Economic Growth Lecture Note 3.

07.02.2008. Christian Groth

# Different concepts of across-country income convergence

Section 1 in this note presents and discusses the different concepts of income convergence, mentioned in B & S (Introduction, Chapter 1, and p. 462 ff.). Section 2 displays some empirical graphs. Appendix A illustrates how easily mistakes can be made.

## 1 Concepts of income convergence

The two most popular income convergence concepts are "$\beta$ convergence" and "$\sigma$ convergence".

### 1.1 $\beta$ convergence vs. $\sigma$ convergence

**Definition 1** *We say that $\beta$ convergence occurs for a given selection of countries if there is a tendency for the poor (those with low income per capita or low output per worker) to subsequently grow faster than the rich.*

By "grow faster" is meant that the growth rate of per capita income (or per worker output) is systematically higher. B & S consider $\beta$ convergence on pp. 14, 17, 45, and 47; the concept is illustrated by scatter plots on pp. 45-47, 470-71, 484 and 489.

In many contexts, a more appropriate convergence concept is the following:

**Definition 2** *We say that $\sigma$ convergence, with respect to a given measure of dispersion, occurs for a given collection of countries if this measure of dispersion, applied to income per capita or output per worker across the countries, declines systematically over time. On the other hand, $\sigma$ divergence occurs, if the dispersion increases systematically over time.*

B & S refer to $\sigma$ convergence on pp. 50 and pp. 462 ff. The reason that $\sigma$ convergence must be considered the more appropriate concept is the following. In the end, it is the question of increasing or decreasing dispersion across countries that we are interested in. From a superficial point of view one might think that $\beta$ convergence implies decreasing dispersion and vice versa, so that $\beta$ convergence and $\sigma$ convergence are more or less equivalent concepts. But since the world is not deterministic, but stochastic, this is not true. Indeed, $\beta$ convergence is only a necessary, not a sufficient condition for $\sigma$ convergence. This is because over time some reshuffling among the countries is always taking place, and this implies that there will always be some extreme countries (those initially far away from the mean) that move closer to the mean, thus creating a negative correlation between initial level and subsequent growth, in spite of equally many countries moving from a middle position toward one of the extremes. In this way $\beta$ convergence may be observed at the same time as there is no $\sigma$ convergence;[1] in fact, $\beta$ convergence may be consistent with $\sigma$ *divergence* (for a formal proof, see B & S, pp. 50-51 and 462 ff.; see also Romer, 2001, p. 32-34).

Hence, it is wrong to conclude from $\beta$ convergence (poor countries tend to grow faster than rich ones) to $\sigma$ convergence (reduced dispersion of per capita income) without any further investigation. The mistake is called "regression towards the mean" or "Galton's fallacy". Francis Galton was an anthropologist (and a cousin of Darwin), who in the late nineteenth century observed that tall fathers tended to have not as tall sons and small fathers tended to have taller sons. From this he falsely concluded that there was a tendency to averaging out of the differences in height in the population. Indeed, being a true aristocrat, Galton found this tendency pitiable. But since his conclusion was mistaken, he did not really have to worry.

Since $\sigma$ convergence comes closer to what we are ultimately looking for, from now, when we speak of just "income convergence", $\sigma$ convergence is understood.

In the above definitions of $\sigma$ convergence and $\beta$ convergence, respectively, we were vague as to what kind of selection of countries is considered. In principle we would like it to be a representative sample of the "population" of countries that we are interested in. The population could be all countries in the world. Or it could be the countries that a century ago had obtained a certain level of development.

[1] Think of the ordinal rankings of the sports teams in a league. The dispersion of rankings is constant by definition. Yet, no doubt there will allways be some tendency for weak teams to rebound toward the mean and of champions to revert to mediocrity. (This example is taken from the first edition of B & S from 1995; I do not know why, but the example has been deleted in the second edition from 2004.)

One should be aware that historical GDP data are constructed retrospectively. Long time series data have only been constructed for those countries that became relatively rich during the after-WWII period. Thus, selecting the countries for which long data series exist as our sample involves a *selection bias* which generates a spurious convergence. A country which was poor a century ago will only appear in the sample if it grew rapidly over the next 100 years. A country which was relatively rich a century ago will appear in the sample unconditionally. This selection bias problem was pointed out by DeLong (1988) in a criticism of false interpretations of Maddison's long data series (Maddison 1982).

