Economic Growth.   Lecture Note 11.

05.04.2010.   Christian Groth

On Alesina and Rodrik: Distributive politics and economic growth

1 Prelude to Alesina and Rodrik (1994)

The point of departure in the Alesina and Rodrik article is the observed negative correlation between inequality in wealth (or income) in a country and the per capita growth rate. The cross-country regression results are reported in tables I and II in Alesina and Rodrik (1994). Then the authors set up a model to check whether the observed phenomenon can be explained through the following two mechanisms:

(a) The *political mechanism*. High inequality leads to strong pressure, by the majority in the population, for redistribution through some form of progressive taxation.

(b) The *economic mechanism*. A high tax rate on capital income leads to a low after-tax rate of return on saving. This results in low aggregate saving and investment and thereby in a low growth.

This lecture note attempts to clarify the logic and the derivations in the Alesina-Rodrik model in more detail than the article does.

But first some supplementary data of interest in relation to the topic is presented (sorry that the labels are in Danish). Benabou (1996) studied the huge difference in the growth performance of South Korea and the Philippines over the period 1960-1990. He observed that the two countries were in 1960 much alike in many respects of importance for growth, for example initial level of per capita income, size of population, degree of urbanization, shares of agriculture and manufacturing in the total economy, and educational level. But there was one respect in which the two countries differed a lot, namely the degree of income inequality as measured by the Gini coefficient.[1] See Table 1.

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[1]Appendix D gives a refresher on the Gini index.

Table 1. Comparison between South Korea and Philippines. Note: column 2-9 give values for 1960.

Tabel X. Vækst og initialbetingelser for Sydkorea og Filippinerne.

	Vækst p.a.	BNP pr. indbygger	Indbyggertal	Andel af befolkning bosat i hovedstad	Landbrugets andel af BNP	Industriens andel af BNP	Landbrugsvarer m.m., andel af samlet eksport	Primær skolegang, drenge	Primær skolegang, piger	Ginikoefficient[c]	
	(pct.)	(US$)[b]	(mio.)	(pct.)	(pct.)	(pct.)	(pct.)	(pct.)	(pct.)	(pct.)	
	1960-90[a]				1960					1965	1988
Sydkorea	6,7	904	25	28	37	20	86	100	99	34,3	33,6
Filippinerne	1,5	1133	28	27	26	28	96	97	89	51,3	45,7

Kilder: Søjle 1-2: Penn World Table 5.6. Søjle 3-7: Lucas (1993), s. 251. Søjle 8-9: Barro og Lee (1993). Søjle 10-11: Benabou (1996).
Anm.: a) Gennemsnitlig vækst i BNP pr. indbygger. b) 1985 PPP korrigerede. c) Målt på husstandsindkomst før skat.

The Gini coefficients in the two last columns of Table 1 are calculated on household income before tax. For comparison with more developed countries, see the numbers in the first column of Table 2.

Table 2. Income inequality and its change over time for some developed countries.

Tabel X. Ulighed i personindkomster i forskellige lande målt ved Ginikoefficienten (niveau og ændring).

	Markedsindkomst		Disponibel indkomst	
	Ginikoefficient	Ændring i pct.	Ginikoefficient	Ændring i pct.
Australien, 1993/94	46,3		30,6	
Ændring 1975-1994		36,6		5,2
Danmark, 1994	42,0		21,7	
Ændring 1983-1994		11,2		-4,9
Finland, 1995	39,2		23,0	
Ændring 1986-1995		11,4		9,7
Frankrig, 1990	-		29,1	
Ændring 1979-1990		-		-1,7
Tyskland, 1994	43,6		28,2	
Ændring 1984-1994		1,2		6,4
Italien, 1993	51,0		34,5	
Ændring 1984-1993		20,8		12,7
Holland, 1994	42,1		25,3	
Ændring 1977-1994		14,2		11,8
Sverige, 1994	48,8		23,4	
Ændring 1975-1994		17,3		0,9
USA, 1995	45,5		34,4	
Ændring 1974-1995		13,1		10,0

Kilde: OECD, *Economic Outlook, Dec. 1997*, s. 51.

Whereas the Gini in South Korea in 1965 as well as 1988 is below these numbers, the Gini in the Philippines is above. (In passing, notice the indication in Table 2 of a tendency since the middle of the 1970s to an increasing Gini in the developed countries, possibly caused by skill-biased technical change.)

2   Elements of the Alesina-Rodrik model

The Alesina-Rodrik model is a reduced-form AK model with *heterogenous* population. Although preferences are the same across individuals, initial resources differ. The setting is a closed economy with government. There is perfect competition in all markets. Firms maximize profits. Population is assumed constant. Households are of the Ramsey type with infinite horizon.

To comply with our usual notation, we let capital letters denote firm inputs as well as aggregate variables, whereas Alesina and Rodrik use small letters for both. We also write the growth rate of a variable x in the usual way, i.e., as $\dot{x}/x$, whereas Alesina and Rodrik write it as $\hat{x}$.

Firm j has the production function

$$Y_{jt} = AK_{jt}^{\alpha}(G_t L_{jt})^{1-\alpha}, \quad 0 < \alpha < 1, A > 0, \qquad j = 1, 2, ..., m, \tag{1}$$

where G_t is a nonrival productive service provided free of charge by the government (e.g., think of non-overloaded infrastructure[2] or technical information services available on TV and the internet).

There is a balanced budget and wealth taxation:

$$G_t = \tau V_t = \tau K_t, \qquad \tau > 0, \tag{2}$$

where V_t is aggregate private financial wealth. In the considered closed economy, where government debt and natural resources (for instance land) are ignored, V_t equals the value of the capital stock, K_t. The tax rate τ is a *wealth tax* (also called a property tax or net worth tax). Unfortunately, Alesina and Rodrik (p. 469) call τ a "capital income tax", which it is not. If τ were a capital income tax, it would result in the after-tax capital income $(1 - \tau)r_t V_t$, where r_t is the real interest rate. The actual after-tax capital income in the model is, however, $(r_t - \tau)V_t$, cf. (12) below.

We follow Alesina and Rodrik and introduce, from the beginning, the slightly artificial assumption that the wealth tax τ *must* be kept constant from now to infinity

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[2]Here ignoring that this should really be modelled as a stock.

(say, for constitutional reasons) and that voters once for all vote about the size of this constant τ, cf. Section 4 and 5 below. No other kinds of taxes are ever considered.

The authors have some vague remarks about the possibility of interpreting K_t as "broad capital". My personal preference, in the context of this model, is to interpret K_t as just physical capital. That makes the whole story more concrete.[3]

The dynamic resource constraint at the aggregate level is

$$\dot{K}_t = Y_t - G_t - C_t, \tag{3}$$

where we follow Alesina and Rodrik and ignore capital depreciation ($\delta = 0$). From now, we suppress the explicit timing of the variables when not needed for clarity.

