Written exam for the M. Sc. in Economics Summer 2004

Economic Growth (Videregående vækstteori)

June 7, 2004

Four hours. No auxiliary material

To be answered in Danish or English[1]

(The problem set is the same whether the student is registered

for the 2003 or 2004 syllabus)

The weighting of the problems is:

Problem 1: 40 %, Problem 2: 30 %, Problem 3A *or* 3B: 20 %, Problem 4: 10 %. It is optional whether to solve Problem 3A or Problem 3B. Problems 1, 2 and 4 are mandatory.

Problem 1. Consider a closed market economy with L utility maximizing households and M profit maximizing firms, operating under perfect competition (L and M are constant, but "large"). There is also a government, supplying a non-rival productive service G per time unit. Each household supplies inelastically one unit of labour per time unit. Aggregate output is Y per time unit, and output is used for private consumption, $C \equiv cL$, the public productive service G and investment I in (physical) capital, i.e., $Y = C + G + I$. The stock of capital K changes according to $\dot{K} = I - \delta K$, where $\delta \geq 0$ is the rate of physical decay of capital. Variables are dated implicitly; t denotes time. The initial value $K(0) > 0$ is given. The capital stock in society is owned, directly or indirectly, by the households. There is perfect competition at the labour market, and the equilibrium real wage is called w. There is a perfect market for loans at the real rate of interest r, and there is no uncertainty. A dot over a variable denotes the time derivative.

The government chooses G so that

$$\frac{G}{Y} = \bar{g}, \tag{*}$$

where the constant $\bar{g}$ is exogenous and is such that positive growth in the economy occurs in equilibrium. The government budget is always balanced. The service G is the only public expenditure, and the tax revenue is

$$\left[\tau(ra + w) + \tau_\ell\right] L = G, \tag{GBC}$$

where a is per capita financial wealth, and τ and τ_ℓ denote the income tax rate and a lumpsum tax, respectively. The tax rate τ is a given constant, whereas τ_ℓ is adjusted when needed for (GBC) to be satisfied.

The production function for firm i is

$$Y_i = AK_i^\alpha (GL_i)^{1-\alpha}, \quad 0 < \alpha < 1, A > 0, \quad i = 1, 2, ..., M.$$

[1] You are also allowed to write in Swedish or Norwegian.

a) Find the capital intensity $k_i \equiv K_i/L_i$ chosen by firm i (expressed as a function of r, δ and G).

b) Find the equilibrium level of aggregate output at some arbitrary point in time. *Hint:* You may use that $k_i = k \equiv K/L$ for all i (why?) to find an expression for Y in which (*) can be inserted to find G. NB! You should end up with Y expressed as a function of only predetermined variables, exogenous variables and parameters.

c) Find the equilibrium real rate of interest and the equilibrium real wage at some arbitrary point in time.

d) Suppose the households, all alike, have an intertemporal utility function with infinite horizon and a constant rate of time preference $\rho > 0$. The instantaneous utility function has elasticity of marginal utility equal to a constant $\theta > 0$. Derive the Keynes-Ramsey rule, given the described taxation system.

e) Write down the transversality condition in a form comparable to the No-Ponzi-Game condition of the household. Comment on the discount rate.

f) Find the growth rate of $k \equiv K/L$ and $y \equiv Y/L$ in this economy (an informal argument, based on your general knowledge, is acceptable). In case, you need to introduce a restriction on some parameters, do it.

g) Comment on the solution to f).

h) Suppose lumpsum taxation is not feasible. Hence, let $\tau_\ell = 0$ for all $t \geq 0$. Is it possible to fix τ at a level (constant over time) such that the government budget is still balanced in equilibrium for all $t \geq 0$?

i) If the welfare of the representative household is the criterion, what proposal to the government do you have w.r.t. the size of $\bar{g}$ and the *form* of taxation (given that taxation must be non-lumpsum)? *Hint:* There may be a problem with the tax candidate that first comes to one's mind.

Problem 2. There is again L households, described in the same way as in Problem 1. But there are no taxes, and the production side of the economy is different. There are two sectors, the "basic-goods sector" and the "innovative sector", which we call Sector 1 and Sector 2, respectively. There is no physical capital in the economy. Instead, households' financial wealth consists of shares in monopoly firms in Sector 2, supplying specialized intermediate goods. These goods are input in Sector 1, where the firms operate under perfect competition. Also the labour market has perfect competition. There is a market for loans at the real rate of interest r. There is no uncertainty. Variables are dated implicitly; t denotes time.

Firm i $(i = 1, 2, ..., M)$ in Sector 1 has the production function

$$Y_i = AL_i^{1-\alpha} \sum_{j=1}^{N} (x_{ij})^{\alpha}, \qquad A > 0,\ 0 < \alpha < 1.$$

Here Y_i, L_i and x_{ij} denote output of the firm, labour input and input of intermediate good j, respectively $(j = 1, 2, ..., N)$.

In Sector 2 R&D activity occurs. New "technical designs", that is, blueprints for making new specialized intermediate goods are invented. Ignoring indivisibility problems, we assume the number of new technical designs invented in the economy per time unit can be written

$$\dot{N} \equiv \frac{dN}{dt} = \beta R, \qquad \beta > 0,\ \beta \text{ constant.}$$

Here, R denotes the aggregate cost (per time unit) of R&D activity in terms of basic goods. For simplicity it is assumed that inventions can go in so many directions that the likelihood of different agents chasing and making the same invention is negligible.

