Chapter 23

The open economy and alternative exchange rate regimes

In this chapter we consider simple open-economy members of the IS-LM family. Section 23.1 revisits the standard static version, the Mundell-Fleming model, in its fixed-exchange-rate as well as floating-exchange-rate adaptations. The Mundell-Fleming model is well-known from elementary macroeconomics and our presentation of the model is merely a prelude to the next sections which address dynamic extensions. In Section 23.2 we show how the *dynamic* closed-economy IS-LM model with rational expectations from the previous chapter can be easily modified to cover the case of a small open economy with fixed exchange rates. In Section 23.3 we go into detail about the more challenging topic of floating exchange rates. In particular we address the issue of exchange rate *overshooting*, first studied by Dornbusch (1976).

Both the original Mundell-Fleming model and the dynamic extensions considered here are *ad hoc* in the sense that the microeconomic setting is not articulated in any precise way. Yet the models are useful and have been influential as a means of structuring thinking about an open economy in the short run.

The models focus on short-run mechanisms in a small open economy. There is a "domestic currency" and a "foreign currency" and these currencies are traded in the foreign exchange market. In this market, nowadays, the volume of trade is gigantic.

The following assumptions are shared by the models in (most of) this chapter:

1. Free mobility across borders of financial capital (i.e., no barriers or restrictions on currency trade).

2. Domestic and foreign bonds are perfect substitutes and hence command the same expected rate of return.

3. Free mobility across borders of goods and services (i.e., no barriers or restrictions on trade in goods and services)

4. No mobility across borders of labor.

5. Domestic and foreign goods are imperfect substitutes.

6. Nominal prices are sluggish and follow an exogenous constant inflation path.

The two first assumptions together make up the case of *perfect mobility of financial capital.* As the two last assumptions indicate, we consider an economy with imperfectly competitive firms. In an asynchronous way, the firms adjust their prices when their unit costs change. The aggregate inflation rate is considered sticky.

We use the same notation as in the previous chapter, with the following clarifications and additions: Y is domestic output (GDP), P the domestic price level, P^* the foreign price level, i the domestic short-term nominal interest rate, i^* the foreign short-term nominal interest rate, and X the nominal exchange rate. Suppose UK is the "home country". Then the exchange rate X indicates the price in terms of British£ (GBP) for one US$ (USD), say. Be aware that a currency trading convention is to announce an exchange rate as, for example, "USD/GBP is X", in words: "USD through GBP is X". The intended meaning is that the exchange rate is X GBP per USD. In ordinary language (as well as in mathematics) a slash, however, means "per". Thus, writing "USD/GBP is X" ought to mean that the exchange rate is X USD per GBP, which is exactly the opposite. Whenever in this text we use a slash, it has this standard mathematical meaning. To counter any risk of confusion, when indicating an exchange rate, we therefore avoid using a slash altogether. Instead we use the unmistakable "per".

When reporting that "the exchange rate is X", our point of view is that of an *importer* in the home country. That is, if UK is the "home country", saying that "the exchange rate is X" shall mean that X GBP must be paid per \$ worth of imports. And saying that "the real exchange rate is x" shall mean that x domestic goods must be paid per imported good. This convention is customary in continental Europe. Note however, that it is the opposite of the British convention which reports the home country's nominal and real "exchange rate" as $1/X$ and $1/x$, respectively.

When considering *terms of trade,* $1/x$, our point of view is that of an *exporter* in the home country. The terms of trade tell us how many foreign goods we get per exported good. In accordance with this, Table 21.1 gives a list of key open economy variables.

