a known client and within intricate legal requirements. It is time-consuming to design, contract, and execute the sequential steps involved in residential construction. Careful guidance and monitoring is needed. These features give rise to fixed costs (to management, architects etc.) and thereby rising marginal costs in the short run. Congestion and bottlenecks may easily arise.

The construction process

Assume the construction industry is competitive. At time t the *representative construction firm* produces B_t units of housing per time unit (B for "building"), thereby increasing the aggregate housing stock according to

$$\dot{H}_t = B_t - \delta H_t, \delta > 0. \tag{15.54}$$

The construction technology is described by a production function $\tilde{F}$:

$$B_t = \tilde{F}(K_t, L_t, \bar{M}; E_t) \equiv \bar{F}(F(K_t, L_t), \bar{M}; E_t) = \bar{F}(I_t, \bar{M}; E_t) \equiv T(I_t; E_t).$$

The last argument of $\tilde{F}$, E_t, is not a production factor but stands for construction experience acquired through accumulated learning in the construction industry. It determines the efficiency of the current technology. The three other arguments of $\tilde{F}$ represent input of capital, K_t, blue-collar labor, L_t, and "management labor", $\bar{M}$, which includes working hours of specialists like architects and lawyers. There are constant returns to scale with respect to these three production factors. We treat $\bar{M}$ as a fixed production factor even in the medium run. Hence the associated fixed cost (salaries) is, in real terms, constant for quite some time. We denote this fixed cost $\bar{f}$.

The remaining two production inputs, capital and blue-collar labor, produces components for residential construction − intermediate goods − in the amount $I_t = F(K_t, L_t)$ per time unit. The production function, F, is "nested" in the "global" production function, $\bar{F}$. Thus construction is modeled as if it makes up a two-stage process. First, capital and blue-collar labor produce intermediate goods for construction. Next, management accomplishes quality checks and "assembling" of these intermediate goods into new houses or at least final new components built into existing houses. The final output is measured in units corresponding to a standard house. This does not rule out that a large part of the output is really in the form of renovations, additions of a room etc.

We treat both blue-collar labor and capital as variable production factors in the short run and assume F has constant returns to scale. The intermediate goods are produced on a routine basis at minimum costs (convex *capital* adjustment costs, as in Chapter 14, are for simplicity ignored). Let the real cost per unit of I_t be denoted c. In our short-to-medium run perspective we treat c as a constant.

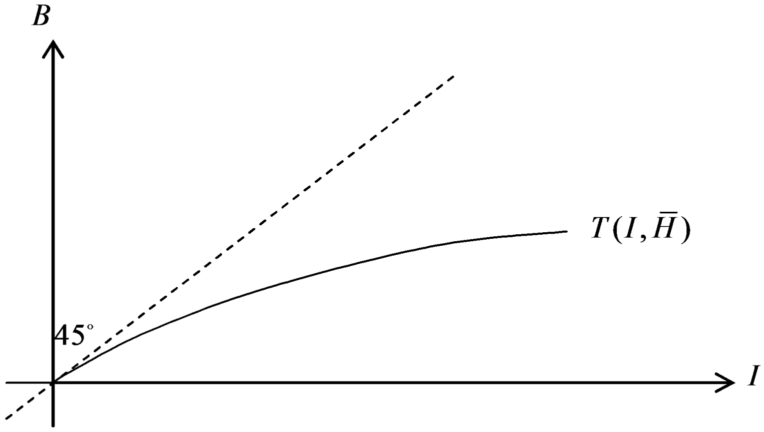

Figure 15.7: The number of new houses as a function of residential investment (for given $E = \bar{H}$).

The marginal productivity of I_t is decreasing in I_t. That is, keeping $\bar{M}$ fixed, the final output, B_t, has *diminishing* returns with respect to the level of *construction activity* per time unit as measured by the flow variable I_t. In the short run thus rising marginal costs obtains, "haste is waste".

To save notation, from now on, with the purpose of suppressing the constant argument $\bar{M}$, we introduce the production function T. Moreover, we suppress the explicit dating of the variables unless needed for clarity. To help intuition, we shall speak of the function T as a *transformation function.* This function is assumed to be strictly concave in I : the larger is I, the smaller is the rate at which a unit increase in I is transformed into new houses.

