12.04.2010. Christian Groth

Problem Set IV

IV.1 *Education in a market economy. A Ramsey one-sector model in MRW style.* This exercise takes a kind of MRW-approach[1] to human capital except that saving and investing in physical and human capital is based on intertemporal optimization. The distinguishing characteristic of the MRW-approach is that human capital accumulation is treated as similar to physical capital accumulation. This corresponds to the approach followed by B & S on pp. 59-61 and in their Chapter 5. We may call it the "human capital parallel to physical capital approach". But whereas B & S consider human and physical capital formation in an isolated family farm or from the perspective of a social planner, the setting considered here is a standard market economy with market prices and firms and households as separate decision units.

We consider a closed market economy with education in private schools that charge a fee from students. Under perfect competition the representative firm chooses capital input, K^d, and labor input, L^d, in order to maximize profit, given the production function

$$Y = F(K^d, \pi L^d), \tag{1}$$

where Y is output, π is "productivity" (or "efficiency") of labor hours, L^d, and F is a neoclassical production function with constant returns to scale. We shall in this exercise assume that $\pi = h$, where h is average human capital in the labor force.

a) Given h and the aggregate supplies of capital, K, and labor, L, respectively, determine the real rental rate, $\hat{r}$, for capital and the real wage, $\hat{w}$, per unit of *effective* labor input in equilibrium.

Aggregate output (= aggregate gross income) is used for consumption, C, investment, I_K, in physical capital and investment, I_H, in human capital, i.e.,

$$Y = C + I_K + I_H.$$

The dating of the variables is suppressed where not needed for clarity. The increase per time unit in the two kinds of capital is given by

$$\begin{aligned} \dot{K} &= I_K - \delta K, \quad \text{and} \\ \dot{H} &= I_H - \delta H, \end{aligned} \tag{2}$$

[1] Cf. Mankiw, Romer, and Weil (1992).

respectively, where $H \equiv hL$. We have, for simplicity, assumed that the depreciation rate, $\delta \geq 0$, is the same for the two kinds of capital.

The representative household (dynasty) has infinite horizon and consists of L members, where $L = L_0 e^{nt}$, $n \geq 0$, $L_0 > 0$. Each family member supplies inelastically one unit of labor per time unit. From (2) and the definition $H \equiv hL$ follows the per capita human capital accumulation equation:

$$\dot{h} = i - (\delta + n)h, \tag{3}$$

where $i \equiv I_H/L$ is the per capita educational cost (in real terms) per time unit.

b) Present a derivation of (3).

Let θ and ρ be positive constants, where $\rho > n$. Let a be per capita financial wealth, r the real interest rate, and $c_t \equiv C_t/L_t$. The representative household chooses a path $(c_t, i_t)_{t=0}^{\infty}$ to maximize

$$\begin{aligned}
U_0 &= \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t} dt \qquad \text{s.t.} && (4)\\
c_t &\geq 0, \ i_t \geq 0, && (5)\\
\dot{a}_t &= (r_t - n)a_t + \tilde{w}_t h_t - i_t - c_t, \qquad a_0 \text{ given}, && (6)\\
\dot{h}_t &= i_t - (\delta + n)h_t, \qquad h_0 > 0 \text{ given}, && (7)\\
\lim_{t\to\infty} a_t e^{-\int_t^\tau (r_s - n)ds} &\geq 0, && (8)\\
h_t &\geq 0 \text{ for all } t. && (9)
\end{aligned}$$

c) Briefly interpret the six elements in this decision problem. Why is there a non-negativity constraint on i_t?

d) Use the Maximum Principle (for the case with two control variables and two state variables) to find the first-order conditions for an interior solution.

e) Derive from the first-order conditions the Keynes-Ramsey rule.

f) Set up a no-arbitrage equation showing a relationship between $\tilde{w}$ and r. You may either use your intuition or derive the relationship from the first-order conditions. In case you use your intuition, check whether it is consistent with the first-order conditions. *Hint:* along an interior optimal path the household should be indifferent between placing the marginal unit of saving in a financial asset yielding the rate of return r or in education to obtain one more unit of human capital.

Assume now for simplicity that the aggregate production function is:

$$Y = AK^\alpha(\pi L)^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1,$$

g) Determine the real interest rate in equilibrium in this case.

Suppose parameters are such that $\dot{c}/c > 0$ and U_0 is bounded.

h) The no-arbitrage equation from f) (which is needed for an *interior* solution to the household's decision problem) requires a specific value of K/H to be present. Determine this value and explain what happens to begin with if the historically given K/H ratio in the economy differs from it; and explain what happens in the long run.

i) Consider a constant subsidy, $s \in (0, 1)$, to education such that per unit of investment in education the private cost is only $1 - s$. That is, i_t in (7) is replaced by $(1 - s)i_t$. Suppose the subsidy is financed by lump-sum taxes. Will such a subsidy affect long-run growth in this model? Explain. *Hint:* In answering, you may use your intuition or make a formal derivation. A quick approach can be based on the no-arbitrage condition in the new situation (for simplicity you may put $\delta = 0$).

j) Since there are no externalities in the model as it stands, it could be argued that there is no need for a subsidy. Going a little outside the model, what kinds of motivations for subsidizing education might be put forward?