Table 21.1	**Open economy glossary**	
Term	*Symbol*	*Meaning*
Nominal exchange rate	X	The price of foreign currency in terms of domestic currency.
Real exchange rate	$x \equiv \frac{XP^*}{P}$	The price of foreign goods in terms of domestic goods (can be interpreted as an indicator of competitiveness).
Terms of trade	$1/x$	In this simple model terms of trade is just the inverse of the real exchange rate (generally, it refers to the price of export goods in terms of import goods).
Purchasing power parity		The nominal exchange rate which makes the cost of a basket of goods and services equal in two countries, i.e., makes $x = 1$.
Uncovered interest parity		The hypothesis that domestic and foreign bonds have the same expected rate of return, expressed in terms of the same currency.
Exports	E	
Imports	IM	
Net exports (in domestic output units)	N	$= E - xIM$.
Net foreign assets	A^f	
Net factor income from abroad (in domestic output units)	$rA^f + w^f L^f$	The present model has $L^f = 0$.
Current account surplus	CAS	$= N + rA^f + w^f L^f$. In this model $L^f = 0$.
Official reserve assets	ORA	
Private net foreign assets	A_p^f	$= A^f - ORA$.
Increase per time unit in some variable z	Δz	
Financial account surplus	FAS	$= -\Delta A_p^f - \Delta ORA = -CAS =$ current account deficit.
Net inflow of foreign exchange	$= CAS$	$= -FAS = -$ (net outflow of foreign exchange)

We simplify by talking of an exchange rate as if the "foreign country" constitutes the rest of the world and the exchange rate is thereby just a bilateral entity. A more precise treatment would center on the *effective exchange rate,* which is a trade-weighted index of the exchange rate vis-a-vis a collection of major trade partners.

23.1 The Mundell-Fleming model

Whether the Mundell-Fleming model is adapted to a fixed or floating exchange rate regime, there is a common set of elements.

23.1.1 The basic elements

Compared with the static closed-economy IS-LM model, the Mundell-Fleming model contains two new elements:

- An extra output demand component, namely a net export function $N(Y, x)$, where $x \equiv XP^*/P$ is the real exchange rate. As a higher income implies more imports, we assume that $N_Y < 0$. And as a higher real exchange rate implies better competitiveness, we assume that $N_x > 0$.[1]
- The uncovered interest parity condition (for short UIP). This says that domestic and foreign financial assets pay the same expected rate of return (measured in the same currency).

Apart from the addition of these open-economy elements, notation is as in the previous chapter. Output demand is given as

$$\begin{aligned} Y^d &= C(Y^p, r^e) + I(Y, r^e) + N(Y, x) + G + \varepsilon_D, \text{ where} \qquad (23.1) \\ 0 &< C_{Y^p} + N_Y < C_{Y^p} \leq C_{Y^p} + I_Y < 1, C_{r^e} + I_{r^e} \leq I_{r^e} < 0, N_x > 0, \end{aligned}$$

and ε_D is a demand shift parameter. This parameter could for instance reflect the level of economic activity in the world economy. Disposable income, Y^p, is

$$Y^p \equiv Y - \mathbb{T}, \qquad (23.2)$$

where $\mathbb{T}$ is real net tax revenue (gross tax revenue minus transfers). We assume a quasi-linear tax revenue function

$$\mathbb{T} = \tau + T(Y), \qquad 0 \leq T' < 1, \qquad (23.3)$$

[1] By assuming $N_x > 0$, it is presupposed that the Marshall-Lerner condition is satisfied, see Appendix A. Throughout the chapter we ignore that it may take one or two years for a rise in x to materialize as a rise in net exports. This is because the price of imports is immediately increased while the quantity of imports and exports only adjust with a time lag (the pattern known as the *J-curve effect*).

where τ is a constant representing the "tightness" of fiscal policy. This parameter, together with the level of public spending, G, describes fiscal policy.