2.1 Factor prices and aggregate production function in equilibrium

At any time t firm j takes the going G as given. Profit maximization under perfect competition leads to

$$\frac{\partial Y_j}{\partial K_j} = \alpha A K_j^{\alpha-1}(GL_j)^{1-\alpha} = \alpha A k_j^{\alpha-1}G^{1-\alpha} = r, \tag{4}$$

$$\frac{\partial Y_j}{\partial L_j} = (1-\alpha)A K_j^{\alpha}G^{1-\alpha}L_j^{-\alpha} = (1-\alpha)A k_j^{\alpha}G^{1-\alpha} = w, \tag{5}$$

where $k_j \equiv K_j/L_j$ and w is the real wage. Since the chosen k_j is seen to be the same for all firms, assuming clearing in factor markets we have

$$k_j = \frac{\sum_j K_j}{\sum_j L_j} = \frac{K}{L}, \qquad j = 1, 2, ..., m, \tag{6}$$

where K is the aggregate amount of capital in the economy and L is the aggregate labor supply, an exogenous constant.[4] Substituting the government budget constraint, cf. (2), and (6) into (4), we find the equilibrium real interest rate

$$r = \alpha A(\frac{K}{L})^{\alpha-1}(\tau K)^{1-\alpha} = \alpha A(L\tau)^{1-\alpha} \equiv r(\tau), \qquad r'(\tau) > 0. \tag{7}$$

Thus, the equilibrium real interest rate is an increasing function of the tax rate, τ. Why? A higher τ provides a higher level of the productive service, G; due to the

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[3]And we do not have to worry about the aggregation procedure upon which a "broad capital" concept would have to rest.

[4]That is, there is no population growth. Alesina og Rodrik normalize L to 1, but that is not a good idea in a model like this, because L is an important parameter in view of the strong scale effect of the model. (Unfortunately, in the early endogenous growth literature it was quite common to hide undesired strong scale effects this way.)

complementarity in the production function, this implies a higher marginal product of capital, hence, in equilibrium, a higher interest rate.

Similarly, the equilibrium real wage w is determined by

$$w = (1-\alpha)A(\frac{K}{L})^\alpha(\tau K)^{1-\alpha} = (1-\alpha)AL^{-\alpha}\tau^{1-\alpha}K \equiv \omega(\tau)K, \qquad \omega'(\tau) > 0. \qquad (8)$$

The interpretation of the positive effect on w of a higher τ is analogous to that for r above. Moreover, note that a greater aggregate capital stock, K, leads to a higher real wage. Two mechanisms lie behind this. First, physical capital is complementary to labor[5] so that more physical capital implies a higher marginal product of labor. Second, more physical capital in the society means more private wealth in the society, hence, for given τ more tax revenue and therefore a higher level of the productive service G.

Since all firms choose the same capital intensity, it is a simple matter to derive an aggregate production function. Indeed, GDP (here $= NDP$) $= Y = \sum_j Y_j = \sum_j y_j L_j = \sum_j A k_j^\alpha G^{1-\alpha} L_j = \sum_j A(\frac{K}{L})^\alpha G^{1-\alpha} L_j = A(\frac{K}{L})^\alpha G^{1-\alpha} \sum_j L_j = A(\frac{K}{L})^\alpha G^{1-\alpha}L = AK^\alpha(\tau K)^{1-\alpha}L^{1-\alpha} = (\tau L)^{1-\alpha}AK$, i.e.,

$$Y = \bar{A}(\tau)K, \qquad \text{where} \qquad \bar{A}(\tau) \equiv (\tau L)^{1-\alpha}A, \qquad \bar{A}'(\tau) > 0. \qquad (9)$$

The model is a reduced-form AK model in that (7) shows that the interest rate is a constant and (9) shows that the output-capital ratio is constant at the aggregate level.

Aggregate factor incomes after tax are

$$
\begin{aligned}
Y_k &= [r(\tau) - \tau]K, \quad \text{and} \\
Y_l &= wL = \omega(\tau)KL,
\end{aligned}
$$

respectively. Is national income equal to aggregate value added (GDP) − as it should be in a closed economy? Let us check:

$$
\begin{aligned}
\text{national income} &= \text{aggregate income} = Y_k + Y_l + \tau K = r(\tau)K + \omega(\tau)KL \\
&= \left[\alpha A(L\tau)^{1-\alpha} + (1-\alpha)AL^{-\alpha}\tau^{1-\alpha}L\right]K \\
&= A(L\tau)^{1-\alpha}K = \bar{A}(\tau)K = Y = \text{GDP}, \qquad (10)
\end{aligned}
$$

hence, OK!

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[5]This is always the case when we have a two-factor neoclassical production function with constant or increasing returns to scale.

2.2 Heterogeneous population

Individual no. i has, at time t, the resources (l_i, k_{it}), where l_i is this person's inelastic exogenous labor supply, which is assumed time-independent,[6] and k_{it} is his or hers financial wealth, which through saving varies over time. We have $\sum_i k_{it} = V_t = K_t$, that is, the sum of individual financial wealth equals aggregate private financial wealth which in turn equals aggregate physical capital. Aggregate labor input is $\sum_i l_i = L$. We choose the measurement unit for labor such that $L =$ population size, so that L is not only aggregate labor supply but also the number of (infinitely-lived) individuals in society.

Consider the factor proportion l_i/k_{it}, $i = 1, 2, ..., L$. Alesina and Rodrik focus on person i's *relative* factor-proportion,

$$\sigma_i \equiv \frac{l_i/L}{k_{it}/K_t} = \frac{l_i/k_{it}}{L/K_t} \geq 0. \tag{11}$$

We shall refer to σ_i as person i's *relative factor endowment*. Here, for simplicity, the model ignores that some individuals might have zero or negative financial wealth. In any case, σ_i can be seen as a measure of *how dependent person i is on labor income relative to capital income* (compared to the average individual in society). In Section 3 we shall see that σ_i is time-independent (this is the reason it has no time index).

We have:

$$\sigma_i \;=\; 0 : \text{ person } i \text{ is a pure capitalist;}$$
$$\sigma_i \;>\; 0 \text{ but low: person } i\text{'s dependency on labor income is positive, but low;}$$
$$\sigma_i \;>\; 0 \text{ and high: person } i\text{'s dependency on labor income is high;}$$
$$\sigma_i \;\to\; \infty : \text{ in the limit person } i \text{ is a pure proletarian.}$$

If all persons had the same factor endowment, we would have

$$(l_i, k_{it}) = (1, \frac{K_t}{L}), \text{ and } \sigma_i = 1, \quad i = 1, 2, ..., L.$$

On the other hand, a person with $\sigma_i = 1$ need not have $(l_i, k_{it}) = (1, \frac{K_t}{L})$; the only thing we can infer from $\sigma_i = 1$ is that person i has the same factor *proportion* as the "average" person, i.e., $l_i/k_{it} = L/K_t$. If all individuals have more or less the same l_i, we can view the relative factor endowment, σ_i, as an indicator of relative *wealth poverty* (where "wealth" refers to financial wealth, not human wealth).

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[6]We say "effective" labor supply to underline that may be the individuals do not differ wrt. hours supplied, but wrt. the productivity of their labor.