IV.2 *Short questions* These questions relate to the model in Problem IV.1.

a) Comment on the model in relation to the concepts of fully endogenous growth and semi-endogenous growth.

b) Comment on the model in relation to the issue of scale effects.

c) What do you guess will be the consequences of assuming $\pi = h^{\varphi}$, $0 < \varphi < 1$? Comment.

IV.3 We consider a market economy. Suppose people are alike and attend school for S years, thereby obtaining individual human capital

$$h = S^{\varphi}, \qquad 0 < \varphi \leq 1. \tag{10}$$

An individual "born" at time 0 chooses S to maximize

$$HW_0 = \int_S^{\infty} \hat{w}_t h e^{-(\bar{r}+m)t} dt, \tag{11}$$

subject to (10). Here $\hat{w}_t$ is the real wage per year *per unit of human capital* at time t, $\bar{r}$ is a constant real interest rate, and m is the "hazard rate" of death faced by the individual. It is assumed that $\hat{w}_t = \hat{w}_0 e^{\bar{g}t}$, where $\bar{g} \geq 0$.

a) Interpret this decision problem, including the parameters that enter.

b) Let the optimal S for this person be denoted S^*. Find S^*; you may directly base your answer on the following hint: $h'(S^*)/h(S^*) = \tilde{r}$, where $\tilde{r}$ is a constant effective discount rate.

c) How does an increase in life expectancy affect the optimal S? What is the intuition?

d) Under perfect competition the representative firm chooses capital input, K_t^d, and labor input (measured in man-years), L_t^d, in order to maximize profit, given the production function

$$Y_t = F(K_t^d, hT_tL_t^d), \tag{12}$$

where Y_t is output, T_t is the exogenous technology level, $T_t = T_0e^{\bar{g}t}$, and F is a neoclassical production function with constant returns to scale. Suppose the country considered is fully integrated in the world market for goods and financial capital with a constant real interest rate equal to $\bar{r}$. Determine the real wage per year at time t for a typical member of the labor force.

e) What is the growth rate over time of this real wage?

f) We now extend the perspective by leaving the assumption of a given constant growth rate for T. In the Bernard and Jones formulation of the technological catching-up hypothesis it was assumed that

$$\frac{\dot{T}_t}{T_t} = \xi\frac{\hat{T}_t}{T_t}, \tag{13}$$

where $\xi > 0$ and $\hat{T}_t = \hat{T}_0e^{\hat{g}t}$ is the world frontier technology level, $\hat{g} > 0$. We assume $T_0 < \hat{T}_0$ and $0 < \xi < \hat{g}$. Will the country be able to catch up in the long run? *Hint:* from Lecture Note 4 we know that $\lim_{t\to\infty} T_t/\hat{T}_t = \xi/\hat{g}$.

g) Suppose the country considered is a developing country and that its catching-up ability is an increasing function of average human capital, i.e., $\xi = \xi(h)$, $\xi' > 0$. Can a general health improvement in the country help in catching up? Why or why not?

IV.4 *Short questions.*

a) In relation to Romer's learning-by-investing model B & S write on p. 219: "One way to eliminate the scale effect [on growth] is to argue that the term A_i in equation (4.22) depends on the economy's average capital per worker, K/L, rather than the aggregate capital stock, K." One might be sceptical towards this way out because it contradicts the nature of technical knowledge. How? Suggest another simple way to eliminate the scale effect on growth in the model.

b) In relation to their model with productive public services in the form of pure public goods B & S write on p. 223: "The failure to [empirically] detect ....scale effects [on growth] likely means that most of the government's services do not have the nonrival character that is assumed in the model." Suggest an alternative interpretation of the failure of the model to comply with the apparent absence of scale effects on growth, an interpretation *maintaining* the nonrival character of the productive public service assumed in the model.

c) "In a representative agent model lump-sum taxation is, from a welfare point of view, always to be preferred to (or at least as good as) a proportional income tax." True or false? Why?

d) "In a reduced-form AK model with productive public services with congestion, a welfare-maximizing government has to implement a policy that reduces the per capita growth rate." True or false? Why?

VI.5 In 1960 per capita GDP in South Korea and Philippines were almost the same. Over the period 1960-1990 the average annual growth rate of per capita GDP was in South Korea 6.7 percent and in Philippines 1.5 percent. Give a brief account of alternative hypothetical explanations of this difference in the growth performance of the two countries.

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