Inserting (23.2) and (23.3) into (23.1), we can write aggregate demand as

$$\begin{aligned} Y^d &= D(Y, r^e, x, \tau) + G + \varepsilon_D, \quad \text{where} \qquad (23.4)\\ 0 &< D_Y = C_{Y^p}(1 - T') + I_Y + N_Y < 1, D_{r^e} = C_{r^e} + I_{r^e} < 0,\\ D_x &> 0, D_\tau = C_{Y^p} \cdot (-1) \in (-1, 0). \end{aligned}$$

The demand for money (domestic currency and checkable deposits in commercial banks) in the home country is, as in the closed economy model,

$$M^d = P \cdot (L(Y, i) + \varepsilon_L), \qquad L_Y > 0, \; L_i < 0, \tag{23.5}$$

where i is the short-term nominal interest rate on the *domestic bond* which is denominated in the *domestic* currency. The symbol ε_L represents a shift parameter which may reflect a shock to liquidity preferences or the payment technology and thereby the money multiplier.

There is a link between r^e and i, namely $r^e = i - \pi^e$, where π^e denotes the expected value of π which is the domestic forward-looking inflation rate. Recall that with continuous interest compounding, the equation $r^e = i - \pi^e$ is an identity. In a discrete time framework the equation is a convenient approximation. Assuming clearing in the output market as well as the money market, we now have:

$$\begin{aligned} Y &= D(Y, i - \pi^e, X\frac{P^*}{P}, \tau) + G + \varepsilon_D, \qquad \text{(IS)}\\ \frac{M}{P} &= L(Y, i) + \varepsilon_L. \qquad \text{(LM)} \end{aligned}$$

In addition to the domestic short-term bond there is a short-term bond denominated in *foreign* currency, henceforth the *foreign bond.* The nominal interest rate on the foreign bond is denoted i^* and is exogenous, $i^* > 0$. The term "bonds" may be interpreted in a broad sense, including large firms' interest-bearing bank deposits.

The no-arbitrage condition between the domestic and the foreign bond is assumed given by the *uncovered interest parity* condition,

$$i = i^* + \frac{\dot{X}^e}{X}, \tag{UIP}$$

where $\dot{X}^e$ denotes the expected increase per time unit in the exchange rate in the immediate future.[2] Imposing (UIP) amounts to assuming that arbitrage quickly

[2] As a preparation for the dynamic extensions to be considered below, we have presented the UIP condition in its continuous time version with continuous interest compounding. In discrete

brings the interest rate on the domestic bond in line with the expected rate of return on investing in the foreign bond, expressed in the domestic currency. This expected rate of return equals the foreign interest rate plus the expected rate of depreciation of the domestic currency.

By invoking the UIP condition the model assumes that asymmetric risk and liquidity aspects can be ignored in a first approximation. So domestic and foreign bonds are considered perfect substitutes. Hence only the expected rate of return matters. If we imagine that in the very short run there is, for example, a "$>$" in (UIP) instead of "$=$", then arbitrage sets in. A massive inflow of financial capital will occur (investors dispose of foreign assets and purchase domestic assets), until "$=$" in (UIP) is re-established. The adjustment will take the form of a lowering of i in case of a fixed exchange rate system. In case of a floating exchange rate system, the adjustment will take the form of an adjustment in X (generally both its level and subsequent rate of change). Only when (UIP) is satisfied, is the system at rest. The primary actors in the foreign exchange market are commercial banks, mutual funds, asset-management companies, insurance companies, exporting and importing corporations, and central banks.

The model assumes that (UIP) holds continuously (arbitrage in international asset markets is very fast). Thus, if for example the domestic interest rate is below the foreign interest rate, it must be that the domestic currency is expected to appreciate vis-à-vis the foreign currency, that is, $\dot{X}^e < 0$. The adjective "uncovered" refers to the fact that the return on the right-hand side of (UIP) is not guaranteed, but only an expectation. On the *covered interest parity*, see Appendix B.

The original Mundell-Fleming model is a static model describing just one short period with the price levels P and P^* set in advance. The model consists of the equations (IS), (LM), and (UIP) with the following partitioning of the variables:

- Exogenous: $P, P^*, G, \tau, \pi^e, i^*, \dot{X}^e, \varepsilon_D, \varepsilon_L$, and either X or M.
- Endogenous: Y, i, and either M or X, depending on the exchange rate regime.

