Economic Growth Exercises

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Problem set I[1]

I.1 *Short questions* (answering requires only a few well chosen sentences).

a) Consider an economy where all firms' technology is described by the same neoclassical production function, $Y_i = F(K_i, L_i)$, $i = 1, 2, ..., N$, with decreasing returns to scale everywhere (standard notation). Suppose there is "free entry and exit" and perfect competition at all markets. Then a paradoxical situation arises in that no equilibrium with a finite number of firms (plants) would exist. Explain.

b) In the Solow and Ramsey growth models, as in many other macro models, the technology is assumed to have constant returns to scale (CRS) with respect to capital and labour taken together. Often the so-called *replication argument* is put forward as a reason to expect CRS should hold in the real world. What is the replication argument? Do you think it is valid in the present case? Explain.

c) What is likely to happen in an economy where, for a certain historical period, there has been something close to constant returns to scale and perfect competition, but then, after a shift to new technologies in the different industries, increasing returns to scale arise?

I.2 In about two decades China has had an average growth rate of real GDP at approximately 9 percent per year.

a) Supposing China's growth performance continues to be like that, how many years does it take for China's GDP to be doubled? You should explain your method.

b) What was in 1987 the ratio of China's GDP per capita to that of the US? *Hint:* see the appendix to Chapter 12 in B & S.

[1] A tilde over a variable generally has the same meaning as the hat in B & S.

c) Suppose China's GDP per capita growth rate per year continues to be as it was on average 1973-1987 and suppose the GDP per capita growth rate per year in the US continues to be as it was on average in 1970-1990. How long time, reckoned from 1987, will it then take for China to catch up with the US? You should explain your method. *Hint:* The needed data for answering this problem can be found in the appendix to Chapter 12 in B & S.

d) Do you find it likely that the actual course of events will be (approximately) like that? Why or why not?

I.3 In a popular magazine on science the following data was reported:

World income per capita relative to income per capita in the US: 1952-96

Year	*Percent*
1952	13.0
1962	13.3
1972	13.0
1982	13.8
1992	15.1
1996	17.7

Source: Knowledge, Technology, & Policy 13, no. 4, 2001, p. 52.
Remark. Countries' per capita income are weighted by population as a fraction of the world population.

a) Briefly, discuss this data relative to concepts of convergence or divergence and relative to your knowledge of the importance of weighting by population size.

b) What is meant by the terms unconditional (or absolute) convergence and conditional convergence?

c) Give a short list of mechanisms that could in principle explain the data above.

I.4 In the data appendix of B & S we see that for Denmark GDP per capita in 1950 is 5227 US$ (1985 prices) and in 1990 it is 14806 US$ (1985 prices). Calculate the average (compound) growth rate in GDP per worker 1950-1990. How does the result compare with the general tendency for developed countries since 1870?

I.5 *Short questions.*

a) "If we know that there is a negative correlation, across countries, between initial per capita income and the growth rate in per capita income, then the dispersion of relative income per capita across the countries must have been diminished." True or not true? Explain.

b) "Considering closed economies that have access to the same technology, we should expect their per capita income levels to converge to the same level." Evaluate this statement.

c) "Considering countries that have similar structural characteristics (say, countries in Western Europe or perhaps the OECD countries), we should expect their per capita income levels to converge (in a literal sense) to the same level over time." Evaluate this statement.

d) "Considering countries that have similar structural characteristics (say, countries in Western Europe or perhaps the OECD countries), we should at most expect their *relative* per capita income levels to converge (in some stochastic sense) to the same level over time." Evaluate this statement.

e) "Considering countries that have similar structural characteristics (say, countries in Western Europe or perhaps the OECD countries), we should expect that the further below its steady state a country is, the higher is its per capita growth rate." Evaluate this statement. If you agree (don't agree), give empirical examples supporting your opinion.

f) "σ convergence implies β convergence." True or not true? Explain.