## 1.2 Measures of dispersion

Our next problem is: *what* measure of dispersion is to be used? Here there are different possibilities. To be precise about this we need some notation. Let

$$\begin{aligned} y &\equiv \frac{Y}{L}, \qquad \text{and} \\ q &\equiv \frac{Y}{P}, \end{aligned}$$

where $Y$ = real GDP, $L$ = labour force and $P$ = population. If the focus is on living standards, $Y/P$, is the relevant variable.[2] But if the focus is on (labour) productivity, it is $Y/L$, that is relevant. Since most growth models focus on $Y/L$ rather than $Y/P$, let os take $y$ as our example.

One might think that the standard deviation of $y$ could be a relevant measure of dispersion when discussing whether $\sigma$ convergence is present or not. The *standard deviation* of $y$ across $n$ countries in a given year is

$$\sigma_y \equiv \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}, \tag{1}$$

where

$$\bar{y} \equiv \frac{\sum_i y_i}{n}, \tag{2}$$

i.e., $\bar{y}$ is the average output per worker. However, if this measure were used, it would be hard to find *any* group of countries for which there is income convergence. This is because $y$ tends to grow over time for most countries, and then there is an inherent tendency for the variance also to grow; hence also the square root of the variance, $\sigma_y$, tends to grow. Indeed,

[2] Or perhaps better, $Q/P$, where $Q \equiv GNP \equiv GDP - rD - wF$. Here, $rD$, denotes net interest payments on foreign debt and $wF$ denotes net labour income of foreign workers in the country.

suppose that for all countries, $y$ is doubled from time $t_1$ to time $t_2$. Then, automatically, $\sigma_y$ is also doubled. But hardly anyone would interpret this as an increase in the income inequality across the countries.

Hence, it is more adequate to look at the standard deviation of *relative* income levels:

$$\sigma_{y/\bar{y}} \equiv \sqrt{\frac{1}{n}\sum_i (\frac{y_i}{\bar{y}} - 1)^2}. \tag{3}$$

This measure is the same as what is called the *coefficient of variation*, $CV_y$, usually defined as

$$CV_y \equiv \frac{\sigma_y}{\bar{y}}. \tag{4}$$

That the two measures are identical can be seen in this way:

$$\frac{\sigma_y}{\bar{y}} \equiv \frac{\sqrt{\frac{1}{n}\sum_i (y_i - \bar{y})^2}}{\bar{y}} = \sqrt{\frac{1}{n}\sum_i (\frac{y_i - \bar{y}}{\bar{y}})^2} = \sqrt{\frac{1}{n}\sum_i (\frac{y_i}{\bar{y}} - 1)^2} \equiv \sigma_{y/\bar{y}}.$$

The point is that the coefficient of variation is "scale free", which the standard deviation itself is not.

Instead of the coefficient of variation, another scale free measure is often used, namely the standard deviation of $\log y$, i.e.,

$$\sigma_{\log y} \equiv \sqrt{\frac{1}{n}\sum_i (\log y_i - \log y^*)^2}, \tag{5}$$

where

$$\log y^* \equiv \frac{\sum_i \log y_i}{n}. \tag{6}$$

Note that $y^*$ is the geometric average, i.e., $y^* \equiv \sqrt[n]{y_1 y_2 \cdots y_n}$. Now, by a first-order Taylor approximation of $\log y$ around $y = \bar{y}$, we have

$$\log y \approx \log \bar{y} + \frac{1}{\bar{y}}(y - \bar{y}).$$

Hence, as a very rough approximation we have $\sigma_{\log y} \approx \sigma_{y/\bar{y}} = CV_y$, though this approximation can be quite poor (cf. Dalgaard and Vastrup, 2001).[3] It may be possible, however, to defend the use of $\sigma_{\log y}$ in its own right to the extent that $y$ tends to be approximately lognormally distributed across countries.

Yet another possible measure of income dispersion across countries is the *Gini index* (more about this in a later lecture note).

[3] In a stochastic context one ought to use higher-order approximations.