We now consider the polar cases of fixed and floating exchange rates, respectively.

time with X_t denoting the exchange rate at the beginning of period t, the UIP condition reads: $1 + i_t = \frac{1}{X_t}(1 + i_t^*)X_{t+1}^e$, which can be written $1 + i_t = (1 + i_t^*)(1+ (X_{t+1}^e - X_t)/X_t)$ $\approx 1 + i_t^* + (X_{t+1}^e - X_t)/X_t$, when i_t^* and $(X_{t+1}^e - X_t)/X_t$ are "small".

23.1.2 Fixed exchange rate

A fixed exchange rate regime amounts to a promise by the central bank to sell and buy unlimited amounts of foreign currency at an announced exchange rate. So X becomes an exogenous constant in the model.[3] The system requires that the central bank keeps foreign exchange reserves to be able to buy the domestic currency on foreign exchange markets when needed to maintain its value.

We assume that the announced exchange rate is at a sustainable level vis-a-vis the foreign currency so that credibility problems can be ignored. So we let $\dot{X}^e = 0$. Then (UIP) reduces to $i = i^*$, and output is determined by (IS), given $i = i^*$. Finally, through movements of financial capital the nominal money supply (hence also the real money supply) adjusts endogenously to the level required by (LM), given $i = i^*$ and the value of Y already determined in (IS). The system thus has a recursive structure.

There is no possibility of an independent monetary policy as long as there are no restrictions on movements of financial capital. The intuition is the following. Suppose the central bank naively attempts to stimulate output by buying domestic bonds, thereby raising the money supply. There will be an incipient fall in i. This induces portfolio holders to convert domestic currency into foreign currency to buy foreign bonds and enjoy their higher interest rate. This tends to raise X_t, however. Assuming the central bank abides by its commitment to a fixed exchange rate, the bank will have to immediately counteract this tendency to depreciation by *buying domestic assets* (domestic currency and bonds) for foreign currency in an amount sufficient to bring the domestic money supply down to its original level needed to restore both the exchange rate and the interest rate at their original values. That is, as soon as the central bank attempts expansionary monetary policy, it has to reverse it.

The model is qualitatively the same as the static IS-LM model for the closed economy with the nominal interest rate fixed by the central bank. The output and money multipliers w.r.t. government spending are, from (IS) and (LM) respectively,

$$\begin{aligned}
\frac{\partial Y}{\partial G} &= \frac{1}{1-D_Y} = \frac{1}{1-C_{Y^p}(1-T')-I_Y-N_Y} > 1, \\
\frac{\partial M}{\partial G} &= PL_Y\frac{\partial Y}{\partial G} = PL_Y\frac{1}{1-C_{Y^p}(1-T')-I_Y-N_Y} > 0.
\end{aligned}$$

The output multiplier can be seen as a measure of how much a unit increase in G raises aggregate demand and thereby stimulates production and income; indeed,

[3] In practice there will be a small margin of allowed fluctuation around the par value. The Danish krone (DKK) is fixed at 746.038 DKK per 100 euro +/- 2.25 percent.

$\Delta Y \approx \partial Y/\partial G \cdot \Delta G = \partial Y/\partial G$ for $\Delta G = 1$. Thereby the transaction-motivated demand for money is increased. This generates an incipient tendency for both the short-term interest rate to rise and the exchange rate to appreciate as portfolio holders worldwide buy the currency of the SOE to invest in its bonds and enjoy their high rate of return. The central bank is committed to a fixed exchange rate, however, and has to prevent the pressure for a higher interest rate and currency appreciation by *buying foreign assets* (foreign currency and bonds) for domestic currency. When the money supply has increased enough to nullify the incipient tendency for a higher domestic interest rate, the equilibrium with unchanged exchange rate is restored. The needed increase in the money supply for a unit increase in G is given by $\Delta M \approx \partial M/\partial G \cdot \Delta G = \partial M/\partial G$ for $\Delta G = 1$. One may say that it is the accommodating money supply that allows the full unfolding of the output multiplier w.r.t. government spending. Nevertheless, owing to the *import leakage* ($N_Y < 0$), both the output and money multiplier are lower than the corresponding multipliers in the closed economy where the central bank maintains the interest rate at a certain target level. The system ends up with higher Y, the same i and X, and lower net exports because of higher imports.

