Economic Growth Exercises

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Problem set II

II.1 *Aggregate saving and the return to saving.* Consider a Ramsey model for a market economy with perfect competition, government consumption, lump-sum transfers and capital income taxation. The government budget is always balanced. The model leads to the following differential equations (standard notation):

$$\dot{\hat{k}} = f(\hat{k}) - \hat{c} - \hat{g} - (\delta + x + n)\hat{k}, \qquad (*)$$

$$\dot{\hat{c}} = \frac{1}{\theta}\left[(1-\tau_a)(f'(\hat{k}) - \delta) - \rho - \theta x\right]\hat{c}. \qquad (**)$$

All parameters are positive, and it is assumed that $\rho > n$, and

$$\lim_{\hat{k}\to 0} f'(\hat{k}) > \delta + \rho + \theta x > \delta + n + x > \lim_{\hat{k}\to\infty} f'(\hat{k}). \qquad (***)$$

The government controls $\hat{g}$ and $\tau_a \in (0,1)$. Until further notice $\hat{g}$ and τ_a are kept constant over time, and the transfers are continuously adjusted when needed for a balanced budget.

a) Briefly interpret (*), (**) and (***), including the parameters.

b) Draw a phase diagram and illustrate the path that the economy follows, for a given $\hat{k}_0 > 0$. Comment.

c) Is it possible for general equilibrium to exist without assuming f satisfies the Inada conditions? Is it possible to determine the time path of the economy without assuming the Inada conditions? Comment.

d) Suppose the economy has been in steady state until time t_0. Then, suddenly τ_a is increased to a higher constant level. Illustrate by a phase diagram what happens in the short run and in the long run. Give an economic interpretation of your result.

e) Does the direction of movement of $\hat{k}$ depend on θ? Comment.

f) Suppose $\theta = 1$. It is well-known that in this case the substitution effect and the income effect on saving of an increase in the (after-tax) rate of return offset each other. Does this imply that aggregate saving does not change in response to the change in fiscal policy? Why or why not? *Hint:* when $\theta = 1$, $c_t = (\rho - n)(a_t + \tilde{w}_t)$, where

$$\tilde{w}_t \equiv \int_t^{\infty} (w_s + v_s) e^{-\int_t^s [(1-\tau_a) r_\tau - n] d\tau} ds;$$

here, v_s is per capita transfers at time s. Four "effects" are in play, not only the substitution and income effects.

II.2 *(command optimum).* Suppose preferences and technology are as in the standard Ramsey model with CRRA utility and exogenous technical progress at the constant rate $x \geq 0$. Suppose resource allocation is not governed by market mechanisms, but by a "social planner" – by which is meant a central authority who is "all-knowing and all-powerful". The social planner is not constrained by other limitations than those from technology and initial resources and can thus ultimately decide on the resource allocation within these confines.

The decision problem of the social planner is (standard notation):

$$\begin{aligned} \max_{(c_t)_{t=0}^{\infty}} U_0 &= \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-(\rho - n)t} dt \qquad \text{s.t.} \\ c_t &> 0, \\ \dot{\hat{k}}_t &= f(\hat{k}_t) - \frac{c_t}{T_t} - (\delta + n + x)\hat{k}_t, \\ \hat{k}_t &\geq 0 \qquad \text{for all } t \geq 0, \end{aligned}$$

where $\delta + x > 0, \rho > n \geq 0$ and $\theta > 0$ (in the case $\theta = 1$, the expression $(c^{1-\theta} - 1)/(1-\theta)$ should be interpreted as $\log c$). Assume $\rho - n > (1-\theta)x$ and that the production function satisfies the Inada conditions.

a) Briefly interpret the problem, including the parameters. Comment on the inequality $\rho - n > (1 - \theta)x$.

b) Derive a characterization of the solution to the problem.

c) Compare the solution to the allocation generated by a market economy described by a Ramsey model with perfect competition and with the same preferences and the same technology as above. Comment.

II.3 *Short questions.*

a) "If the technology is Cobb-Douglas, then Harrod-neutrality implies Hicks-neutrality and vice versa." True or false? Comment.

b) Suppose labour income is taxed at the rate $\tau_w \in (0, 1)$. Assume that the revenue from this income tax is redistributed back to the households as lump-sum transfers. Consider the assertion: "In the Ramsey model a time-varying labour income tax τ_w will distort the allocation of resources." True or false? Comment.

c) Consider a Ramsey model with a consumption tax rate $\tau_c > 0$ such that the household pays $(1 + \tau_c)c$ for the consumption level c. Let τ_c be constant over time. The government revenue from the tax is used for financing lump-sum transfers to the households. Suppose someone states the following: "In the described framework, an unanticipated once-for-all increase in the consumption tax rate leads to lower consumption in the short run and higher capital intensity in the long run." True or false? Comment.