## 1.3 Weighting by size of population

Another important issue is whether the applied dispersion measure is based on a *weighting of the countries by size of population*. For the world as a whole, when no weighting by size of population is used, then there is a slight tendency to income divergence according to the $\sigma_{\log q}$ criterion (B & S, p. 2-3), where $q$ is per capita income ($\equiv Y/P$). As seen by Fig. 4 below, this tendency is not so clear according to the $CV_q$ criterion. Anyway, when there *is* weighting by size of population, then in the last twenty years there has been a tendency to income convergence at the global level (Sala-i-Martin 2006). With weighting by size of population (5) is modified to

$$\sigma^w_{\log q} \equiv \sqrt{\sum_i w_i (\log q_i - \log q^*)^2},$$

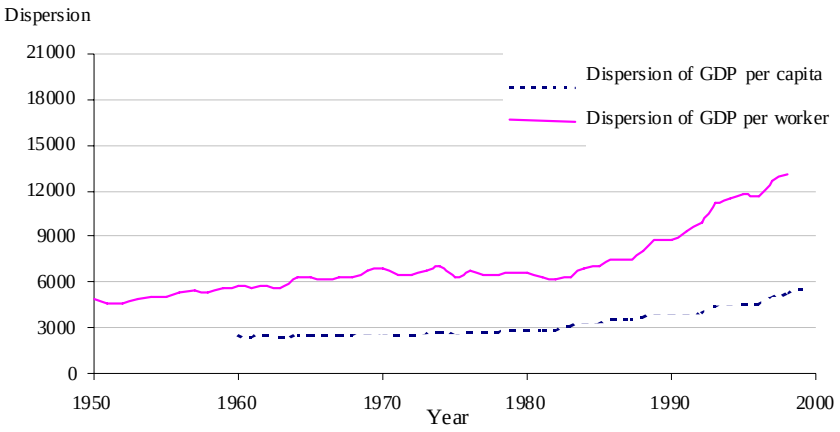
where

$$w_i = \frac{L_i}{L} \quad \text{and} \quad \log q^* \equiv \sum_i w_i \log q_i.$$

## 1.4 Unconditional vs. conditional convergence

Yet another distinction in the study of income convergence is that between unconditional (or absolute) and conditional convergence. We say that a large heterogeneous group of countries (say the countries in the world) show *unconditional* income convergence if income convergence occurs for the whole group without conditioning on specific characteristics of the countries. If income convergence occurs only among a subgroup of the countries, namely such countries that in advance share the same "structural characteristics", then we say there is *conditional* income convergence.

What the precise meaning of "structural characteristics" is, will depend on what model of the countries the researcher has in mind. According to the Solow model, a set of relevant "structural characteristics" are: the aggregate production function, the initial level of technology, the rate of technical progress, the capital depreciation rate, the saving rate, and the population growth rate. But the Solow model, as well as its extension with human capital (Mankiw et al., 1992), is a model of a closed economy with exogenous technical progress. The model deals with "within-country" convergence in the sense that the model predicts that a closed economy being initially below or above its steady state path, will over time converge towards its steady state path. It is far from obvious that this kind of model is a good model of across-country convergence in a globalized world where




Remarks: Germany is not included in GDP per worker. GDP per worker is missing for Sweden and Greece in 1950, and for Portugal in 1998. The EU comprises Belgium, Denmark, Finland, France, Greece, Holland, Ireland, Italy, Luxembourg, Portugal, Spain, Sweden, Germany, the UK and Austria.
Source: Pwt6, OECD Economic Outlook No. 65 1999 via Eco Win and World Bank Global Development Network Growth Database.

Figure 1: Standard deviation of GDP per capita and per worker across 12 EU countries, 1950-1998.

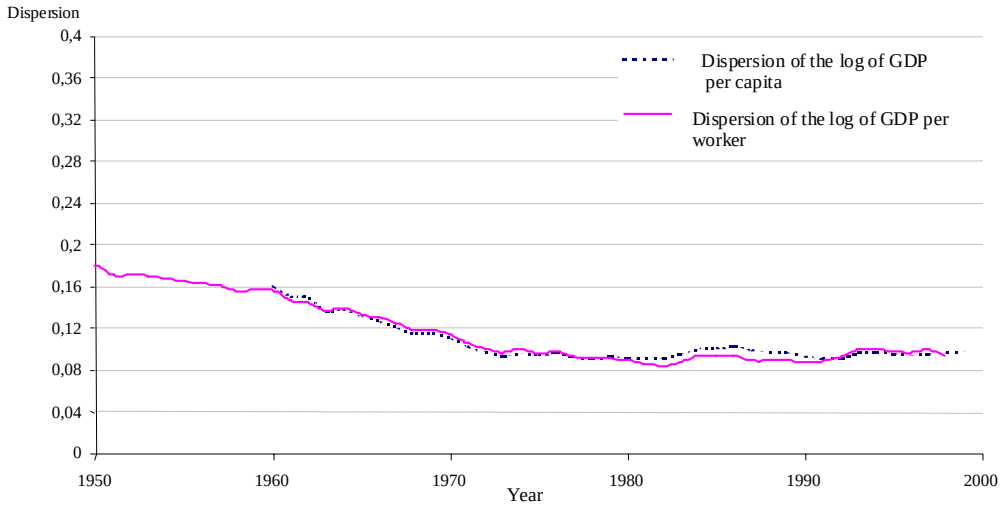
capital and labour mobility are important and some countries are pushing the technological frontier further out, while others try to imitate and catch up.