2.3 Economic behavior of individual i

It is assumed that all individuals have infinite horizon and the same preferences, given by a logarithmic instantaneous utility function and a constant rate of time preference $\rho > 0$. The economic decision problem for individual i, as seen from time 0, is:

$$\max_{(c_{it})_{t=0}^{\infty}} U_i = \int_0^{\infty} (\log c_{it}) e^{-\rho t} dt \quad \text{s.t.}$$

$$c_{it} > 0,$$

$$\dot{k}_{it} = [r(\tau) - \tau] k_{it} + \omega(\tau) K_t l_i - c_{it}, \qquad k_{i0} \text{ given}, \qquad (12)$$

$$\lim_{t \to \infty} k_{it} e^{-\int_0^t [r(\tau) - \tau] ds} \geq 0, \qquad (13)$$

where τ taken as given. Here k_{it} is seen as the individual's financial wealth which is here, for completeness, allowed to be negative; by imposing the No-Ponzi-Game condition (13), solvency is ensured. (If there were no loan market and k_{it} were interpreted as physical capital directly possessed by individual i, (13) should be replaced by the requirement $k_{it} \geq 0$ for all $t \geq 0$.)

From the FOCs we get the Keynes-Ramsey rule

$$\frac{\dot{c}_{it}}{c_{it}} = r(\tau) - \tau - \rho \equiv \gamma(\tau). \qquad (14)$$

The transversality condition is

$$\lim_{t \to \infty} k_{it} e^{-\int_0^t [r(\tau) - \tau] ds} = 0. \qquad (15)$$

By (14) we see that the growth *rate* of individual consumption is the same for all individuals (because they have the same θ and ρ).

3 Economic development for a given tax rate

Combining the aggregate accumulation equation (3) with (2), (10), the Keynes-Ramsey rule (14) and the transversality condition (15), it can (in the usual way for AK-style models) be proved that

$$\frac{\dot{k}_{it}}{k_{it}} = \frac{\dot{c}_{it}}{c_{it}} = \gamma(\tau) = r(\tau) - \tau - \rho = \frac{\dot{K}_t}{K_t} = \frac{\dot{w}_t}{w_t}. \qquad (16)$$

Indeed, this is what we should expect, since we have an AK structure (constant after-tax interest rate and constant output-capital ratio, cf. (10)).[7] Since (16) shows that

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[7]Of course, it is also important that this AK structure is embedded in a kind of Ramsey-style household sector. If the AK structure were part of an overlapping generations framework, the conclusion (16) would *not* be valid.

individual financial wealth grows at the same rate for all individuals, the relative factor endowment, σ_i, of every individual will be constant over time (as was hinted at above).

To ensure positive growth we assume:

$$\alpha A(L\tau)^{1-\alpha} - \tau > \rho. \tag{A1}$$

Given (A1), $\gamma(\tau) = r(\tau) - \tau - \rho > 0.$[8] Moreover,

$$\gamma'(\tau) = r'(\tau) - 1 = (1-\alpha)\alpha A L^{1-\alpha}\tau^{-\alpha} - 1 \gtreqqless 0 \quad \text{for} \quad r'(\tau) \gtreqqless 1,$$
$$\text{i.e., for } \tau \lesseqqgtr \left[(1-\alpha)\alpha A L^{1-\alpha}\right]^{1/\alpha} \equiv \tau^*. \tag{17}$$

Fig. 1 illustrates. An increase in the tax rate τ has two effects that go in opposite directions. On the one hand, a higher τ means a higher level of productive services, G, and therefore a higher Y/K (see (9)), which implies a higher growth potential. On the other hand, a higher τ means both more preempting of output and more 'distortion' of saving incentives[9] and therefore *lower* growth. Starting from a low τ, when τ increases, the first effect dominates until $\tau = \tau^*$. A further increase in τ lowers growth because the combined preempting and distortionary effect dominates. This pattern reflects the positive, but falling marginal productivity of G, cf. (1).

4 Voters' political preferences

First we determine the consumption function of individual i. Given the AK structure of the model, we know that the key to the consumption function lies in the first equality in (16). Indeed, from that equation combined with (12) follows

$$\frac{\dot{k}_{it}}{k_{it}} = r(\tau) - \tau + \omega(\tau)K_t l_i - \frac{c_{it}}{k_{it}} = \gamma(\tau) = r(\tau) - \tau - \rho, \quad \text{hence,}$$
$$\frac{c_{it}}{k_{it}} = \omega(\tau)K_t l_i + \rho \quad \text{or}$$
$$c_{it} = [\omega(\tau)L\sigma_i + \rho]\, k_{it}, \tag{18}$$

where we have used that $K_t l_i / k_{it} \equiv L\sigma_i$ from the definition of σ_i in (11).

4.1 The tax rate preferred by a given individual

We now ask: given the utility function U_i, what level of τ would person i prefer? The answer can be found in the following way.

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[8]In this model we need no particular parameter restriction to ensure bounded utility, since the instantaneous utility function is logarithmic. With logarithmic utility the (absolute) elasticity of marginal utility (θ) equals 1 and so the usual requirement $\rho > (1-\theta)\gamma(\tau)$ is automatically satisfied as long as $\rho > 0$.

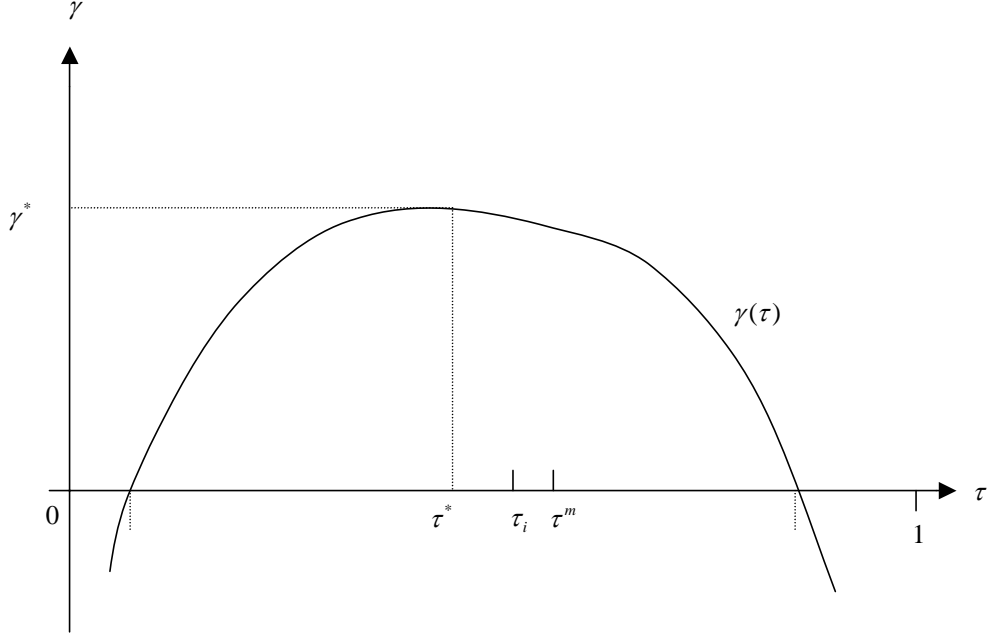[9]It is the *after-tax* rate of return to saving that matters for saving.

