Economic Growth. Lecture Note 13.

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A note on B & S-style one- and two-sector models with human capital

According to our general definition the term *human capital* refers to the stock of skills embodied in a human being and acquired through formal learning (education) and on-the-job training. Human capital is not only a *rival* and *excludable* good. Because it is *embodied* in a human being, it is lost upon death. It cannot be transferred by inheritance.

In my view the main problem with the exposition of human capital accumulation in B & S is that they treat human capital accumulation as something similar to physical capital accumulation.

The approach to understanding the role of education and human capital in a growth context I favor is the approach taken by Jones (2002, 2007), among others. This is the approach where human capital accumulation combined with physical capital accumulation is not in itself a sufficiently powerful growth engine to generate sustained per capita growth. There are both theoretical and empirical reasons to see the role of human capital as rather being to *assist* technological *innovations* and innovation-driven per-capita growth.

At the microeconomic level this approach rests on the *Mincerian specification* of the formation of human capital, which is recapitulated in Jones (2007). This brief article[1] by Jones, together with the application of the theory in Jones (2002),[2] constitute the *primary* material on human capital in this course.

But since the approach to human capital taken by B & S in their Chapter 5 is not uncommon in endogenous growth theory, this lecture note briefly comments on their presentation the issues, especially their Section 5.1 and 5.2.1.

[1] See our course pack.
[2] See our course pack.

1 "Broad capital" in the one-sector model with physical and human capital

The focus is on a closed economy and formal education rather than on-the-job training.

The interpretation of the model in Section 1 of Chapter 5 is that the manufacturing sector and the educational sector have the same technology. The key equations are (5.2) and (5.3), to which one should add the constraints $I_K \geq 0$ and $I_H \geq 0$ (cf. B & S, p. 243).

Since the manufacturing sector and the educational sector have the same technology, we can aggregate to a one-sector model with physical and human capital. We have

$$\frac{\partial Y}{\partial K} = \alpha A K^{\alpha-1} H^{1-\alpha} = \alpha A (\frac{K}{H})^{\alpha-1}.$$

The analysis shows that *after an initial period of complete specialization* (implied by $K_0/H_0 \neq \alpha/(1-\alpha)$ and the non-negativity constraints on I_K and I_H),

$$\frac{K_t}{H_t} = \frac{\alpha}{1-\alpha}, \tag{1}$$

a constant. Inserting into the formula for $\partial Y/\partial K$, we get

$$\frac{\partial Y}{\partial K} = \alpha^\alpha (1-\alpha)^{1-\alpha} A, \tag{2}$$

a constant. Hence,

$$r^* = \alpha^\alpha (1-\alpha)^{1-\alpha} A - \delta,$$

and

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(\alpha^\alpha (1-\alpha)^{1-\alpha} A - \delta - \rho) \equiv \gamma^*.$$

Aggregate output is

$$Y = A K^\alpha H^{1-\alpha} = A(\frac{K}{H})^{\alpha-1} K = A\alpha^{\alpha-1}(1-\alpha)^{1-\alpha} K \equiv \bar{A} K. \tag{3}$$

Here B & S write (p. 242): "Thus the model is equivalent to the AK model that we studied in Chapter 4. .... The results for r^* and γ^* ... are essentially the same as those obtained for the AK model developed in chapter 4." In my view these are dangerous formulations that can easily be misunderstood. For example, in the AK model of Section 4.1 we have $\partial Y/\partial K = A$. But in the present model, in spite of (3), we do *not* have $\partial Y/\partial K = \bar{A}$. Instead, in view of (2), $\partial Y/\partial K = \alpha\bar{A} < \bar{A}$.

This is a reminder about a general feature. One should be aware that the different reduced-form AK models (models with constant marginal and average product of capital at the aggregate level, as in Sections 4.2, 4.3, 4.4 and 5.1) *usually* have a real interest rate below the level corresponding to the simple or "true" AK model of Section 4.1 in B & S.