# 2 A bird's-eye view at the data

In the following no serious econometrics is attempted. We use the term "trend" in an admittedly loose sense.

Fig. 1 shows the time profile for the standard deviation of $y$ itself for 12 EU countries, whereas Fig. 2 and Fig. 3 show the time profile of the standard deviation of $\log y$ and the time profile of the coefficient of variation, respectively. Comparing the upward trend in Fig. 1 with the downward trend in the two other figures we have an illustration of the fact that the movement of the standard deviation of $y$ itself does not capture income convergence. To put it another way: although there seems to be conditional income convergence with respect to the two scale-free measures, Fig. 1 shows that this tendency to convergence is *not* so strong as to produce a narrowing of the absolute distance between the EU countries

Fig. 4 shows the time path of the coefficient of variation across 121 countries in the



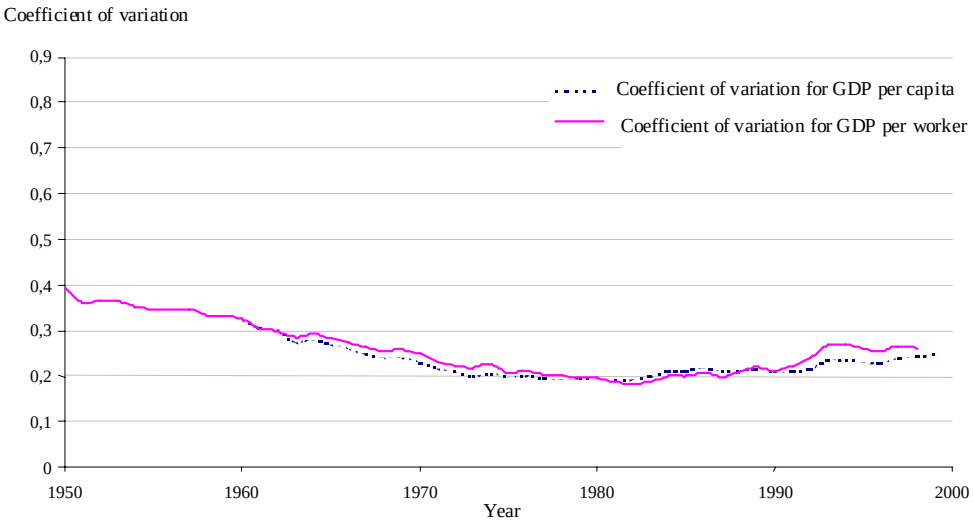

Remarks: Germany is not included in GDP per worker. GDP per worker is missing for Sweden and Greece in 1950, and for Portugal in 1998. The EU comprises Belgium, Denmark, Finland, France, Greece, Holland, Ireland, Italy, Luxembourg, Portugal, Spain, Sweden, Germany, the UK and Austria.
Source: Pwt6, OECD Economic Outlook No. 65 1999 via Eco Win and World Bank Global Development Network Growth Database.

Figure 2: Standard deviation of the log of GDP per capita and per worker across 12 EU countries, 1950-1998.

world, 22 OECD countries and 12 EU countries, respectively. We see the lack of unconditional income convergence, but the presence of conditional income convergence. One should not over-interpret the observation of convergence for the 22 OECD countries over the period 1950-1990. It is likely that this observation suffer from the selection bias problem mentioned in Section 1. A country that was poor in 1950 will typically have become a member of OECD only if it grew relatively fast afterwards.

