_j}\frac{dP_j}{dX_j}) \\ &\equiv P_j(1 + \frac{1}{E_{X_j,P_j}}) = MC = 1, \end{aligned}$$

where E_{X_j,P_j} is the elasticity of demand wrt. the price. By (11), this elasticity is, $-1/(1-\alpha)$, and we get the profit maximizing price as

$$P_j = \frac{1}{1-(1-\alpha)} = \frac{1}{\alpha} \equiv P > 1. \tag{12}$$

Owing to monopoly power, the price is above $MC = 1$; the mark-up is $1/\alpha$. And since the elasticity of demand wrt. the price is independent of the quantity demanded and since MC is constant, the chosen price is time independent. Moreover the price is the same for all $j = 1, 2, ..., N$.

Substitution into (11) gives

$$X_j = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} \equiv X_m, \tag{13}$$

where the subscript m indicates "monopolist supply".[5] Profits are

$$\pi_j = (\frac{1}{\alpha} - 1)X_j = (\frac{1}{\alpha} - 1)LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} \equiv \pi. \tag{14}$$

[5] Another way of deriving (12) and (13) is the following. By (11), $P_j = \alpha A(X_j/L)^{\alpha-1}$. Substituting this into (10) and setting $d\pi_j/dX_j = 0$ then yields (13) and (12).

The firm j then obtains a market value (present value of future profits) equal to

$$V_j(t) = \pi \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} d\tau \equiv V(t). \tag{15}$$

Thus, all the monopolist firms in Sector 2 charge the same price, $1/\alpha$, sell the same quantity X_m, earn the same profit, π, and have the same market value, $V(t)$.

4 When is R&D active?

When is it worth doing R&D? Clearly, when the value of an innovation, $V(t)$, is not lower than the cost, η, that is, when $V(t) \geq \eta$. And in an *equilibrium* with $\dot{N} > 0$ (positive R&D investment), the value of an innovation, $V(t)$, must equal the cost:

$$V(t) = \eta. \tag{16}$$

Indeed, if $V(t) > \eta$, everybody wants to invest in R&D and there will be infinite demand for loanable funds to finance R&D (pay for the investment R). This excess demand drives the interest rate up and thereby the present value, $V(t)$, of future profits down. On the other hand, if $V(t) < \eta$, nobody will invest in R&D; the supply of loanable funds (saving) from the households will find no demand and so the interest rate will fall − until $V(t) = \eta$.

We now combine (16) with the general no-arbitrage condition in the absence of uncertainty. The rate of return to financial wealth placed in an equity share of an innovative firm is $\left[\pi(t) + \dot{V}(t)\right]/V(t)$, and on the loan market it is $r(t)$. Thus, the no-arbitrage condition is

$$\frac{\pi(t) + \dot{V}(t)}{V(t)} = r(t) \text{ for all } t. \tag{17}$$

In an equilibrium with active R&D we have (16) and so $\dot{V}(t) = 0$. Substituting into (17) gives the equilibrium interest rate:

$$r(t) = \pi/\eta = (\frac{1}{\alpha} - 1)\frac{L}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \equiv r, \tag{18}$$

where we have used (14). The interest rate is thus time independent. This is a first sign that the model may end up as a kind of reduced-form AK model.

Potential innovators finance their R&D investment by issuing equity shares. Households place their saving in these shares to earn a return on saving. The return comes when the successful innovators distribute their profits to the share holders.

5 The households

There are L households, all alike, with infinite horizon. A household chooses $(c(t))_{t=0}^{\infty}$ to maximize

$$\begin{aligned} U_0 &= \int_0^\infty \frac{c(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \quad \text{s.t.} \\ c(t) &> 0, \\ \dot{a}(t) &= ra(t) + w(t) - c(t), \qquad a(0) \text{ given}, \\ \lim_{t\to\infty} a(t)e^{-rt} &\geq 0, \end{aligned} \tag{19}$$

where $a(t)$ equals per capita financial wealth. In equilibrium

$$a(t) = \frac{V(t)N(t)}{L},$$

because the savings of the households is placed in equity shares in the monopoly firms.

