Economic Growth Exercises

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Problem set III

III.1 Consider a closed market economy with L utility maximizing households and M profit maximizing firms, operating under perfect competition (L and M are constant, but "large"). There is also a government that free of charge supplies a non-rival productive service G per time unit. Each household has an infinite horizon and supplies inelastically one unit of labor per time unit. Aggregate output is Y per time unit and output is used for private consumption, $C \equiv cL$, the public productive service, G, and investment, I, in (physical) capital, i.e., $Y = C + G + I$. The stock of capital, K, changes according to $\dot{K} = I - \delta K$, where $\delta \geq 0$ is the rate of physical decay of capital. Variables are dated implicitly. The initial value $K_0 > 0$ is given. The capital stock in society is owned, directly or indirectly (through bonds or shares), by the households. There is perfect competition at the labor market. The equilibrium real wage is called w. There is a perfect market for loans with a real interest rate, r, and there is no uncertainty. A dot over a variable denotes the time derivative.

The government chooses G so that

$$G = \bar{g}Y,$$

where the constant $\bar{g}$ is exogenous and is such that positive growth in the economy occurs in equilibrium. The government budget is always balanced and the service G is the only public expenditure. Only households are taxed. The tax revenue is

$$\left[\tau(ra + w) + \tau_\ell\right] L = G, \tag{GBC}$$

where a is per capita financial wealth, and τ and τ_ℓ denote the income tax rate and a lump-sum tax, respectively. The tax rate τ is a given constant, $0 \leq \tau < 1$, whereas τ_ℓ is adjusted when needed for (GBC) to be satisfied.

The production function for firm i is

$$Y_i = AK_i^\alpha (GL_i)^{1-\alpha}, \quad 0 < \alpha < 1, A > 0, \quad i = 1, 2, ..., M. \tag{*}$$

a) Comment on the nature of G.

It can be shown that in equilibrium

$$\begin{aligned} r &= \alpha \bar{A} - \delta, \quad \text{where} \quad k \equiv K/L \quad \text{and} \quad \bar{A} \equiv A^{\frac{1}{\alpha}} (\bar{g}L)^{\frac{1-\alpha}{\alpha}}, \\ Y &= \sum_i Y_i = \sum_i y_i L_i = y \sum_i L_i = yL = Ak^{\alpha} G^{1-\alpha} L = A^{1/\alpha} (\bar{g}L)^{(1-\alpha)/\alpha} kL \equiv \bar{A}K. \end{aligned}$$

b) Briefly explain these results in words.

Suppose the households, all alike, have a constant rate of time preference $\rho > 0$ and an instantaneous utility function with (absolute) elasticity of marginal utility equal to a constant $\theta > 0$.

c) Set up the optimization problem of a household and derive the Keynes-Ramsey rule, given the described taxation system.

d) Write down the transversality condition in a form comparable to the No-Ponzi-Game condition of the household. Comment.

e) Find the growth rate of $k \equiv K/L$ and $y \equiv Y/L$ in this economy (an informal argument, based on your general knowledge about reduced-form AK models, is enough). In case, you need to introduce a restriction on some parameters to ensure existence of equilibrium with growth, do it.

f) Comment in relation to the scale effect issue.

Suppose lump-sum taxation is not feasible. Hence, let $\tau_\ell = 0$ for all $t \geq 0$.

g) Examine whether it is possible to fix τ at a level (constant over time and < 1) such that the government budget is still balanced in equilibrium for all $t \geq 0$? *Hint:* if you need a new restriction on parameters to ensure $\tau < 1$, introduce it.

h) If the welfare of the representative household is the criterion, what proposal to the government do you have wrt. the size of $\bar{g}$?

i) With respect to the *form* of taxation (given that a direct lump-sum tax is not feasible), let us see if we can suggest an appropriate tax scheme:

1. is an income tax non-distortionary? Why or why not?
2. will a pure labor income tax work? *Hint:* perhaps the needed labor income tax rate is too large in some sense.
3. will a consumption tax work?

III.2 *A subsidy to saving in Romer's learning-by-investing model.* Consider a closed market economy with perfect competition where firm no. i has the production function

$$Y_{it} = F(K_{it}, T_t L_{it}),$$

where F is a neoclassical production function with CRS and satisfying the Inada conditions (standard notation). It is assumed that the technology level T_t satisfies

$$T_t = K_t^{\lambda}, \qquad 0 < \lambda \leq 1.$$

Time, t, is continuous. There is no uncertainty. At the aggregate level,

$$\dot{K}_t \equiv \frac{dK_t}{dt} = Y_t - C_t - \delta K_t, \qquad \delta > 0, \quad K_0 > 0 \text{ given.}$$

a) Determine the equilibrium real interest rate, r, and the aggregate production function. Comment.

From now we assume $\lambda = 1$.

b) Determine the equilibrium real interest rate, r, and the aggregate production function in this case. Comment.

   There is a representative Ramsey household with instantaneous utility function of CRRA type:

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \qquad \theta > 0,$$

   where c is per capita consumption ($c \equiv C/L$). The rate of time preference is a constant $\rho > 0$. There is no population growth ($n = 0$).

c) Determine the growth rate of c and name it γ.

From now, assume (A1) $F_1(1, L) - \delta > \rho$ and (A2) $(1 - \theta)\gamma < \rho$.

d) What could be the motivation for these two assumptions?

e) Determine the growth rate of $k \equiv K/L$ and $y \equiv Y/L$. A detailed derivation involving the transversality condition need not be given; instead you may refer to a general property of AK and reduced-form AK models in a Ramsey framework where (A2) holds.

f) Set up and solve the social planner's problem, assuming the same criterion function as that of the representative household. *Hint*: the linear differential equation $\dot{x}(t) + ax(t) = ce^{ht}$, with $h \neq -a$ has the solution:

$$x(t) = (x(0) - \frac{c}{a+h})e^{-at} + \frac{c}{a+h}e^{ht}.$$

g) Now consider again the decentralized market economy, but suppose there is a government that wants to establish the social planner's allocation by use of a subsidy, σ, to private saving such that the after-subsidy-rate of return on private saving is $(1+\sigma)r$. Let the subsidy be financed by a lump-sum tax. Determine σ such that the social planner's allocation is established, if this is possible. Comment.

III.3 In endogenous growth theory two alternative kinds of scale effects may be present. Give a brief account. Link two alternative learning-by-investing models to these two kinds of scale effects.