To summarize: the amount of new houses built per time unit is

$$\begin{aligned} B &= T(I, E), \quad \text{where} \\ T(0, E) &= 0, \quad T_I(0, E) = 1, \quad T_I > 0, \quad T_{II} < 0, \quad T_E \geq 0. \end{aligned} \tag{15.55}$$

A higher level of construction activity per time unit means that a larger fraction of I is "wasted" because of control, coordination, and communication difficulties. Hence $T_{II}(I, E) < 0$, i.e.,.T is strictly concave in I

The second argument in the transformation function is the construction experience, E. More experience means that the intermediate goods can be designed in a better way thus implying higher productivity of a given I than otherwise, hence $T_E \geq 0$.[13] As an indicator of cumulative experience it would be natural to use

[13]In a long-run perspective, the increasing scarcity of available land may hamper the productivity of the intermediate goods, for given I and E. This is ignored in our medium-run perspective. All the same, in the real world construction technology improves over time and the limited availability of land can to some extent be dealt with by building taller structures.

cumulative gross residential investment, $\int_{-\infty}^{t} B_s ds$, reflecting cumulative learning by doing. It is simpler, however, to use cumulative *net* residential investment, H_t. We thus assume that E_t is (approximately) proportional to H_t.[14] Normalizing the factor of proportionality to one, we have

$$E_t = H_t.$$

For fixed $E = \bar{H}$, Fig. 15.7 shows the graph of $T(I, \bar{H})$ in the $(I,\ B)$ plane. The assumptions $T_I(0, \bar{H}) = 1$ and $T_{II} < 0$ imply $T_I(I, \bar{H}) < 1$ for $I > 0$, as visualized in the figure. An example satisfying all the conditions in (15.55) is a CES function,[15]

$$T(I, H) = A(aI^{\beta} + (1-a)H^{\beta})^{1/\beta}, \quad \text{with } 0 < A < 1, 0 < a < 1, \text{ and } \beta < 0.$$

From the perspective of Tobin's q-theory of investment, we may let the "waste" be represented by a kind of adjustment cost function $G(I, H)$ akin to that considered in Chapter 14. Then $T(I, H) \equiv I - G(I, H)$. In Chapter 14 convex adjustment costs were associated with the installation of firms' fixed capital and acted as a reduction in the firms' output available for sale. In construction we may speak of analogue costs acting as a reduction in the productivity of the intermediate goods in the construction process. It is easily seen that, on the one hand, all the properties of G *required* in Chapter 14 when $I \geq 0$ are maintained. On the other hand, not all properties required of T in (15.55) need be satisfied in Tobin's q-theory (see Appendix B).

Profit maximization

The representative construction firm takes the current economy-wide experience $E = H$ as given. The gross revenue of the firm is pB and costs are cI. Given the market price p, the firm maximizes profit:

$$\begin{aligned} \max_I \Pi &= pB - cI \qquad \text{s.t.} \qquad B = T(I, H) \text{ and} \\ I &\geq 0. \end{aligned}$$

Inserting $B = T(I, H)$, we find that an interior solution satisfies

$$\frac{d\Pi}{dI} = pT_I(I, H) - c = 0, \text{ i.e., } \frac{p}{c} T_I(I, H) = 1. \tag{15.56}$$

[14] At least in an economic growth context, where H would almost never be decreasing, this approximation of the learning effect would not seem too coarse.

[15] As shown in the appendix to Chapter 4, by defining $T(I, H) = 0$ when $I = 0$ or $H = 0$, the domain of the CES function can be extended to include all $(I, H) \in \mathbb{R}_+^2$ also when $\beta < 0$, while maintaining continuity.

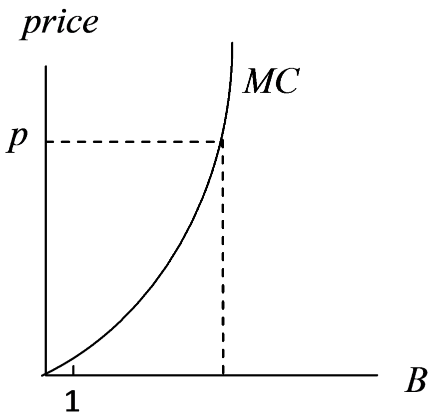

Figure 15.11: Marginal costs in house construction (housing stock given).

C. Interpreting construction behavior in a marginal cost perspective (Section 15.2.2)

We may look at the construction activity of the representative construction firm from the point of view of increasing marginal costs in the short run. First, let $\mathcal{TC}$ denote the total costs per time unit of the representative construction firm. We have $\mathcal{TC} = \bar{f} + \mathcal{TVC}$, where $\bar{f}$ is the fixed cost to management and $\mathcal{TVC}$ is the total variable cost associated with the construction of B $(= T(I, H))$ new houses per time unit, given the economy-wide stock H. All these costs are measured in real terms. We have $\mathcal{TVC} = cI$. The input of intermediates, I, required for building B new houses per time unit is an increasing function of B. Indeed, the equation

$$B = T(I, H), \tag{*}$$

where $T_I > 0$, defines I as an implicit function of B and H, say $I = \varphi(B, H)$. By implicit differentiation in (*) we find

$$\varphi_B = \partial I/\partial B = 1/T_I(\varphi(B, H), H) > 1, \quad \text{when } I > 0$$

So $\mathcal{TVC} = cI = c\varphi(B, H)$, and short-run marginal cost is

$$\mathcal{MC}(c, B, H) = \frac{\partial \mathcal{TVC}}{\partial B} = c\varphi_B = \frac{c}{T_I(\varphi(B, H), H)} > c, \quad \text{when } I > 0. \tag{**}$$

CLAIM
(i) The short-run marginal cost, $\mathcal{MC}$, of the representative construction firm is increasing in B.
(ii) The construction sector produces new houses up the point where $\mathcal{MC} = p$.
(iii) The cost of building one new house per time unit is approximately c.