As usual the first-order conditions for the household decision problem lead to the Keynes-Ramsey rule

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(r-\rho). \tag{20}$$

Inserting (18) gives

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[\left(\frac{1}{\alpha}-1\right)\frac{L}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} - \rho\right] \equiv \gamma, \tag{21}$$

a constant. The necessary transversality condition is that the No-Ponzi-Game condition (19) is satisfied with equality.

6 General equilibrium

To ensure that the equilibrium path considered is really one with $\dot{N} > 0$, we need the parameter restriction

$$(\frac{1}{\alpha}-1)\frac{L}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} > \rho. \tag{A1}$$

To ensure a bounded utility integral we need, in addition, the restriction

$$\rho > (1-\theta)\gamma, \tag{A2}$$

where γ is given in (21).

6.1 The aggregate production function

We will show that the aggregate production function in the basic-goods sector is AK-style. Indeed, since $P_j = P$ for all j, firm i in Sector 1 chooses

$$x_{ij} = x_i, \text{ for all } j,$$

in view of (7). Hence, (1) can be simplified to

$$Y_i = ANx_i{}^{\alpha}L_i{}^{1-\alpha} = AN(\frac{x_i}{L_i})^{\alpha}L_i. \tag{22}$$

Now, from (8),

$$\frac{x_{ij}}{L_i} = \frac{x_i}{L_i} = (\alpha A/P)^{\frac{1}{1-\alpha}} = (\alpha^2 A)^{\frac{1}{1-\alpha}}, \tag{23}$$

since $P = 1/\alpha$. The reason that the input ratio x_i/L_i is the same for all firms in the basic-goods sector is that they face the same input prices and have the same CRS production function. Substituting (23) into (22) and summing gives

$$\begin{aligned} Y &= \sum_{i=1}^{M} Y_i = AN(\alpha^2 A)^{\frac{\alpha}{1-\alpha}} L && (24)\\ &= A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}} LN \equiv \bar{A}N. && (25)\end{aligned}$$

We see that aggregate output of Sector 1 is a constant multiplied by the number of intermediate goods varieties (in some sense an index of the endogenous level of technical knowledge in society). This confirms that we have an AK-style model with N (the number of varieties or the level of technical knowledge) acting in the role of K.

The last part of (22) suggests another way of writing aggregate output of Sector 1. Indeed, total supply of each intermediate good j is X_m and total demand is $\sum_i x_i$. Thus, in equilibrium, $X_m = \sum_i x_i = (\alpha^2 A)^{\frac{1}{1-\alpha}} \sum_{i=1}^{M} L_i = (\alpha^2 A)^{\frac{1}{1-\alpha}} L$, where the second equality follows from (23). So

$$\frac{X_m}{L} = (\alpha^2 A)^{\frac{1}{1-\alpha}} = \frac{x_i}{L_i}.$$

Substituting into (22) gives

$$Y_i = AN(\frac{X_m}{L})^{\alpha}L_i.$$

Now, summing over all i gives

$$\begin{aligned} Y &= \sum_{i=1}^{M} Y_i = AN(\frac{X_m}{L})^{\alpha}L = AX_m^{\alpha}L^{1-\alpha}N && (26)\\ A(NX_m)^{\alpha}N^{1-\alpha}L^{1-\alpha} &= AX^{\alpha}(NL)^{1-\alpha}, && (27)\end{aligned}$$

since aggregate input of intermediate goods is $X = NX_m$.

The aggregate production function has now been expressed in three alternative forms. The form (25) is useful by directly displaying an "AK structure". The form (26) serves the same purpose in a more compact way. Finally, the form (27) has the virtue of making it explicit how the *total* input, X, of intermediate goods enters, given the static efficiency condition that X_j is the same for all j. This X is for instance useful as one of the control variables when setting up the social planner's problem for this economy. Note also that the form (27) displays CRS wrt. producible inputs, X and N, in the "growth engine" (the growth-driving sector).[6] Already this indicates that the model is, from a technological point of view, capable of generating fully endogenous growth.