Figure 1:

Since k_i grows at the constant rate $\gamma(\tau)$, we have from (18)

$$c_{it} = [\omega(\tau)L\sigma_i + \rho]\, k_{it} = [\omega(\tau)L\sigma_i + \rho]\, k_0 e^{\gamma(\tau)t} = c_0 e^{\gamma(\tau)t}. \tag{19}$$

Consider the problem:

$$\max_{\tau} U_i = \int_0^\infty (\log c_{it})e^{-\rho t}dt \quad \text{s.t. (19).} \tag{20}$$

(In Alesina and Rodrik, 1994, p. 474, is added the "constraint" $\dot{K} = \gamma(\tau)K$, but that condition is superfluous, since K does not enter the problem.) The problem (20) is not a dynamic optimization problem, but a purely static problem. Indeed, inserting (19) gives

$$
\begin{aligned}
U_i &= \int_0^\infty \left[\log c_{i0} + \gamma(\tau)t\right] e^{-\rho t}dt \\
&= \log c_{i0} \int_0^\infty e^{-\rho t}dt + \gamma(\tau) \int_0^\infty t e^{-\rho t}dt \\
&= \log\left\{[\omega(\tau)L\sigma_i + \rho]\, k_{i0}\right\}\frac{1}{\rho} + \gamma(\tau)\frac{1}{\rho^2} \quad \text{(by partial integration)} \\
&= \frac{1}{\rho}\left[\log\left\{[\omega(\tau)L\sigma_i + \rho] + \log k_{i0} + \frac{\gamma(\tau)}{\rho}\right\}\right] \equiv U_i(\tau). \tag{21}
\end{aligned}
$$

So $U_i(\tau)$ is the *indirect* utility function. To maximize $U_i(\tau)$ take the derivative wrt.

9

τ :

$$U_i'(\tau) = \frac{1}{\rho}\left[\frac{\omega'(\tau)L\sigma_i}{\omega(\tau)L\sigma_i+\rho}+\frac{\gamma'(\tau)}{\rho}\right] = 0 \Rightarrow$$

$$-\gamma'(\tau)\left[\omega(\tau)L\sigma_i+\rho\right] = \rho\omega'(\tau)L\sigma_i \Rightarrow$$

$$\tau\left[1-\alpha(1-\alpha)AL^{1-\alpha}\tau^{-\alpha}\right] = \rho(1-\alpha)\theta_i(\tau), \tag{22}$$

by (8) and the definition

$$\theta_i \equiv \frac{\omega(\tau)L\sigma_i}{\omega(\tau)L\sigma_i+\rho} = \frac{\omega(\tau)L\sigma_i k_{it}}{\left[\omega(\tau)L\sigma_i+\rho\right]k_{it}} = \frac{i\text{'s labor income}}{i\text{'s consumption}}. \tag{23}$$

Three questions arise. Does (22) have a solution in τ? If so, is it unique? Finally, how does it depend on individual i's relative dependency on labor income, σ_i? To answer these questions, we substitute (8) into (23) and the result into (22) to get

$$\tau\left[1-\alpha(1-\alpha)AL^{1-\alpha}\tau^{-\alpha}\right] = \rho(1-\alpha)\frac{(1-\alpha)AL\tau^{1-\alpha}L\sigma_i}{(1-\alpha)AL\tau^{1-\alpha}L\sigma_i+\rho}.$$

Multiplying by $\tau^{\alpha-1}$ on both sides and ordering, we have

$$\left[(1-\alpha)A(L\tau)^{1-\alpha}\sigma_i+\rho\right]\left[\tau^\alpha-\alpha(1-\alpha)AL^{1-\alpha}\right] = \rho(1-\alpha)^2AL^{1-\alpha}\sigma_i. \tag{24}$$

In Appendix A it is shown that (24) has a solution in τ; it is unique since $\partial\left[LHS(24)\right]/\partial\tau > 0$, whereas the RHS of (24) does not depend on τ. In Appendix B it is shown that $U_i''(\tau) < 0$. In view of this concavity of $U_i(\tau)$, the value of τ satisfying (24) *is* person i's preferred tax rate. This value we shall denote τ_i.

By (24), the only personal characteristic on which the preferred tax rate, τ_i, depends is the relative factor-endowment, σ_i. Thus, we shall consider the preferred tax rate as a function of σ_i, $\tau_i = \tau(\sigma_i)$. We can show (see Appendix C) that

$$\frac{\partial\tau_i}{\partial\sigma_i} = \tau'(\sigma_i) > 0. \tag{25}$$

This is the key result. It says that the higher is individual i's relative dependency on labor income, σ_i, the higher is the tax rate preferred by this individual. The reason is that the less you depend, relatively, on capital income, the less you are hit by the wealth tax, but you get the full benefit from the public service G. Viewing σ_i as an indicator of relative (financial) wealth poverty, it is clear that the higher this is, the less important for the individual is the return on wealth as part of total income and the less important is the growth rate of wealth for the individual.[10]

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[10] In practice labor income is more evenly distributed than financial wealth and income from financial wealth. Hence, an interesting exercise would be to examine the extent to which the story would, qualitatively, be the same with an *income* tax (whether proportional or progressive).

From (24) we see that τ_i for a *pure capitalist* ($\sigma_i = 0$) satisfies

$$
\begin{aligned}
\tau_i^\alpha &= \alpha(1-\alpha)AL^{1-\alpha}, \quad \text{i.e.,} \\
\tau_i &= \left[\alpha(1-\alpha)AL^{1-\alpha}\right]^{1/\alpha} \equiv \tau^*,
\end{aligned}
\tag{26}
$$

from (17). That is, $\tau(0) = \tau^*$. The intuition is that since a pure capitalist according to (18) has the consumption function $c_i = \rho k_i$, she wants the highest possible growth in k_i, that is, she wants $\gamma(\tau)$ maximized.

All *other* members of society have $\tau_i > \tau^*$. This is because they depend on labor income and thus have $\sigma_i > 0$, so that, by (25), $\tau_i > \tau^*$. The intuitive explanation follows from the consumption function (19) which for convenience we repeat here:

$$
c_{it} = \left[\omega(\tau)L\sigma_i + \rho\right]k_{i0}e^{\gamma(\tau)t}.
$$

An increase in τ has both a level effect, via $\omega(\tau)$, and a growth effect, via $\gamma(\tau)$. Start with a low τ and let τ be increased. To begin with, both effects lead in the direction of preferring an even higher τ. This is because (i) $\omega'(\tau) > 0$ for all τ (cf. Fig. 2); and (ii) $\gamma'(\tau) > 0$ for $\tau < \tau^*$. At $\tau = \tau^*$, a person who is dependent on labor income will still benefit from an increase in τ, since $\gamma'(\tau) = 0$ whereas $\omega'(\tau) > 0$ at $\tau = \tau^*$. But with further increases in τ sooner or later a point is reached where the contribution to utility of the still positive level effect is exactly offset by the utility cost of the negative growth effect, since $\gamma'(\tau) < 0$ for $\tau > \tau^*$. That point represents the preferred τ_i, cf. Fig. 2.

