Economic Growth Exercises

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Problem Set V

V.1 *Education in a market economy. A one-sector model*[1] Consider a closed market economy with education in private schools. Under perfect competition the representative firm chooses capital input, K^d, and labour input, L^d, in order to maximize profit, given the production function

$$Y = F(K^d, hL^d), \tag{1}$$

where Y is output, h is a measure of labour productivity, and F is a neoclassical production function with constant returns to scale. We shall assume that h reflects average human capital acquired through formal education.

a) Given h and the aggregate supplies of capital, K, and labour, L, respectively, determine the real rental rate, $\tilde{r}$, for capital and the real wage, $\hat{w}$, per unit of *effective* labour input in equilibrium.

Aggregate output (= aggregate gross income) is used for consumption, C, investment, I_K, in physical capital and investment, I_H, in human capital, i.e.,

$$Y = C + I_K + I_H.$$

The dating of the variables is suppressed where not needed for clarity. The increase per time unit in the two kinds of capital is given by

$$\begin{aligned} \dot{K} &= I_K - \delta K, \quad \text{and} \\ \dot{H} &= I_H - \delta H, \end{aligned} \tag{2}$$

respectively, where $H \equiv hL$. We have, for simplicity, assumed that the depreciation rate, $\delta \geq 0$, is the same for the two kinds of capital.

The representative household (family) has infinite horizon and consists of L members, where $L = L_0 e^{nt}$, $n \geq 0$, $L_0 > 0$. Each family member supplies inelastically one unit of labour time per time unit. From (2) and the definition $H \equiv hL$ follows the per capita human capital accumulation equation:

$$\dot{h} = i - (\delta + n)h, \tag{3}$$

where $i \equiv I_H/L$ is the per capita educational cost (in real terms) per time unit.

[1] Whereas B & S, Section 5.1, considers human capital formation in an isolated family farm or from the perspective of a social planner, the setting considered here is a standard market economy with perfect competition.

b) Present a derivation of (3).[2]

Let θ and ρ be positive constants, where $\rho > n$. Let $a \equiv$ per capita financial wealth, $c_t \equiv C_t/L_t$, $r \equiv$ the real rate of interest. The representative household chooses a path $(c_t, i_t)_{t=0}^{\infty}$ to maximize

$$\begin{aligned}
U_0 &= \int_0^\infty \frac{c_t^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} dt \qquad \text{s.t.} && (4)\\
c_t &\geq 0, \; i_t \geq 0, && (5)\\
\dot{a}_t &= (r_t - n)a_t + \hat{w}_t h_t - i_t - c_t, \qquad a_0 \text{ given}, && (6)\\
\dot{h}_t &= i_t - (\delta + n)h_t, \qquad h_0 > 0 \text{ given}, && (7)\\
\lim_{t\to\infty} a_t e^{-\int_t^\tau (r_s - n)ds} &\geq 0, && (8)\\
h_t &\geq 0 \text{ for all } t. && (9)
\end{aligned}$$

c) Briefly interpret the six elements in this decision problem.

d) Use the Maximum Principle (for the case with two control variables and two state variables) to find the first-order conditions for an interior solution and the transversality conditions.

e) Derive from the first-order conditions the Keynes-Ramsey rule.

f) Set up a the no-arbitrage equation showing a relationship between $\hat{w}$ and r. You may either use your intuition or derive the relationship from the first-order conditions. In case you use your intuition, check whether it is consistent with the first-order conditions. *Hint:* along an interior optimal path the household should be indifferent between placing the marginal unit of saving in a financial asset yielding the rate of return r or in education to obtain one more unit of human capital.

Assume now for simplicity that the aggregate production function is:

$$Y = AK^\alpha (hL)^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1,$$

g) Determine the real interest rate in equilibrium at time t in this case.

Suppose parameters are such that $\dot{c}/c > 0$ and U_0 is bounded.

[2] Note that the appearance of the $-nh$ term in (3) indicates that the present model follows the Mankiw-Romer-Weil (1992) approach and treat human capital in a completely parallel way to physical capital. This is also the approach followed by B & S on pp. 59-61. We may call it the "human capital parallel to physical capital approach". There is an alternative approach, used by Lucas (1988) and others (see Lecture Note 16), where the two forms of capital are *not* treated in this parallel way, because human capital is embodied in an individual and is not a tangible thing which can immediately be transferred to others. We may call this approach the "human capital as not transferable approach". Which of these two approaches B & S use in their Chapter 5 is not visible because they assume $n = 0$.

h) The no-arbitrage equation from f) (which is needed for an *interior* solution to the household's decision problem) requires a specific value of K/H to be present. Determine this value and explain what happens to begin with if the historically given K/H ratio in the economy differs from it; and explain what happens in the long run.

i) Consider a constant subsidy, s, to education such that per unit of investment in education the private cost is only $1 - s$. That is, i_t in (7) is replaced by $(1 - s)i_t$. Suppose the subsidy is financed by lump-sum taxes. Will such a subsidy affect long-run growth in this model? Explain. *Hint:* In answering, you may use your intuition or make a formal derivation. A quick approach can be based on the no-arbitrage condition in the new situation (for simplicity you may put $\delta = 0$).

V.2 *Short questions* These questions relate to the model in Problem V.1.

a) Comment on the model in relation to the concepts of fully endogenous growth and semi-endogenous growth.

b) Comment on the model in relation to the issue of scale effects.

c) What do you guess will be the consequences of replacing h in (1) by h^{φ}, $0 < \varphi < 1$? Comment.

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