Nevertheless, in the present case it is possible to introduce a concept of *broad capital* such that formally we get a "true" AK model with respect to *this* concept. The point is that (3) only holds as long as (1) holds. A marginal increase in K without any change in H, decreases H/K and so we get $\partial Y/\partial K < \bar{A}$.

Let us define "broad capital" as

$$\tilde{K} \equiv K + H.$$

Then, inserting (1), we get

$$\tilde{K} = K + \frac{1-\alpha}{\alpha}K = \frac{1}{\alpha}K,$$

so that $K = \alpha\tilde{K}$ and $H = (1-\alpha)\tilde{K}$. Thus

$$\begin{aligned} Y &= AK^{\alpha}H^{1-\alpha} = A(\alpha\tilde{K})^{\alpha}(1-\alpha)^{1-\alpha}\tilde{K}^{1-\alpha} = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{K} \\ &\equiv \alpha\bar{A}K \equiv \tilde{A}\tilde{K}. \end{aligned}$$

From this we find the marginal product wrt. *broad* capital as

$$\frac{\partial Y}{\partial \tilde{K}} = \tilde{A} = \alpha\bar{A} = \frac{\partial Y}{\partial K}.$$

2 Two-sector model with physical and human capital

In Section 5.2.1 B & S discuss a general two-sector model with physical and human capital. Let the manufacturing sector be called Sector 1 and the educational sector Sector 2. Summing the inputs of physical capital and human capital, respectively, in the two sectors, we have

$$\begin{aligned} K_1 + K_2 &= K, \\ H_1 + H_2 &= H \equiv hL. \end{aligned}$$

We follow B & S and assume L is constant (but that is not essential).

Output in the manufacturing sector is

$$Y = AK_1^{\alpha}H_1^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1. \tag{4}$$

This output is used for consumption and investment in physical capital. Thus,

$$\dot{K} = Y - cL - \delta K.$$

Output (gross investment in human capital) in the educational sector is

$$\dot{H} + \delta H = BK_2^{\eta} H_2^{1-\eta}, \qquad B > 0, 0 < \eta < 1. \tag{5}$$

The rate of depreciation for human capital is assumed to be the same as that for physical capital. The representative household has the intertemporal utility function

$$U_0 = \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} Le^{-\rho t} dt, \qquad \theta > 0, \rho > 0.$$

B & S discuss this framework without showing any solution. It can be shown, however, (by a method similar to that used in a later lecture note on the Uzawa-Lucas model) that a social planner - or the market economy under perfect competition - will end up with the steady state solution

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left\{ \alpha A \left[\frac{1-\eta}{\alpha} \frac{B}{A} \left(\frac{\eta}{1-\eta} \frac{1-\alpha}{\alpha} \right)^{\eta} \right]^{\frac{1-\alpha}{1-\alpha+\eta}} - \delta - \rho \right\} \equiv \gamma^*.$$

We see that $\partial \gamma^* / \partial B > 0$. Thus, in this model higher B, i.e., higher productivity in the educational sector, results in a higher long-run growth rate. Since $AA^{-\frac{1-\alpha}{1-\alpha+\eta}} = A^{\frac{\eta}{1-\alpha+\eta}}$, we also have $\partial \gamma^* / \partial A > 0$. These results are due to the fact that the two sectors together constitute the "growth engine" of this economy.

Now, assume there is a government that levies a production tax at the constant rate $\tau \in (0, 1)$ and uses the tax revenue to finance lump-sum transfers to the households. There are no other government activities, and the government budget is always balanced. Then for all t,

$$\tau Y_t = T_t$$

where T_t is aggregate transfers. Thus, aggregate private gross income after tax is $(1 - \tau) Y_t + T_t$. Comparing with (4) we see that the tax acts like a reduction in A. In view of $\partial \gamma^* / \partial A > 0$, the implication is that the production tax reduces long-run growth in a model like this.

In contrast, in the Uzawa-Lucas model (B & S, Section 6.2.2) the educational sector does not use physical capital, but only human capital. Then the production (manufacturing) sector is not part of the growth engine and a production tax τ will have no effect on the long-run growth rate.

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