The fact that the preferred tax rate as a function of the relative factor-endowment share, $\tau_i = \tau(\sigma_i)$, has the properties that $\tau(0) = \tau^*$ and $\tau'(\sigma_i) > 0$, implies that $\tau(1) > \tau^*$. That is, an "average person", i.e., an individual with $(l_i, k_{it}) = (1, K_t/L)$ and therefore $\sigma_i = 1$, prefers a tax rate *above* the growth-maximizing tax rate. The median voter is likely to have a preferred tax rate, τ^m, above that the "average person" because the distribution of financial wealth is right-skewed $(k^m < K/L)$, see below.

The two next subsections, which deal with interpretation and put things into perspective, may be skipped in a first reading.

4.2   An egalitarian society

We may define an egalitarian society as a society where, from the beginning, everybody has the same factor endowment, i.e.,

$$
l_i = 1, k_{i0} = \frac{K_0}{L}, \quad \text{and therefore } \sigma_i = 1, \quad i = 1, 2, ..., L.
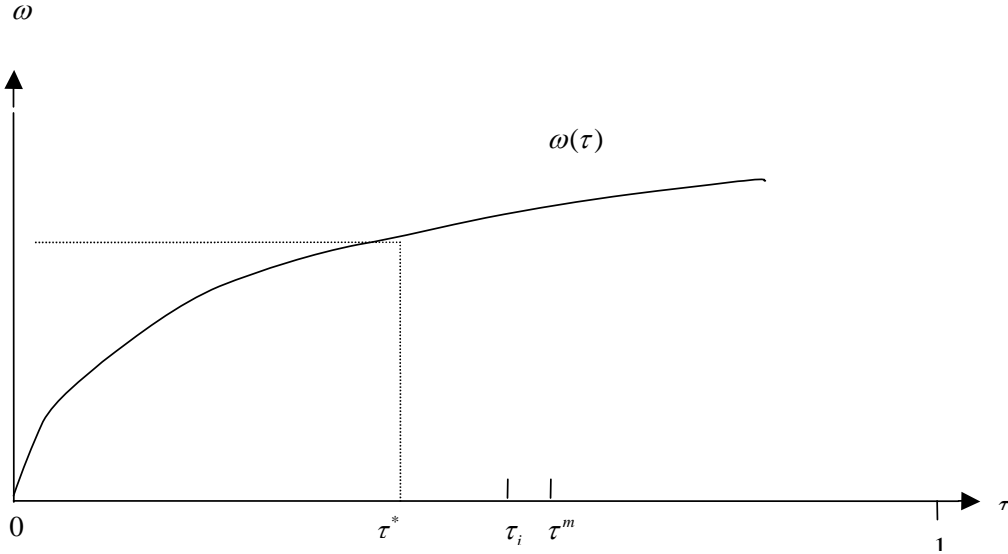\tag{27}
$$

Figure 2:

Every individual is now a "representative agent" and it is fairly unambiguous what is meant by a "welfare-maximizing" government. Since everybody have the same resources, it is natural to give all individuals the same weight in the social welfare function, that is, the government should attempt to maximize discounted utility of the representative agent subject to technology and initial resources. In this way, if we include competitive markets and a wealth tax system as part of the given constraints, the "second best" solution is to choose $\tau = \tau(1)$, that is, choose τ so that U_i is maximized. This is "only" a second-best solution because the optimization is made *within* the constraint that wealth taxation *has* to be used. The social planner's allocation (see below) involves higher welfare, but its implementation in the decentralized economy would require lump-sum taxation.

The fact that $\tau(1) > \tau^*$ combined with G being financed by a wealth tax implies that second best welfare maximization is *not* coincident with second best growth maximization.[11]

The aggregate production function can be written $Y = AK^{\alpha}(GL)^{1-\alpha}$, cf. the derivation of (9). Hence, $\partial Y/\partial G = (1-\alpha)Y/G$, and, since $C + \dot{K} = Y - G$, static efficiency requires $\partial Y/\partial G - 1 = 0$, i.e.,

$$\frac{G}{Y} = 1 - \alpha.$$

In the market economy with wealth taxation and $\tau = \tau^*$ we have, however, $G = \tau^* K$,

---

[11] This is contrary to what we found in the model with *congestion* in Lecture Note 10.

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and therefore

$$\frac{G}{Y} = \tau^* \frac{K}{Y} = \frac{\tau^*}{A(\tau^* L)^{1-\alpha}} = (\tau^*)^\alpha A^{-1} L^{\alpha-1} = \alpha(1-\alpha)AL^{1-\alpha}A^{-1}L^{\alpha-1} = \alpha(1-\alpha) < 1-\alpha$$

from (9) and (17). Even when the market economy with wealth taxation maximizes growth, the economy "under-invests" in the productive public service. The explanation is that the service is not financed with a lump-sum tax, but with a distorting tax which reduces the after-tax return on saving.

4.3   A social planner

In a non-egalitarian society it is less evident what welfare maximization means. How should different individuals with different initial resources and different opportunities be weighted in the social welfare function? There is no unambiguous answer to this question. There are conflicting economic interests and this is where political preferences and political struggle enter the picture.

As a thought experiment, consider the case where the social planner's objective function is identical to the objective function of the "average individual" in society, i.e., identical to U_i for an individual satisfying (27). The planner will, of course, ensure static efficiency by choosing $G/Y = 1 - \alpha$; in addition, balancing the initial consumption level against growth, the planner will choose a growth rate

$$\gamma_{SP} = \alpha A^{1/\alpha} \left[(1-\alpha)L \right]^{(1-\alpha)/\alpha} - \rho \equiv \alpha \tilde{A} - \rho,$$

cf. Lecture Note 10, Section 3, with $\theta = 1$ and $\delta = 0$.

We will compare this outcome to that of the market economy with wealth taxation. *First*, to maximize growth in *that* system, we need $\tau = \tau^*$ and get, from (16), (17), and (7), the growth rate

$$
\begin{aligned}
\gamma^* &= \gamma(\tau^*) = \alpha AL^{1-\alpha} \left[\alpha(1-\alpha)AL^{1-\alpha} \right]^{(1-\alpha)/\alpha} - \left[\alpha(1-\alpha)AL^{1-\alpha} \right]^{1/\alpha} - \rho \\
&= \alpha^2 AL^{1-\alpha} \left[\alpha(1-\alpha)AL^{1-\alpha} \right]^{(1-\alpha)/\alpha} - \rho \\
&= \alpha^{\frac{1+\alpha}{\alpha}} A^{\frac{1}{\alpha}} \left[(1-\alpha)L \right]^{\frac{1-\alpha}{\alpha}} - \rho \equiv \alpha^{\frac{1+\alpha}{\alpha}} \tilde{A} - \rho < \alpha \tilde{A} - \rho = \gamma_{SP}.
\end{aligned}
$$

Thus, because of the distortionary effects of the wealth tax, the highest possible growth rate with this tax system is smaller than that chosen above by the social planner who cares about the average individual.