II.4 *Short questions.*

a) Consider a Ramsey model with a government that provides necessary infrastructure services for the private sector. The government wants to have a balanced budget always. Suppose the government wants to use non-distortionary tax instruments to finance its expenditures, but lump-sum taxes are not at its disposal. Can you suggest some ways out for the government? Comment in relation to the real world.

b) We now extend the Ramsey model considered under a). Now also land is a necessary production factor. Is there any way this can affect your answer or comment to a)? Comment.

c) Consider a Ramsey-style framework extended with household heterogeneity in assets and exogenous labour productivity. All households have the same preferences (same θ and ρ). Suppose someone states the following: "In the described framework, the relative asset positions move over time towards the relative productivity positions." True or false? Comment.

II.5 *An Arrow-style learning-by-investing model.* Consider a closed market economy with perfect competition where firm no. i has the production function

$$Y_{it} = F(K_{it}, T_t L_{it}),$$

where F is a neoclassical production function with CRS and satisfying the Inada conditions (standard notation). It is assumed that

$$T_t = K_t^{\lambda}, \qquad 0 < \lambda < 1.$$

Time, t, is continuous. There is no uncertainty. At the aggregate level,

$$\dot{K}_t \equiv \frac{dK_t}{dt} = Y_t - C_t - \delta K_t, \qquad \delta > 0.$$

a) Briefly, interpret the three equations.

Let the household sector be described by the standard Ramsey framework with CRRA instantaneous utility and no uncertainty. Let the population growth rate, n, be a non-negative constant.

b) Find an expression for the equilibrium interest rate in terms of predetermined variables.

c) Show that the aggregate production function can be written $Y_t = F(K_t, T_t L_t) = T_t L_t f(\tilde{k}_t)$, where $\tilde{k}_t \equiv K_t/(T_t L_t)$.

The representative household has instantaneous utility function of CRRA type:

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \qquad \theta > 0,$$

where c is per capita consumption ($c \equiv C/L$). The rate of time preference is a constant $\rho > n$.

d) Derive two coupled differential equations in $\tilde{k}$ and $\tilde{c} \equiv c/T$ (standard notation) and construct a phase diagram. *Hints*: it is useful to determine the growth rate of C, K, and T in steady state and then the real interest rate in steady state; if you for some reason need a parameter inequality to ensure existence of general equilibrium, introduce it; as to the sign of the slope of the $\dot{\tilde{c}} = 0$ locus, it is enough to consider the slope in a small neighborhood of the steady state.

e) What can be said about stability of the steady state?

f) Determine the steady-state growth rate of $y \equiv Y/L$ and c, respectively? Comment in relation to different types of endogenous growth.

g) Comment in relation to a possible need for government intervention.

II.6 *A Romer (1986)-style learning-by-investing model.* The model is the same as in Problem II.5, except for two important features. Here $\lambda = 1$ and $n = 0$.

a) Determine the real interest rate, r, and the aggregate production function. Comment.

b) Determine the steady-state growth rate of $y \equiv Y/L$ and c? Comment in relation to different types of endogenous growth.

c) Comment in relation to the scale effect issue and the assumption that $n = 0$.

d) Set up and solve the social planner's problem, assuming the same criterion function as that of the representative household. *Hint*: the linear differential equation $\dot{x}(t) + ax(t) = ce^{ht}$, with $a \neq 0$ and $h \neq -a$ has the solution:

$$x(t) = (x(0) - \frac{c}{a+h})e^{-at} + \frac{c}{a+h}e^{ht}.$$

e) Now consider again the decentralized market economy, but suppose there is a government that wants to establish the social planner's allocation by use of a subsidy, s, to reduce firm's capital costs to

$$(1 - s)(r + \delta)$$

per unit of capital per time unit. Let the subsidy be financed by a lump-sum tax. Determine s such that the social planner's allocation is established, if this is possible. Comment.

f) Suppose the government contemplates an alternative policy, namely a subsidy σ to saving such that the rate of return on saving is $(1 + \sigma)r$. Also this subsidy is to be financed by a lump-sum tax. Can this policy work? Comment.

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