I.6 Consider a standard Solow model for a closed economy with perfect competition. The rate of Harrod-neutral technical progress is 1.8 percent per year, the rate of population growth is 0.5 percent per year, capital depreciates at the rate 0.6 per year, and in steady state the share of labour income in steady state is 2/3.

a) Find the speed of adjustment (also called the speed of convergence) and the half-life. *Hint:* Given the intensive production function $f(\hat{k})$, in a neighbourhood of the steady state, the speed of adjustment is approximately $(1 - \frac{\hat{k}^* f'(\hat{k}^*)}{f(\hat{k}^*)})(x + n + \delta)$.

b) Comment on the result you have got in relation to your knowledge of direct estimates of empirical adjustment speeds.

c) What is the long-run per capita growth rate implied by the model?

d) What is the doubling-time of income per capita implied by the model?

I.7 Set up a Solow-type model where, though there is no technical progress, sustained per capita growth occurs. Comment. *Hint:* a simple approach can be based on the production function $Y = BK^{\alpha}L^{1-\alpha} + AK$, where $A > 0, B > 0, 0 < \alpha < 1$.

I.8 Consider a closed economy with technology described by the aggregate production function

$$Y = F(K, L),$$

where F is a neoclassical production function with CRS and satisfying the Inada conditions, Y is output, K is capital input and L is labour input = labour force = population (there is always full employment). A constant fraction, s, of *net* income is saved (and invested). Capital depreciates at the constant rate $\delta > 0$.

a) Assuming a constant population growth rate n, derive the fundamental differential equation of the model and illustrate the dynamics by a phase diagram. Comment.

b) Assume instead that the population growth rate n is a smooth function of per capita income, i.e., $n = n(y)$, where $y \equiv Y/L$. At very low levels of per capita income, n is zero, at higher per capita income, n is a hump-shaped function of y, and at very high levels of y, n tends to zero, that is, for some $\bar{y} > 0$ we have

$$n'(y) \gtreqless 0, \text{ for } y \lesseqgtr \bar{y}, \text{ respectively,}$$

whereas $n(y) \approx 0$ for y considerably above $\bar{y}$. Show that this may give rise to a dynamics quite different from that of the Solow model. Comment.

I.9 *Short questions.* We assume a selection of countries (considered, for simplicity, as closed economies) can be described by the Ramsey model with Harrod-neutral technical progress at a constant positive rate. For each country parameters and initial conditions are such that an equilibrium exists (B&S notation).

a) "The model predicts that for countries with the same technology (same F, T_0, x and δ), differences in per capita growth rates are only temporary and due to the transitional dynamics." True or false? Comment.

b) "The model predicts that for countries with the same technology, differences in per capita income are only temporary and due to the transitional dynamics." True or false? Comment.

c) "The Ramsey model predicts that for countries with similar structural characteristics, the further away from its steady state a country is, the higher is its per capita growth rate." True or not true? Comment.

I.10 *Short questions* Consider the Ramsey model for a market economy with perfect competition.

a) Write down the dynamic budget constraint and the NPG condition for the representative household expressed in absolute terms (not per capita terms).

b) Derive the corresponding dynamic budget constraint and NPG condition expressed in per capita terms.

c) "Only if the production function is Cobb-Douglas does the Ramsey model predict that the share of labour income in national income is constant in the long run." True or false? Give a reason for your answer.

d) Are the predictions of the Ramsey model (with exogenous Harrod-neutral technical progress) consistent with Kaldor's "stylized facts"? Give a reason for your answer.

I.11 *Short questions*

a) Could a path *below* the saddle path in the $(\tilde{k}, \tilde{c})$ space be precluded as an equilibrium path with perfect foresight in the Ramsey model? Give a reason for your answer.

b) Could a path *above* the saddle path in the $(\tilde{k}, \tilde{c})$ space be precluded as an equilibrium path with perfect foresight in the Ramsey model? Give a reason for your answer.

c) Answer questions a) and b), now presuming that we are dealing with the solution of the problem from the point of view of the social planner in the Ramsey model.

d) "When the steady state of a dynamic system is a saddle point, then the system is saddlepoint stable." True or false? Why?