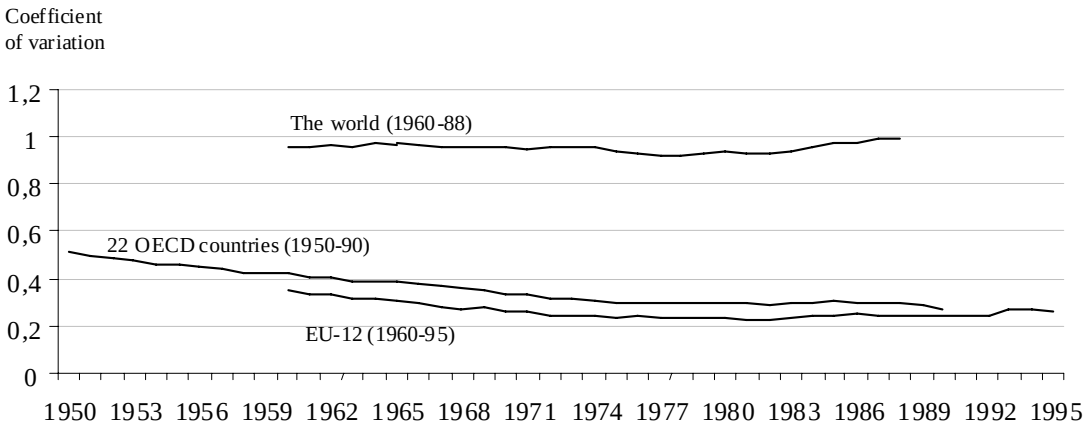
# 3 Other concepts

Of course, just considering the time profile of the first and second moments of a distribution may sometimes be a poor characterization of the evolution of the distribution. For example, there are signs that the distribution has polarized into *twin peaks* of rich and poor countries (Quah, 1996; Jones, 1997). Related to this observation is the notion of club convergence. If income convergence occurs *only* among a subgroup of the countries that to some extent share the same initial conditions, then we say there is *club-convergence.* This concept is relevant in a setting where there are *multiple* steady states toward which countries can




Remarks: Germany is not included in GDP per worker. GDP per worker is missing for Sweden and Greece in 1950, and for Portugal in 1998. The EU comprises Belgium, Denmark, Finland, France, Greece, Holland, Ireland, Italy, Luxembourg, Portugal, Spain, Sweden, Germany, the UK and Austria.
Source: Pwt6, OECD Economic Outlook No. 65 1999 via Eco Win and World Bank Global Development Network Growth Database.

Figure 3: Coefficient of variation of GDP per capita and GDP per worker across 12 EU countries, 1950-1998.




Remarks: 'The world' comprises 121 countries (not weighed by size) where complete time series for GDP per capita exist. The OECD countries exclude South Korea, Hungary, Poland, Iceland, Czech Rep., Luxembourg and Mexico.
EU-12 comprises: Benelux, Germany, France, Italy, Denmark, Ireland, UK, Spain, Portugal og Greece.
Source: Penn World Table 5.6 and OECD Economic Outlook, Statistics on Microcomputer Disc, December 1998.

Figure 4: Coefficient of variation of income per capita across different sets of countries.

be heading. Existence of multiple steady states is a phenomenon that can easily arise in overlapping-generations models. Then initial conditions matter for which of these steady states tends to be realized. Similarly, we may say that *conditional club-convergence* is present, if income convergence occurs *only* among a subgroup of the countries that share the same structural characteristics (this may to some extent be true for the OECD countries) *and* to some extent have similar initial conditions.

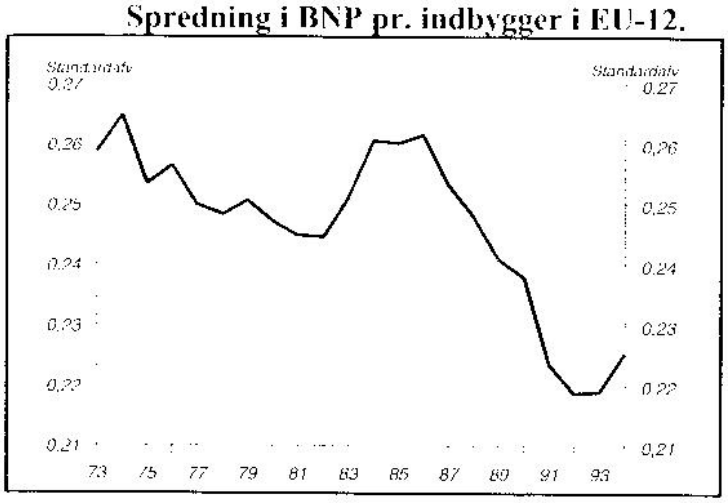
Instead of focusing on income convergence, one could study *TFP convergence.* Sometimes the less demanding concept of *growth rate convergence* is the focus.