Second, as we will see below, the politico-economic equilibrium has $\tau > \tau^*$. So the decentralized economy ends up having a growth rate even less than γ^* and therefore considerably less than γ_{SP}.
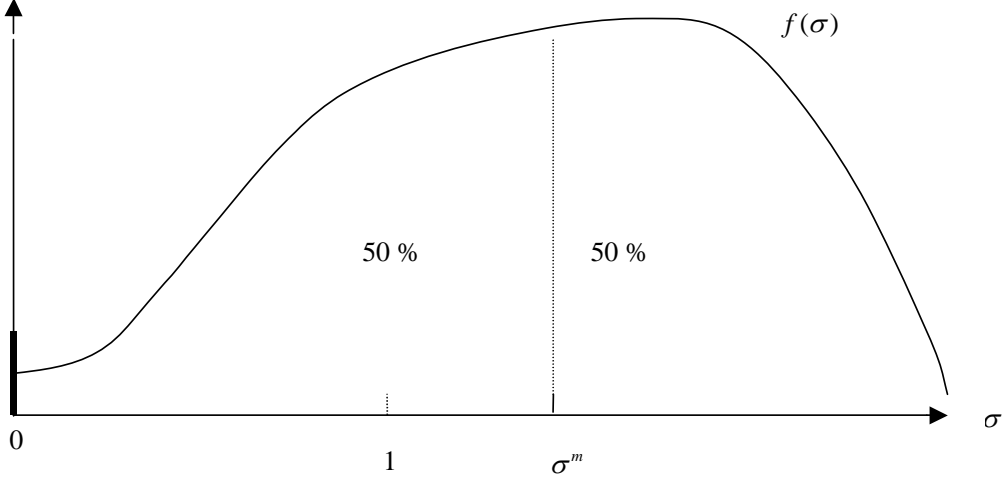
Figure 3:

5 Policy choice when the majority decides

Fig. 3 depicts a possible distribution of the population according to the relative factor endowment $\sigma_i \equiv l_i K_t/(k_{it}L)$ of the members of society. The density function is denoted $f(\sigma)$ and the median value in the distribution is denoted σ^m. Empirically, we would expect the median to be *higher* than the average factor proportion, i.e., $\sigma^m > 1$. To understand this, it is important to realize that Fig. 3 maps the distribution differently from how wealth distributions are usually graphed. Along the horizontal axis Fig. 3 has $\sigma_i \equiv l_i K_t/(k_{it}L)$ which amounts to relative (financial) wealth *poverty,* if all individuals have more or less the same l_i. Indeed, real-world distributions typically have the property that the wealth-poorest half of the population own far less than half of aggregate wealth, whereas labor incomes are less unequally distributed. Thus it is not far out to assume that the l_i's are all approximately the same, while the k_i's differ a lot, so that $\sigma_i \approx 1 \cdot K_t/(k_{it}L)$. Since individual wealth, k_{it}, is in the denominator of the σ's, we get $\sigma^m > 1$ as indicated in Fig. 3.

Now, let us see whether we can apply the median-voter theorem. By construction:

1. there is only one political issue, namely the size of the wealth tax rate;

2. preferences are "single-peaked" $(U_i'(\tau) = 0, U_i''(\tau) < 0)$;

3. there is a monotonic relationship between the relative factor endowment of the individuals and their preferred tax rate $(\partial \tau_i/\partial \sigma_i > 0)$.

Thus the preconditions for applying the median-voter theorem are satisfied (see, e.g., Mas-Colell et al., 1995). The political solution is that the chosen tax rate will be

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the tax rate preferred by the *median voter*, τ^m. One can say that τ^m is the only tax rate that does not have a majority against it. Or that in comparisons of tax rates in pairs, τ^m beats all other proposals.

In the analysis of the economic behavior of the single individual, we assumed that the individual faced a time-independent tax rate τ. And when we considered the political choice, it was a once-for-all choice of a forever constant tax rate. A less extreme interpretation would be that after every electoral period there is a new voting. We could imagine that if the individuals *expect* an unchanged tax rate after every election in the future, they will vote sequentially in the same way. So the actual tax rate will remain unchanged. Thereby their expectations are confirmed. Indeed, in every new election, the situation is as in the previous election, since the real interest rate and the wealth distribution are constant over time, whereas the real wage grows at the same constant rate as consumption and capital.

In the terminology of *political economy* (the study of the interaction between the economic and the political sphere of society) the allocation based on the tax rate $\tau = \tau^m$, with a resulting per capita growth rate $\gamma = \gamma(\tau^m)$, is a *politico-economic equilibrium* of the model.

6 Conclusion

We saw that the politically chosen tax rate, τ^m, is higher than the growth-maximizing one, which is τ^*; further, τ^m is higher, the higher is the median voter's relative wealth poverty, σ^m. Since $\gamma'(\tau) < 0$ for $\tau > \tau^*$, it follows that the realized growth rate is lower, the higher is σ^m. To the extent that $\sigma^m - 1$ is an indicator of wealth (or income) inequality,[12] the conclusion from the model is that countries (at least more or less democratic countries) with high wealth and income inequality tend to have lower per capita growth. In the last section of their article Alesina and Rodrik provide some empirical evidence (cross-country regression analysis) to support this conclusion.

It is the combination of a political mechanism and an economic mechanism that gives the negative correlation between inequality and growth. The political mechanism is that high inequality leads to political pressure for *some* form of progressive taxation. The economic mechanism is that a rise in the marginal tax rate distorts saving and investment incentives and thence leads to a lower growth rate. In models where the diminishing marginal productivity of capital is not fully offset by other factors, the

---

[12]Recall an "average person" has $\sigma_i = (\ell_i/L)/(k_i/K) = 1$. If the distribution of wealth is *very* right-skewed, the median person's wealth, k^m, is *much* lower than the average, which is K/L.

decrease in the growth rate is only a temporary phenomenon (but with a permanent *level* effect). In the present model, which is an AK-style model, the growth effect is permanent.

A thorough empirical investigation would test these two mechanisms *separately.* Alesina and Rodrik do not do this, but Perotti (1996) does. Perotti concludes that the evidence does *not* support it. Perotti rather emphasizes social and political instability as hampering growth. All in all, the relationship between inequality and growth is a disputed topic. Indeed, Forbes (2000) rejects that there should be a robust negative correlation between the two.

Postscript   From Ramsey-style models with infinite horizon (like the present model) one gets the impression that capital taxation (in the form of wealth taxes or capital income taxes) and low rates of return on saving are always bad for economic growth. For a more varied view, see Saez (2002) and Salanié (2003). In an overlapping generations framework one may get different results because in that framework much depends on the saving by the "young" (those in the labor force), who have relatively *little* capital income (cf. Salanié, 2003). It should be emphasized that by overlapping generations framework we do not necessarily mean Diamond's two-period OLG model (discussed in Romer, Advanced Macroeconomics, and in B & S, Appendix to Chapter 3). The coarse notion of time in this model (a 30-years period length) probably plays down the role of the rate of return two much. An alternative is Blanchard's continuous time OLG model (see, e.g., B & S, Chapter 3.6) which in several respects give results somewhere in between the Ramsey framework and the Diamond framework.