I.12 *(productivity slowdown)* Consider a Ramsey model of a market economy with perfect competition in all markets. The model produces two coupled differential equations:

$$\dot{\tilde{k}} = f(\tilde{k}) - \tilde{c} - (\delta + x + n)\tilde{k}, \qquad (*)$$

$$\dot{\tilde{c}} = \frac{1}{\theta}\left[f'(\tilde{k}) - \delta - \rho - \theta x\right]\tilde{c}. \qquad (**)$$

a) Equation (*) generally holds for a closed economy with a per capita production function f, a capital depreciation rate δ, a growth rate n of the labour force and a growth rate x of labour efficiency. Briefly explain this.

b) Equation (**) emerges from assumptions specific to the Ramsey model. Give a brief account of this.

c) From now it is assumed that $f(0) > 0$. State in words the economic interpretation of this assumption.

d) Construct a phase diagram to illustrate the dynamics of the model.

e) Assume that $\tilde{k}_0 > \tilde{k}^*$, where $\tilde{k}_0$ is the initial (effective) capital intensity and $\tilde{k}^*$ is the capital intensity in the economically interesting steady state (which is thus assumed to exist). Show in the phase diagram the evolution over time brought about by the model. Next, show in a graph having time on the horizontal axis (i.e., a "time profile diagram") the evolution of $\tilde{k}$, $\tilde{c}$, r and w/T (standard notation).

f) Suppose the economy considered is one of the "emerging economies" (newly industrialized economies) in the world (and ignore that our Ramsey model does not fit entirely well, in view of its closed-economy character). If time $t = 0$ corresponds to, say, the year 1980, is there then a sense in which the presumption that $\tilde{k}_0 > k^*$ seems just as plausible as the opposite, i.e., $\tilde{k}_0 < k^*$?

From now we assume that the economy considered has been in steady state until time t_0. Then x unexpectedly shifts down to a lower constant level x'. Immediately after time t_0 the economic agents revise their expectations accordingly.

g) Using a phase diagram, illustrate how $\tilde{k}$ and $\tilde{c}$ evolve in the economy for $t \geq t_0$. As for $\tilde{c}$, the sign of the immediate change cannot be determined without more information; why not? But the direction of the movement in the future can be determined unambiguously, why?

h) Show, using a time profile diagram, how $\tilde{k}$, $\tilde{c}$, r and w/T in the economy evolve for $t \geq t_0$.

i) What is the growth rate of output per worker and the real wage, respectively, in the long run? Are these growth rates overall diminishing or increasing in the adjustment process towards the new steady state? Give a reason for your answer.

I.13 *Stocks versus flows* Two basic elements in growth models are often presented in the following way. The aggregate production function is described by

$$Y = F(K, L, T), \tag{1}$$

where Y is output, K is capital input, L is labour input and T is the "level of technology". And accumulation of the stock of capital in the (closed) economy is described by

$$\dot{K} = \frac{dK}{dt} = Y - C - \delta K, \tag{2}$$

where δ is the (exogenous) rate of (physical) depreciation of capital.

a) What dimensions should be attached to output, capital input and labour input in a production function?

b) What is the dimension attached to K in the accumulation equation?

c) Is there any consistency problem in the notation? Explain.

d) Suggest an interpretation that ensures that there is no consistency problem.

e) Suppose there are two countries. They have the same technology and the same number of man-hours per worker per year. Country A does not use shift work, but country B uses shift work, two work teams per day. Adapt the formula (1) so that it can be applied to both countries.

f) Suppose F is a neoclassical production function with CRS w.r.t. K and L and that at time t, the two countries have the same capital stock and the same labour force. Compare their output levels. Comment.

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