Economic Growth. Exercises

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Problem Set VII

VII.1 *Hartwick's rule.* Consider an economy with technology and resource constraints described by:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} R_t^{\gamma}, \qquad \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, \tag{1}$$

$$\dot{K}_t = Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \; K_0 > 0 \text{ given}, \tag{2}$$

$$\dot{S}_t = -R_t, \qquad \int_0^{\infty} R_t dt \leq S_0, \quad S_0 > 0 \text{ given}, \tag{3}$$

$$L_t = L_0 e^{nt}, \qquad n \geq 0, \tag{4}$$

where Y_t is aggregate output and A_t, K_t, L_t, and R_t are TFP, input of capital, input of labor, and input of a non-renewable resource, respectively, at time t. The labor input equals the size of the labor force (= population), and S_t is the stock of the non-renewable resource (e.g., oil reserves) in the ground.

a) Interpret the model relations (1) - (4). Can we from the given information establish whether there are costs associated with extraction of the resource from the ground? Why or why not?

Hartwick, and before him Solow, considered the following case: no technical progress, no capital depreciation, constant population. Like Solow and Hartwick we shall study the question whether under these conditions positive per capita consumption can be sustained and if so, what is the maximum sustainable level of per capita consumption.

b) There are two features of the set-up that make these questions non-trivial and for example rule out sustainable economic growth? What are these two features?

c) Let $L = 1$ and write down the simplified model under the stated conditions.

d) Since sustainability of consumption clearly requires sustainability of output, we have immediately that continued building up of capital is needed. Why?

e) Maintaining both output and consumption constant implies that the time path for K has a simple form. What form?

f) Given this form, write down the required time path for R_t in terms of K_0 and the so far unknown investment level.

g) Find the value of R_0 required for the resource stock to be neither exhausted in finite time, nor in part be left unexploited forever. *Hint:* $\int_0^\infty x^a dx = \frac{1}{a+1} x^{a+1}\Big|_0^\infty$ for $a \neq -1$.

h) To ensure the requirement under g), a particular technology condition must be satisfied. What is this condition and why (intuitively) is it necessary?

i) Assuming the condition is satisfied, express the implied time path for C_t in terms of the investment level, the given initial conditions, and parameters.

j) Choose the investment level so as to maximize sustainable consumption.

k) Comment on the resulting investment rule, expressed in terms of the income of the resource owners under perfect competition. What is the intuition behind the rule?

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