Economic Growth Exercises

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Problem set IV

IV.1 Consider a closed market economy with L utility maximizing households and M profit maximizing firms, operating under perfect competition (L and M are constant, but "large"). There is also a government, supplying a non-rival productive service G per time unit. Each household has an infinite horizon and supplies inelastically one unit of labour per time unit. Aggregate output is Y per time unit and output is used for private consumption, $C \equiv cL$, the public productive service, G, and investment, I, in (physical) capital, i.e., $Y = C + G + I$. The stock of capital, K, changes according to $\dot{K} = I - \delta K$, where $\delta \geq 0$ is the rate of physical decay of capital. Variables are dated implicitly. The initial value $K(0) > 0$ is given. The capital stock in society is owned, directly or indirectly (through bonds or shares), by the households. There is perfect competition at the labour market. The equilibrium real wage is called w. There is a perfect market for loans at the real rate of interest r, and there is no uncertainty. A dot over a variable denotes the time derivative.

The government chooses G so that

$$G = \bar{g}Y,$$

where the constant $\bar{g}$ is exogenous and is such that positive growth in the economy occurs in equilibrium. The government budget is always balanced and the service G is the only public expenditure. The tax revenue is

$$\left[\tau(ra + w) + \tau_\ell\right] L = G, \tag{GBC}$$

where a is per capita financial wealth, and τ and τ_ℓ denote the income tax rate and a lump-sum tax, respectively. The tax rate τ is a given constant, $0 \leq \tau < 1$, whereas τ_ℓ is adjusted when needed for (GBC) to be satisfied.

The production function for firm i is

$$Y_i = AK_i^\alpha (GL_i)^{1-\alpha}, \quad 0 < \alpha < 1, A > 0, \quad i = 1, 2, ..., M. \tag{*}$$

a) Briefly interpret (*) and comment on the nature of G.

It can be shown that in equilibrium

$$\begin{aligned} Y &= \sum_i Y_i = \sum_i y_i L_i = y \sum_i L_i = yL = Ak^\alpha G^{1-\alpha} L = A^{1/\alpha}(\bar{g}L)^{(1-\alpha)/\alpha} kL \equiv \bar{A}K, \quad \text{and} \\ r &= \alpha - \delta \equiv \alpha\bar{A} - \delta, \quad \text{where} \quad k \equiv K/L \quad \text{and} \quad \bar{A} \equiv A^{\frac{1}{\alpha}}(\bar{g}L)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

b) Briefly explain these results in words.

Suppose the households, all alike, have a constant rate of time preference $\rho > 0$ and an instantaneous utility function with (absolute) elasticity of marginal utility equal to a constant $\theta > 0$.

c) Derive the Keynes-Ramsey rule, given the described taxation system.

d) Write down the transversality condition in a form comparable to the No-Ponzi-Game condition of the household. Comment.

e) Find the growth rate of $k \equiv K/L$ and $y \equiv Y/L$ in this economy (an informal argument, based on your general knowledge about reduced-form AK models is enough). In case, you need to introduce a restriction on some parameters to ensure existence of equilibrium with growth, do it.

f) Comment in relation to the scale effect issue.

Suppose lump-sum taxation is not feasible. Hence, let $\tau_\ell = 0$ for all $t \geq 0$.

g) Examine whether it is possible to fix τ at a level (constant over time and < 1) such that the government budget is still balanced in equilibrium for all $t \geq 0$? *Hint:* if you need a new restriction on parameters to ensure $\tau < 1$, introduce it.

h) If the welfare of the representative household is the criterion, what proposal to the government do you have w.r.t. the size of $\bar{g}$ and the *form* of taxation (given that lump-sum taxation is not feasible)? *Hint:* there may be a problem with the tax candidate that first comes to one's mind.

IV.2 *Short questions.*

a) In relation to Romer's learning-by-investing model B & S write on p. 219: "One way to eliminate the scale effect [on growth] is to argue that the term A_i in equation (4.22) depends on the economy's average capital per worker, K/L, rather than the aggregate capital stock, K." Suggest yet another simple way to eliminate the scale effect on growth in the model. Comment.

b) In relation to their model with productive public services in the form of pure public goods B & S write on p. 223: "The failure to [empirically] detect ....scale effects [on growth] likely means that most of the government's services do not have the non-rival character that is assumed in the model." Suggest an alternative interpretation of the failure of the model to comply with the seeming absence of scale effects on growth, an interpretation *maintaining* the nonrival character assumed in the model.

c) "In a representative agent model lump-sum taxation is, from a welfare point of view, always to be preferred to (or at least as good as) a proportional income tax." True or false? Why?

d) "In a reduced-form AK model with productive public services with congestion a welfare-maximizing government has to implement a policy that reduces the per capita growth rate." True or false? Why?