The output multipliers w.r.t. a demand shock, an interest rate shock, and a liquidity preference shock, respectively, are

$$\begin{aligned}
\frac{\partial Y}{\partial \varepsilon_D} &= \frac{\partial Y}{\partial G} = \frac{1}{1 - C_{Y^p}(1-T') - I_Y - N_Y} > 1, \\
\frac{\partial Y}{\partial i^*} &= \frac{D_{r^e}}{1 - D_Y} = \frac{C_{r^e} + I_{r^e}}{1 - C_{Y^p}(1-T') - I_Y - N_Y} < 0, \\
\frac{\partial Y}{\partial \varepsilon_L} &= 0.
\end{aligned}$$

This last result reflects that after the liquidity preference shock, the money market equilibrium is restored by full adjustment of the money supply ($dM = Pd\varepsilon_L$) at unchanged interest rate.

23.1.3 Floating exchange rate

In a floating exchange rate regime, also called a flexible exchange rate regime, the exchange rate is allowed to respond endogenously to the market forces, supply and demand, in the foreign exchange market. The model treats the money stock, M, as exogenous. More precisely, the money supply is targeted by the central bank and the model assumes this is done successfully. The exchange rate adjusts so that the available supplies of money and domestic bonds are willingly held.

In the present static portrait of the floating exchange rate regime, $\dot{X}^e$ is treated as exogenous. Following Mundell (1963), we imagine that the system has settled down in a steady state with $\dot{X}^e = 0$. The model is again recursive. First (UIP) yields $i = i^*$. Then output is determined by (LM), given $i = i^*$. And finally the required exchange rate is determined by (IS) for a given level of P^*/P. So the story behind the equilibrium described by the model is that the exchange rate has adjusted to a level such that aggregate demand and output is at the point where, given the real money supply, the transactions-motivated demand for money establishes clearing in asset markets for a nominal interest rate equal to the foreign nominal interest rate.

There is no possibility of *fiscal policy* affecting output as long as there are no restrictions on movements of financial capital. The interpretation is the following. Consider an expansive fiscal policy, $dG > 0$ or $d\tau < 0$. The incipient output stimulation increases the transaction demand for money and thereby the interest rate. The rise in the interest rate is immediately counteracted, however, by inflow of foreign exchange induced by the high interest rate. This inflow means higher demand for the domestic currency, which thereby appreciates, thus lowering competitiveness and net exports. The appreciation continues until competitiveness has decreased enough[4] to bring the interest rate back to its initial level. This state of affairs is obtained when the exchange rate has reached a level at which the fall in net exports matches the rise in G or fall in τ, thereby bringing aggregate demand and output back to their initial levels. In effect, the system ends up with unchanged output and interest rate, a lower exchange rate, X, and lower net exports.[5]

On the other hand, *monetary policy* is effective. An increase in the money supply (through an open-market operation) generates an incipient fall in the interest rate. This triggers a counteracting outflow of financial capital, whereby the domestic currency depreciates, i.e., X rises. The depreciation continues until the real exchange rate has increased enough to induce a rise in net exports and output large enough for the transaction demand for money to match the larger money supply and leave the interest rate at its original level. The system ends up with higher Y, the same i, higher X, and higher net exports.[6]

From (LM) and (IS), respectively, we find the output and exchange rate mul-

[4] Recall that, as we have defined the exchange rate, "up is down and down is up" or, perhaps with a little more transparency, "currency up is exchange rate down".

[5] This canonical result relies heavily on the idealized assumption that foreign and domestic bonds are perfect substitutes and move without restraint of any kind.