The above considerations are only of a very elementary nature. The reader is referred to the large existing literature on concepts and econometric methods of relevance for characterizing the evolution of world income distribution (for a survey, see Islam 2003).

# 4 Appendix: An exercise

Sometimes misleading figures about income convergence are published. For example, Fig. 5 shows a copy of a figure from a publication by the Danish Ministry of Finance, 1996. In English, the headline reads "Standard deviation in GDP per capita in EU-1". Similarly, Fig. 6 shows a copy of a figure from the Danish Economic Council, 1997. In English, the headline reads: Standard deviation in GDP per capita and per worker across the EU countries.

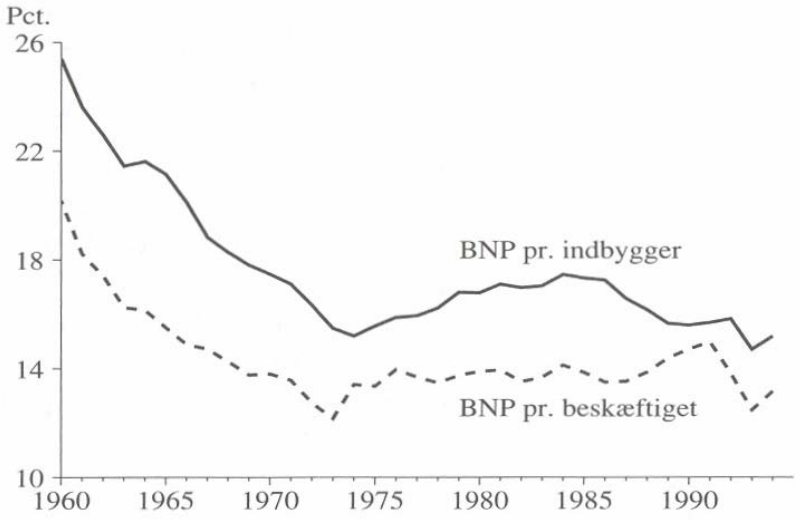
As an exercise, I ask the reader to critically evaluate at least one aspect of the figures. When doing this, the reader may take as given that the above figures are correct.




Kilde: OECD: *Economic Outlook 59*, 1996, samt egne beregninger

Figure 5: Source: Finansredegørelse 96. Bilag p. 53. Finansministeriet, Dec. 1996.

*Figur IV.4 Spredningen i EU-landenes indkomst pr. indbygger og pr. beskæftiget*




Anm.: Spredningen er beregnet som den vægtede spredning i EU-landenes købekraftskorrigerede BNP pr. indbygger og pr. beskæftiget. Som landevægte er anvendt hhv. befolkningstal og antal beskæftigede.

Kilde: OECD, *Economic Outlook* og egne beregninger.

Figure 6: Source: Det økonomiske Råd, Dansk økonomi Forår 1997, p. 147.

# 5 Literature


Barro, Robert J., & Xavier Sala-i-Martin, 2004, *Economic Growth, 2. ed.,* MIT Press, New York.

Cowell, Frank A., 1995, *Measuring Inequality. 2. ed.,* London.

Dalgaard, Carl-Johan, 1999, Om den globale indkomstfordeling. Bjørn Lomborgs mening og en anden mening, *Samfundsøkonomen*, nr. 4, 1999.

Dalgaard, Carl-Johan, and Jacob Vastrup, 2001, On the measurement of $\sigma$-convergence, *Economics letters,* vol. 70, 283-87.

*Dansk økonomi. Efterår 2001,* (Det økonomiske Råds formandskab) Kbh. 2001.

Deininger, K., and L. Squire, 1996, A new data set measuring income inequality, *The World Bank Economic Review*, 10, 3.

Delong, B., 1988,

*Handbook of Economic Growth*, vol. 1A and 1B, ed. by S. N. Durlauf and P. Aghion, Amsterdam 2005.

*Handbook of Income Distribution*, vol. 1, ed. by A.B. Atkinson and F. Bourguignon, Amsterdam 2000.

Islam, N., 2003, What have we learnt from the convergence debate? *Journal of Economic Surveys 17*, 3, 309-62.

Maddison, A., 1982,

Mankiw, N.G., D. Romer and D.N. Weil, 1992,

Romer, D., 2001, *Advanced Macroeconomics*, 2. ed., New York.

Sala-i-Martin, X., 2006, The World Distribution of Income, *Quarterly Journal of Economics 121*, No. 2.

On measurement problems, see: http://www.worldbank.org/poverty/inequal/methods/index.htm