6.2 The balanced growth path

In view of the AK-style structure, our conjecture is that already from the date 0 the equilibrium path is a path where the produced inputs, N and X, grow at the same rate as c. We now show that this is in fact true.

Substituting (25) into (2), we find an expression for R, which inserted into (4) gives

$$\dot{N}(t) = \frac{1}{\eta}\left[(\bar{A} - X_m)N(t) - c(t)L\right]. \tag{28}$$

Using (21) and constancy of L we can write this first-order linear differential equation on the form:

$$\dot{N}(t) = \frac{1}{\eta}(\bar{A} - X_m)N(t) - \frac{L}{\eta}c(0)e^{\gamma t}. \tag{29}$$

The initial level of consumption, $c(0)$, is endogenous. From our general knowledge of AK-style models, we know that to satisfy the transversality condition of the household, $c(0)$ must be such that the state variable, here N, grows at the same rate as c. Thus, from (28), the requirement is

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\eta}\left(\bar{A} - X_m - \frac{c(t)L}{N(t)}\right) = \gamma. \tag{30}$$

Thus $c(t)L = \left(\bar{A} - X_m - \eta\gamma\right)N(t)$ for all $t \geq 0$, which for $t = 0$ determines the required $c(0)$ since $N(0)$ is predetermined. Also $X(t) = X_m N(t)$ grows at the constant rate γ already from the first date.

[6] In this model the basic-goods sector (which ultimately is also the basis for the R&D activity) is the "growth engine". Generally, the "growth engine" of an endogenous growth model is defined as the set of sectors which use their own output as input.

Labour productivity can be defined as

$$y \equiv Y/L = \bar{A}N/L. \tag{31}$$

It grows at the same rate as N, the rate γ. Thus, the model generates fully endogenous growth and there are no transitional dynamics.

7 Comparative analysis

$\partial\gamma/\partial\rho = -1/\theta < 0$. Higher impatience $\Rightarrow$ lower propensity to save $\Rightarrow$ less investment in R&D.

$\partial\gamma/\partial\theta = -\frac{1}{\theta^2}\left[\left(\frac{1}{\alpha}-1\right)\frac{L}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} - \rho\right] = -\frac{\gamma}{\theta} < 0$. Higher desire for consumption smoothing $\Rightarrow$ attempt to transform some of the higher future consumption possibility into higher consumption today, hence lower saving which in turn implies less investment in R&D.

$\partial\gamma/\partial A = \frac{L}{\theta\alpha\eta}A^{\frac{\alpha}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} > 0$. Higher factor productivity $\Rightarrow$ higher return on saving $\Rightarrow$ more saving at the aggregate level (the negative substitution effect and wealth effect on consumption dominates the positive income effect) $\Rightarrow$ more investment in R&D. As usual, the constant A need not have a narrow technical interpretation. It can reflect the quality of the institutions in society (rule of law etc.) and the level of "social capital". By social capital is meant society's stock of social networks and shared norms that support and maintain confidence, credibility, willingness to respect social norms, trust and trustworthiness.

$\partial\gamma/\partial\eta = -\frac{1}{\theta}\left(\frac{1}{\alpha}-1\right)\frac{L}{\eta^2}A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} < 0$. Higher R&D costs result in lower R&D investment.

$\partial\gamma/\partial L = \frac{1}{\theta\eta}\left(\frac{1}{\alpha}-1\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} > 0$. A larger population L implies lower per capita cost η/L associated with producing a given amount of new technical knowledge, which in turn improves productivity for *all.* This is an implication of knowledge being nonrival good. In a larger society, with larger markets, the incentive to do R&D is therefore higher. In the present version of the R&D model the result is a higher growth rate permanently. This is the controversial "strong" scale effect (scale effect on growth), typical for innovation-based growth models with fully endogenous growth. This strong scale effect as well as the fully endogenous growth property are due to a "hidden" knife-edge condition in the specification of the "growth engine", essentially a knife-edge condition in the production function for basic goods. We will return to this later in the course.