[6] The implicit assumption that a higher X does not affect the price level P is of course problematic. If intermediate goods are an important part of imports, then a higher exchange rate would imply higher unit costs of production. And since prices tend to move with costs, this would imply a higher price level. If imports consist primarily of final goods, however, it is easier to accept the logic of the model.

tipliers w.r.t. the money supply:

$$\frac{\partial Y}{\partial M} = \frac{1}{PL_Y} > 0, \tag{23.6}$$

$$\frac{\partial X}{\partial M} = \frac{1 - C_{Y^p}(1-T') - I_Y - N_Y}{D_x P^* L_Y} > 0. \tag{23.7}$$

Finally, the output multipliers w.r.t. a demand shock and a liquidity preference shock, respectively, are

$$\frac{\partial Y}{\partial \varepsilon_D} = \frac{\partial Y}{\partial G} = 0,$$

$$\frac{\partial Y}{\partial \varepsilon_L} = -\frac{1}{L_Y} < 0.$$

Like increased public spending, a positive output demand shock does not affect output. It is neutralized by appreciation of the domestic currency. A positive liquidity preference shock reduces output. The mechanism is that the increase in money demand triggers an incipient rise in the domestic interest rate. The concomitant appreciation of the currency, resulting from the induced inflow of financial capital, reduces net exports.

23.1.4 Perspectives

The message of this simple model is that for a small open economy, a fixed exchange rate regime is better at stabilizing output if money demand shocks dominate, and a floating exchange rate system is better if most shocks are output demand shocks. In any case, there is an asymmetry. Under a fixed exchange rate system, two macroeconomic short-run policy instruments are given up: exchange rate policy and monetary policy. Under a floating exchange rate system and perfect capital mobility, only one macroeconomic short-run policy instrument is given up: fiscal policy. Yet, the historical experience seems to be that an international system of floating exchange rates ends up with higher volatility in both real and nominal exchange rates than one with fixed exchange rates (Mussa, 1990; Obstfeld and Rogoff, 1996; Basu and Taylor, 1999).

The result that fiscal policy is impotent in the floating exchange rate regime does not necessarily go through in more general settings. For example, in practice, domestic and foreign financial claims may often *not* be perfect substitutes. Indeed, the UIP hypothesis tends to be empirically rejected at short forecast horizons, while it does somewhat better at horizons longer than a year (see the literature notes at the end of the chapter). And as already hinted at, treating the price level as exogenous when import prices change is not satisfactory.

Anyway, even the static Mundell-Fleming model provides a basic insight: the *impossible trinity.* A society might want a system with the following *three* characteristics:

- free mobility of financial capital (to improve resource allocation);
- independent monetary policy (to allow a stabilizing role for the central bank);
- fixed exchange rate (to avoid exchange rate volatility).

But it can have only two of them. A fixed exchange rate system is incompatible with the second characteristic. And a flexible exchange rate system contradicts the third.

In the next sections we extend the model with dynamics and rational expectations. We first consider the fixed exchange rate regime, next the flexible exchange rate regime.

23.2 Dynamics under a fixed exchange rate

We ignore the shift parameters ε_D and ε_L. On the other hand we introduce an additional asset, a long-term bond that is indexed w.r.t. domestic inflation. In the fixed exchange rate regime this extension is easy to manage. In addition we assume rational expectations. The formal structure of the model then becomes exactly the same as that of the dynamic IS-LM model for a closed economy with short- and long-term bonds, studied in the previous chapter.