7   Appendix

A. Proof that (24) has a solution in τ and that it is unique
From (24) we have $\frac{\partial [LHS(24)]}{\partial \tau}$

$$
\begin{aligned}
&= \left[(1-\alpha)A(L\tau)^{1-\alpha}\sigma_i + \rho\right]\alpha\tau^{\alpha-1} + \left[\tau^\alpha - \alpha(1-\alpha)AL^{1-\alpha}\right](1-\alpha)^2 AL^{1-\alpha}\tau^{-\alpha}\sigma_i \\
&= \left[(1-\alpha)A(L\tau)^{1-\alpha}\sigma_i + \rho\right]\alpha\tau^{\alpha-1} + \left[1 - \alpha(1-\alpha)AL^{1-\alpha}\tau^{-\alpha}\right](1-\alpha)^2 AL^{1-\alpha}\sigma_i \\
&> 0, \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (28)
\end{aligned}
$$

where the inequality holds at least in a neighborhood of the optimal τ. The positive sign in (28) arises in the following way. The term in the first brackets is always positive. The term in the second brackets is equal to the left-hand-side of (22), which is non-negative, since the right-hand-side of (22) is non-negative. The positively sloped curve

in Fig. 4 shows the left-hand-side of (24) as a function of τ, and the horizontal line in the figure shows the right-hand-side of (24). The figure illustrates the determination of τ_i, person i's preferred tax rate.

B. Proof that $U_i''(\tau) < 0$

From (21) we get

$$U_i''(\tau) = \frac{1}{\rho} \left(\frac{(\omega(\tau)L\sigma_i + \rho)\omega''(\tau)L\sigma_i - (\omega'(\tau)L\sigma_i)^2}{(\omega(\tau)L\sigma_i + \rho)^2} + \frac{\gamma''(\tau)}{\rho} \right) < 0,$$

since $\gamma''(\tau) \le 0$ for $\tau = \tau_i \ge \tau^*$, and $\omega''(\tau) < 0$.

C. Proof that $\frac{\partial \tau_i}{\partial \sigma_i} > 0$

Totally differentiating (24) gives

$$\left[(1-\alpha)A(L\tau)^{1-\alpha}\sigma_i + \rho \right] \alpha\tau^{\alpha-1} d\tau$$
$$+ \left[\tau^\alpha - \alpha(1-\alpha)AL^{1-\alpha} \right] (1-\alpha)AL^{1-\alpha} \left[\tau^{1-\alpha}d\sigma_i + \sigma_i(1-\alpha)\tau^{-\alpha}d\tau \right]$$
$$= \rho(1-\alpha)^2 AL^{1-\alpha} d\sigma_i \Rightarrow$$
$$\left\{ \left[(1-\alpha)A(L\tau)^{1-\alpha}\sigma_i + \rho \right] \alpha\tau^{\alpha-1} + \left[1 - \alpha(1-\alpha)AL^{1-\alpha}\tau^{-\alpha} \right] (1-\alpha)^2 AL^{1-\alpha}\sigma_i \right\} d\tau$$
$$= \left\{ \rho(1-\alpha)^2 AL^{1-\alpha} - \left[1 - \alpha(1-\alpha)AL\tau^{-\alpha} \right] \tau(1-\alpha)AL^{1-\alpha} \right\} d\sigma_i \Rightarrow$$
$$\frac{\partial \tau}{\partial \sigma_i} = \frac{\rho(1-\alpha)^2 AL^{1-\alpha} - \left[1 - \alpha(1-\alpha)AL\tau^{-\alpha} \right] \tau(1-\alpha)AL^{1-\alpha}}{\left[(1-\alpha)A(L\tau)^{1-\alpha}\sigma_i + \rho \right] \alpha\tau^{\alpha-1} + \left[1 - \alpha(1-\alpha)AL^{1-\alpha}\tau^{-\alpha} \right] (1-\alpha)^2 AL^{1-\alpha}\sigma_i}$$
$$= \frac{\rho(1-\alpha)^2 AL^{1-\alpha} - \rho(1-\alpha)\theta_i(\tau)(1-\alpha)AL^{1-\alpha}}{\frac{\partial LHS(24)}{\partial \tau}} \quad \text{(from (22) and (28))}$$
$$= \frac{\rho(1-\alpha)^2 AL^{1-\alpha}(1 - \theta_i(\tau))}{\frac{\partial LHS(24)}{\partial \tau}} > 0 \quad \text{(since } \theta_i(\tau) < 1, \text{ and } \frac{\partial LHS(24)}{\partial \tau} > 0 \text{ by (28))}.$$

D. The Gini index of inequality

The Lorenz curve Let P denote the from-below-accumulated share (P for "proportion") of population. Thus, if we consider the 20 % poorest, $P = 0.20$. For the $100 \times P$ % poorest persons in the population, the *Lorenz function* (for the income distribution) gives these persons' share of aggregate income (due to negative capital income).
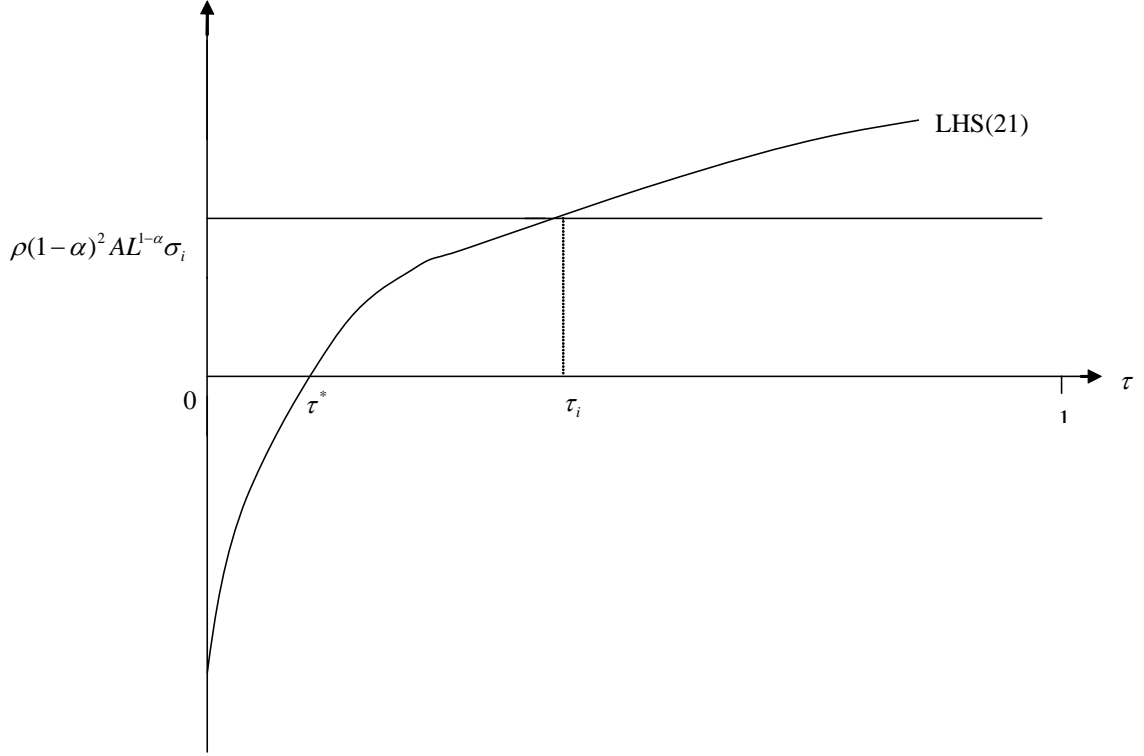
Figure 4:

To be more precise. Let

$$
\begin{aligned}
y &= \text{personal income,} \\
x &= \text{a given level of income,} \\
F(x) &= \text{fraction of population with } y \leq x, \\
f(x) &= F'(x) = \text{density function} \\
\bar{y} &= \int_0^\infty y f(y) dy = \text{average income}
\end{aligned}
\tag{29}
$$

In the formula for $\bar{y}$ we have ignored that some persons may have *negative* income.