After an invention has been made, the inventor begins supplying the new intermediate good. The inventor retains a perpetual monopoly over the production and sale of the invented intermediate good (say by concealment of the new technical design or by taking out a patent, which we, for simplicity, may assume free of charge). Once invented, an intermediate good of type j costs one basic good to produce and nothing else.

Aggregate output of basic goods, $Y \equiv \sum_{i=1}^{M} Y_i$, is used partly for consumption, $C \equiv cL$, partly for input in R&D activity and partly for input in the production of specialized intermediate goods. Hence, we have

$$Y = C + R + X,$$

where $X \equiv \sum_j \sum_i x_{ij}$.

a) Inventor j (firm j in Sector 2) earns a profit, π_j, per unit of time:

$$\pi_j = (\frac{1}{\alpha} - 1)X^m \equiv \pi^m,$$

where

$$X^m = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}.$$

Explain this result (you don't have to derive it formally).

b) Write down an expression for the market value $V(t)$ of firm j in Sector 2.

c) Find what r is in equilibrium with $\dot{N} > 0$. *Hint:* In equilibrium with $\dot{N} > 0$, $V(t)$ satisfies a simple relation. This can be combined with the no-arbitrage condition (or Fisher equation). Comment.

d) Find the rate of growth of c and N, respectively, in an equilibrium with $\dot{N} > 0$ (an informal argument, based on your general knowledge, is acceptable). In case, you need to introduce restrictions on some parameters, do it. How do A, β and L affect growth?

e) Find an expression for aggregate output of basic goods as a function of X, N and L and use this expression to explain why, from a technological point of view, the model is capable of generating endogenous growth?

f) Is it theoretically possible that preferences can nevertheless, in this framework, preclude growth? Be as specific and precise as you can in answering this question.

Problem 3A. *A government sector in the model of Problem 2.* Here we extend the model of Problem 2 with a government sector. Suppose there is a government that does two things. First, it employs a given constant fraction, s, of the labour force as civil servants, whose services do not affect marginal utility of private consumption; the wages of civil servants is the same as that of other workers. Second, to finance the wage payments to civil servants, the government levies a tax on monopoly profits at a constant rate τ, $0 < \tau < 1$, so that after-tax profit of inventor j is $(1-\tau)\pi_j$.

a) Assuming technology and preferences are consistent with an equilibrium with $\dot{N} > 0$, determine the growth rate of c and N in such an equilibrium. *Hint:* Employment in the basic-goods sector is now $L' = (1-s)L$.

b) Determine the equilibrium real wage in the economy, for given $s < \alpha/(1+\alpha)$.

c) Find the tax rate τ, such that the government budget is balanced.

d) The government sector affects growth through several channels. How?

Problem 3B. *Stochastic erosion of monopoly power.* Here we extend the model of Problem 2 by introducing uncertainty as to how long the monopoly position of an inventor lasts.

a) Sooner or later inventor j looses the monopoly (patent protection is only temporary, imitators find out how to make close substitutes). When this happens, intermediate good j becomes competitive, i.e., it is supplied in the amount

$$X^c = L(\alpha A)^{\frac{1}{1-\alpha}}.$$

Explain this result and comment on the size relation between X^c and X^m.

b) Suppose the erosion of monopoly power can be described by a Poisson process. That is, if T denotes the remaining lifetime of monopoly j, then the probability that $T > z$ is e^{-pz}, where $p > 0$ is a given Poisson intensity (the same for all monopolies). Further, the cessations of the different monopolies are stochastically independent. N is "large" so that by holding shares in many different firms, the households face no risk. The market value of monopoly j at time t can be written

$$\begin{aligned} V(t) &= E_t \int_t^\infty \pi_j(\tau) e^{-\int_t^\tau r(s)ds} d\tau \qquad (*) \\ &= \pi^m \int_t^\infty e^{-\int_t^\tau (r(s)+p)ds} d\tau, \qquad (**) \end{aligned}$$

where r is the rate of interest on safe loans. Explain this result, either just in words or by deriving it formally.

c) Determine r and the growth rate of c in equilibrium with $\dot{N} > 0$. *Hint:* In equilibrium with $\dot{N} > 0$, $V(t)$ satisfies a simple relation. This can be combined with the no-arbitrage condition (or Fisher equation), which with this kind of uncertainly is $(\pi^m + \dot{V} - pV)/V = r$.

d) Let N^c denote the number of intermediates that have become competitive and let $N^m \equiv N - N^c$. It can be shown that N^c/N approaches a constant $(N^c/N)^*$ over time. This constant is an increasing function of p, and so is Y in equilibrium. Explain the intuition behind these two features. Combined with the result from c), in what sense does this illustrate a classical dilemma of patent legislation?

Problem 4. *Short questions.*

a) "A relatively homogeneous group of countries such as for example the EU countries tend to experience income convergence in the sense that the standard deviation of income per capita across the countries diminishes over time." True or not true as an empirical statement? Explain.

b) "If there is 'learning-by-investing', standard growth accounting tend to understate total factor productivity growth." True or not true? Explain.

c) One of the weaknesses of models of the kind presented in Problem 2 is revealed by observing what would happen to the growth rate of the economy, if the labour force was growing. Briefly comment on this and mention some R&D-based models that do not share this weakness.