With R_t denoting the real long-term interest rate at time t (defined as the internal real rate of return on an inflation indexed consol), aggregate demand is

$$\begin{aligned} Y_t^d &= C(Y_t^p, R_t) + I(Y_t, R_t) + N(Y_t, x) + G \equiv D(Y_t, R_t, x, \tau) + G, \\ & \quad \text{where} \\ 0 &< D_Y = C_{Y^p}(1 - T') + I_Y + N_Y < 1, D_R = C_R + I_R < 0, D_x > 0, \\ -1 &< D_\tau = -C_{Y^p} < 0, \end{aligned}$$

where x is the real exchange rate, XP_t^*/P_t, with X representing the given and constant nominal exchange rate and the price ratio P_t^*/P_t assumed constant. The latter assumption is equivalent to assuming the domestic inflation rate to equal the foreign inflation rate for all t. Moreover, this common inflation rate is assumed equal to a constant, π.

To highlight the dynamics between fast-moving asset markets and slower-moving goods markets, the model replaces (IS) by the error-correction specification

$$\dot{Y}_t \equiv \frac{dY_t}{dt} = \lambda(Y_t^d - Y_t) = \lambda(D(Y_t, R_t, x, \tau) + G - Y_t), \qquad Y_0 > 0 \text{ given}, \quad (23.8)$$

where $\lambda > 0$ is the constant adjustment speed. Because changing the level of production is time consuming, Y_0 is historically given.

We assume the fixed exchange rate policy is sustainable. That is, the level of X is such that no threatening cumulative current account deficits in the future are glimpsed. In view of rational expectations we then have $\dot{X}_t^e = 0$ for all $t \geq 0$. In effect the uncovered interest parity condition reduces to

$$i_t = i^*, \quad (23.9)$$

where the exogenous foreign interest rate, i^*, is for simplicity assumed constant.

The remaining elements of the model are well-known from Chapter 22:

$$\frac{M_t}{P_t} = L(Y_t, i^*), \qquad L_Y > 0,\ L_i < 0. \quad (23.10)$$

$$R_t = \frac{1}{q_t}, \quad (23.11)$$

$$\frac{1 + \dot{q}_t^e}{q_t} = r_t^e, \quad (23.12)$$

$$r_t^e \equiv i^* - \pi_t^e, \qquad \pi_t \equiv \frac{\dot{P}_t}{P_t}, \quad (23.13)$$

$$P_t = P_0 e^{\pi t}, \quad (23.14)$$

where q is the real price of the long-term bond (the consol), the superscript e denotes expected value. If a shock occurs, it fits intuition best to interpret the time derivatives in (23.8), (23.12), and (23.13) as right-hand derivatives, e.g., $\dot{Y}_t \equiv \lim_{\Delta t \to 0^+}(Y(t + \Delta t) - Y(t))/\Delta t$. The variables τ, G, i^*, x, P_0, and π are exogenous constants. The first five of these are positive, and we assume $\pi < i^*$.

As there is no uncertainty in this model (no stochastic elements), the assumption of rational expectations amounts to perfect foresight. We thus have $\dot{q}_t^e = \dot{q}_t$ and $\pi_t^e = \pi$ for all t. Therefore, the equations (23.13) and (23.9) imply $r_t^e = r_t$ $= i^* - \pi > 0$ for all t. Combining this with (23.11) and (23.12), we end up with

$$\dot{R}_t = (R_t - i^* + \pi)R_t. \quad (23.15)$$

Assuming no speculative bubbles, the no-arbitrage condition (23.12) is equivalent to a saying that the consol has market value equal to its *fundamental value*:

$$q_t = \int_t^\infty 1 \cdot e^{-\int_t^s r_\tau d\tau} ds, \qquad \text{so that} \tag{23.16}$$
$$R_t = \frac{1}{q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} = \int_t^\infty \frac{e^{-\int_t^s r_\tau d\tau}}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} r_s ds.$$

In other words: the long-term rate, R_t, is an average of the (expected) future short-term rates, r_τ, with weights proportional to the discount factor $e^{-\int_t^s r_\tau d\tau}$, cf. the appendix of Chapter 22..