The *Lorenz function* is defined as a function $L : [0,1] \rightarrow \mathbb{R}$, satisfying

$$
P = F(x) \Rightarrow L(P) = \frac{\int_0^x y f(y) dy}{\bar{y}}.
\tag{30}
$$

The *Lorenz curve* is the graph $(P, L(P))$, cf. Fig. 5. Thus, the Lorenz curve traces out the income share of the $100 \times P$ % purest people in the population for P going from 0 to 1.
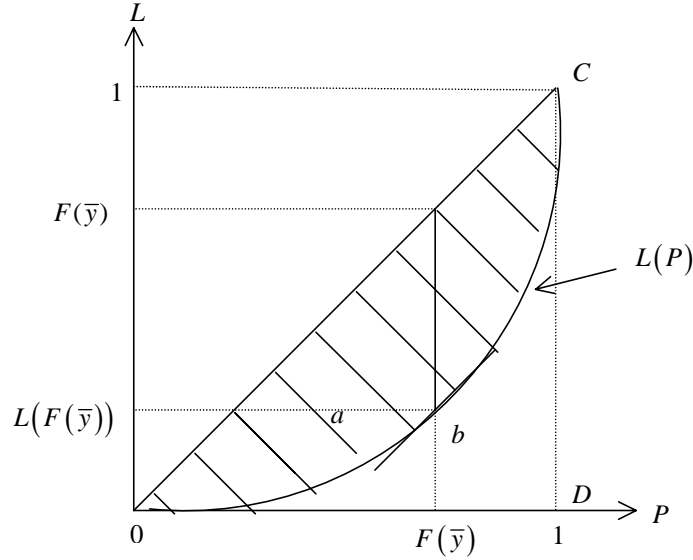
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Figure 5: The Lorenz curve $L(P)$ (showing the income share of the $100 \times P$ % purest people in the population).

The basic formulas can also be presented in the following way. Let

$$
\begin{aligned}
N &= \text{measure of population size} \\
f(y)dy &\simeq \text{fraction of population with income in the interval } [y, y + dy], \quad (31) \\
Nf(y)dy &\simeq \text{measure of the population with income in the interval } [y, y + dy] \quad (32) \\
Y &= N \int_0^\infty y f(y)dy = \text{aggregate income in society,} \\
\bar{y} &\equiv \int_0^\infty y f(y)dy = \frac{Y}{N} = \text{ average income,}
\end{aligned}
$$

The *Lorenz function* $L : [0,1] \to R$ is then defined by

$$
P = F(x) \Rightarrow L(P) = \frac{N \int_0^x y f(y)dy}{Y}. \tag{33}
$$

The Gini index   The *Gini index* (also called the Gini coefficient or the Gini ratio) is defined on the basis of the Lorenz curve and is a measure of the *degree of income inequality* in society. The Gini index is defined as

$$
Gini \equiv \frac{\int_0^1 (P - L(P))dP}{1/2} = \frac{\text{hatched area in Fig. 1}}{\text{area of triangle } OCD} = \frac{a}{a+b}. \tag{34}
$$

We divide by $1/2$ (or with the area of triangle OCD) in order to get a number between zero and 1. A high value of *Gini* means high degree of inequality in the distribution

of income. If everybody had the same income, then the Lorenz curve would coincide with the 45° line in Fig. 1, and the Gini index would be zero.

If the patched area ("the cigar") in Fig. 1 is called a, and the area of triangle OCD is $a + b$, then we can also write (34) as

$$Gini = \frac{a}{a+b} = \frac{a}{1/2} = 2a = 2(\frac{1}{2} - b) = 1 - 2b, \tag{35}$$

where we have used that $a + b = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$, so that $a = \frac{1}{2} - b$.

One can show that

$$Gini = \frac{1}{2\bar{y}} \int_0^\infty \int_0^\infty |y - x| \, f(x)f(y)dxdy. \tag{36}$$

Thus the *Gini index* can be interpreted as (half of) the expected income difference between two randomly chosen individuals (expressed as a fraction of the average income in the total population).

As an alternative to the above continuous formulation, one can start from a numbering of the persons, $i = 1, 2, ..., N$, with corresponding incomes $y_1, y_2, ..., y_N$. Then it can be shown that the discrete definition of the Gini index corresponding to (34) and (36) above is

$$Gini \equiv \frac{1}{2\bar{y}} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{|y_i - y_j|}{N^2}, \tag{37}$$

which is a number between 0 and $\frac{N-1}{N}$. This formula gives us the Gini index as (half of) the average income difference for all pairs of individuals divided by the average income in society.

Notice that the Gini index is *scale invariant* in the sense that if all members of society have their income multiplied by the same positive factor, then the value of the Gini index is not changed. This is a "natural" requirement for a measure of income inequality (the *relative* income differences should matter, not the absolute).

For calculation of the Gini index in practice one can use the covariance routine, a standard tool in any statistics software package. Indeed, it can be shown that

$$Gini = \frac{2}{\bar{y}}Cov\left[y, F(y)\right], \tag{38}$$

that is, the Gini index is $2 \times$ the relative covariance between the income levels and their "rank".

When measuring income inequality it is important to be explicit about the unit of observation: (a) is it income of an individual or a household, (b) is it income before taxation and transfers or after, and (c) is it annual income or life time income? Another

interesting variable to consider is financial wealth (and in less developed countries land ownership). The inequality in the distribution of financial wealth can be measured by a Gini index in a way analogue to the above. For almost all (or simply all?) countries for which there are data the Gini for wealth inequality is considerably higher than that for inequality in annual income.

There exists a huge literature on advantages and disadvantages of different measures of income and wealth inequality and of their possible role in social welfare functions. The Gini index is just one of these measures. The reader is referred to the list of references.

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Further material

http://www.bris.ac.uk/Depts/Economics/Growth/refs/inequal.htm

On measurement problems:

http://www.worldbank.org/poverty/inequal/methods/index.htm

On theories à la Alesina & Rodrik (1994):

http://www.worldbank.org/poverty/inequal/ econ/index.htm

On the Kuznets curve:

http://www.worldbank.org/poverty/inequal/econ/distrib.htm

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