Economic Growth Exercises

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Problem set III

III.1   *A subsidy to saving in Romer's learning-by-investing model.* Consider a closed market economy with perfect competition where firm no. i has the production function

$$Y_{it} = F(K_{it}, T_t L_{it}),$$

where F is a neoclassical production function with CRS and satisfying the Inada conditions (standard notation). It is assumed that the technology level T_t satisfies

$$T_t = K_t^\lambda, \qquad 0 < \lambda \leq 1.$$

Time, t, is continuous. There is no uncertainty. At the aggregate level,

$$\dot{K}_t \equiv \frac{dK_t}{dt} = Y_t - C_t - \delta K_t, \qquad \delta > 0, \qquad K_0 > 0 \text{ given.}$$

a) Determine the equilibrium real interest rate, r, and the aggregate production function. Comment.

From now we assume $\lambda = 1$.

b) Determine the equilibrium real interest rate, r, and the aggregate production function in this case. Comment.

There is a representative Ramsey household with instantaneous utility function of CRRA type:
$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \qquad \theta > 0,$$
where c is per capita consumption ($c \equiv C/L$). The rate of time preference is a constant $\rho > 0$. There is no population growth ($n = 0$).

c) Determine the growth rate of c and name it γ.

From now, assume (A1) $F_1(1, L) - \delta > \rho$ and (A2) $(1 - \theta)\gamma < \rho$.

d) What could be the motivation for these two assumptions?

e) Determine the growth rate of $k \equiv K/L$ and $y \equiv Y/L$. A detailed derivation involving the transversality condition need not be given; instead you may refer to a general property of AK and reduced-form AK models in a Ramsey framework where (A2) holds.[1]

f) Set up and solve the social planner's problem, assuming the same criterion function as that of the representative household. *Hint*: the linear differential equation $\dot{x}(t) + ax(t) = ce^{ht}$, with $a \neq 0$ and $h \neq -a$ has the solution:

$$x(t) = (x(0) - \frac{c}{a+h})e^{-at} + \frac{c}{a+h}e^{ht}.$$

g) Now consider again the decentralized market economy, but suppose there is a government that wants to establish the social planner's allocation by use of a subsidy, σ, to private saving such that the after-subsidy-rate of return on private saving is $(1 + \sigma)r$. Let the subsidy be financed by a lump-sum tax. Determine σ such that the social planner's allocation is established, if this is possible. Comment.

III.2   In endogenous growth theory two alternative kinds of scale effects may be present. Give a brief account. Link two alternative learning-by-investing models to these two kinds of scale effects.

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[1] The two defining characteristics of AK and reduced-form AK models are that the real interest rate is a constant and the aggregate production function implies a constant output-capital ratio.