The evolution of the economy over time is described by the two differential equations, (23.8) and (23.15), in the endogenous variables, Y_t and R_t. Since the exchange rate X is an exogenous constant, the UIP condition is upheld by movements of financial capital providing the needed continuous adjustment of the endogenous money supply so as to satisfy $M_t = P_t L(Y_t, i^*) = P_0 e^{\pi t} L(Y_t, i^*)$, in view of (23.10) and (23.14). It is presupposed that the central bank keeps foreign exchange reserves to be able to buy the domestic currency on foreign exchange markets when needed to maintain its value.

The dynamics is essentially the same as that of a closed economy with a target short-term interest rate fixed by the central bank. In a phase diagram the $\dot{R} = 0$ locus is horizontal and coincides with the saddle path. In the absence of speculative bubbles and expected future changes in i^*, we thus get $R_t = r_t = i^* - \pi > 0$ for all $t \geq 0$. The only difference compared with the closed economy is that the short-term interest rate is not a policy variable any more, but an exogenous variable given from the world financial market.

As an example of an adjustment process, consider a fiscal tightening (increase in τ or decrease in G). This will immediately decrease output demand. Thereby output gradually falls to a new lower equilibrium level. The fall in output implies lower money demand because the amount of money-mediated transactions becomes lower. The lower money demand generates an incipient tendency for the short-term interest rate to fall and the domestic currency to depreciate. This tendency is immediately counteracted, however. To take advantage of a higher foreign interest rate, portfolio holders worldwide convert home currency into foreign currency at the given exchange rate in order to buy foreign bonds. Owing to its commitment to a fixed exchange rate, the central bank now intervenes by selling foreign currency and domestic bonds. As soon as i is restored at its original value, i^*, the downward pressure on the value of the domestic currency is nullified. By assumption, the price level stays on the time path (23.14), whereby r and R remain essentially unaffected and equal to the constant $i^* - \pi$ during the output contraction. The figures 20.12 and 20.13 of the previous chapter illustrate.

Another kind of demand shock is a shift in the exports demand due to, say, a reduced economic growth in the world economy.

23.3 Dynamics under a floating exchange rate: overshooting

The exogeneity – and in fact absence – of expected exchange rate changes in the static Mundell-Fleming model of a floating exchange rate regime is unsatisfactory. By a dynamic approach we can open up for an endogenous and time-varying $\dot{X}^e$.

The floating exchange rate regime requires one more differential equation compared to the fixed exchange rate system. To avoid the complexities of a three-dimensional dynamic system, we therefore simplify along another dimension by dropping the distinction between short-term and long-term bonds. Hence, output demand is again described as in (23.1) and depends negatively on the expected short-term real interest rate, r^e. We ignore the disturbance term ε_D.

Apart from the exchange rate now being endogenous, money supply exogenous, and long-term bonds absent, the model is similar to that of the previous section. At the same time the model is close to a famous contribution by the German-American economist Rudiger Dornbusch (1942-2002), who introduced forward-looking rational expectations into a floating exchange rate model (Dornbusch, 1976). Dornbusch thereby showed that exchange rate "overshooting" could arise. This was seen as a possible explanation of the rise in both nominal and real exchange rate volatility during the 1970s after the demise of the Bretton-Woods system. In his original article, Dornbusch wanted to focus on the dynamics between fast moving asset prices and sluggishly changing goods prices. He assumed output to be essentially unchanged in the process. In the influential Blanchard and Fischer (1989) textbook this was modified by letting output adjust gradually to spending, while goods prices were in the short run simply unaffected by demand shifts. This seems a more apt approximation, since the empirics tell us that in response to demand shifts, output moves faster than goods prices. We follow this approach and name it the *Blanchard-Fischer version* of Dornbusch's overshooting model.

23.3.1 The model

This modified Dornbusch model has three building blocks. The first building block is the output error-correction process,

$$\begin{aligned} \dot{Y}_t &= \lambda(Y_t^d - Y_t), \qquad \text{where} \\ Y_t^d &= D(Y_t, r_t^e, x_t, \tau) + G, \end{aligned} \tag{23.17}$$