Proof. (i) By (**) and (*),

$$\frac{\partial \mathcal{MC}}{\partial B} = \frac{-cT_{II}(\varphi(B,H),H)\varphi_B}{T_I(\varphi(B,H),H)^2} = \frac{-cT_{II}(\varphi(B,H),H)}{T_I(\varphi(B,H),H)^3} > 0,$$

since $T_I > 0$ and $T_{II} < 0$. (ii) Follows from (**) and the first-order condition (15.56) found in the text. (iii) The cost of building ΔB, when $B = 0$, is $\mathcal{MC}(c, \Delta B, H) \approx [c/T_I(0,H)] \cdot \Delta B = c\Delta B = c$ when $\Delta B = 1$, where we have used (**). □

That it is profitable to produce new houses up the point where $\mathcal{MC} = p$ is illustrated in Fig. 15.11.

D. Solving the no-arbitrage equation for p_t in the absence of house price bubbles (Section 15.2.4)

By definition, if there are no housing bubbles, the market price of a house equals its *fundamental value*, i.e., the present value of expected (possibly imputed) after-tax rental income from owning the house. Denoting the fundamental value $\hat{p}_t$, we thus have

$$\begin{aligned}\hat{p}_t &= (1-\tau_R)\int_t^\infty R(H_s)e^{-(\tau_p+\delta)(s-t)}e^{\tau_R\delta(s-t)}e^{-(1-\tau_r)r(s-t)}ds, \quad (15.64)\\ &= (1-\tau_R)\int_t^\infty R(H_s)e^{-[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p](s-t)}ds,\end{aligned}$$

where the three discount rates appearing in the first line are, first, $\tau_p + \delta$, which reflects the rate of "leakage" from the investment in the house due to the property tax and wear and tear, second, $\tau_R\delta$, which reflects the tax allowance due to wear and tear, and, finally, $(1-\tau_r)r$, which is the usual opportunity cost discount. In the second row we have done an addition of the three discount rates so as to have just one discount factor easily comparable to the discount factor appearing below.

In Section 15.2.4 we claimed that in the absence of housing bubbles, the linear differential equation, (15.61), implied by the no-arbitrage equation (15.53) under perfect foresight, has a solution p_t equal to the fundamental value of the house, i.e., $p_t = \hat{p}_t$. To prove this, we write (15.61) on the standard form for a linear differential equation,

$$\dot{p}_t + ap_t = -(1-\tau_R)R(H_t), \quad (15.65)$$

where

$$a \equiv -\left[(1-\tau_r)r + (1-\tau_R)\delta + \tau_p\right] < 0. \quad (15.66)$$

The general solution to (15.65) is

$$p_t = \left(p_{t_0} - (1-\tau_R) \int_{t_0}^{t} R(H_s) e^{a(s-t_0)} ds \right) e^{-a(t-t_0)}.$$

Multiplying through by $e^{a(t-t_0)}$ gives

$$p_t e^{a(t-t_0)} = p_{t_0} - (1-\tau_R) \int_{t_0}^{t} R(H_s) e^{a(s-t_0)} ds.$$

Rearranging and letting $t \to \infty$, we get

$$p_{t_0} = (1-\tau_R) \int_{t_0}^{\infty} R(H_s) e^{a(s-t_0)} ds + \lim_{t\to\infty} p_t e^{a(t-t_0)}.$$

Inserting (15.66), replacing t by T and t_0 by t, and comparing with (15.64), we see that

$$p_t = \hat{p}_t + \lim_{T\to\infty} p_T e^{-[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p](T-t)}.$$

The first term on the right-hand side is the fundamental value of the house at time t. The second term on the right-hand side thus amounts to the difference between the market price of the house and its fundamental value. By definition, this difference represents a bubble. In the absence of the bubble, the market price, p_t, therefore coincides with the fundamental value.

On the other hand, we see that a bubble being present requires that

$$\lim_{T\to\infty} p_T e^{-[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p](T-t)} > 0.$$

In turn, this requires that the house price is explosive in the sense of ultimately growing at a rate not less than $(1-\tau_r)r + (1-\tau_R)\delta + \tau_p$. The candidate for a bubbly path ultimately moving North-East portrayed in Fig. 15.9 in fact has this property. Indeed, by (15.61), for such a path we have

$$\dot{p}_t/p_t = [(1-\tau_r)r + (1-\tau_R)\delta + \tau_p] - (1-\tau_R)R(H_t)/p_t \to (1-\tau_r)r + (1-\tau_R)\delta + \tau_p \text{ for } t \to \infty,$$

since $p_t \to \infty$ and $R'(H_t) < 0$.

15.5 Exercises

(15.61)