Written re-exam for the M. Sc. in Economics, Winter 2016-17

Advanced Macroeconomics

Master's course

February 15, 2017

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 5 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.The weighting of the problems is:

Problem 1: 55 %, Problem 2: 35 %, Problem 3: 10 %.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 The Blanchard OLG model for a closed economy is described by the two differential equations

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{\lambda + b}{b}\tilde{c}_t - (\delta + g + b - m)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given,} \tag{*}$$

$$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho + \lambda - g\right]\tilde{c}_t - b(\rho + m)\tilde{k}_t, \tag{**}$$

and the condition that for any fixed pair (t_0, v), where $t_0 \geq 0$ and $v \leq t_0$,

$$\lim_{t\to\infty} a_{t,v} e^{-\int_{t_0}^{t}(f'(\tilde{k}(s))-\delta+m)ds} = 0. \tag{***}$$

Notation: $\tilde{k}_t \equiv K_t/(T_tL_t)$ and $\tilde{c}_t \equiv C_t/(T_tN_t) \equiv c_t/T_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively; N_t is population, L_t is labor supply, and T_t is the technology level, all at time t; f is a production function in intensive form, satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$, and the Inada conditions. Finally, $a_{t,v}$ is financial wealth at time t of an individual born at time v. The remaining symbols stand for parameters and we assume all these are strictly positive. Furthermore, $\rho \geq b - m \geq 0$ and $\lambda < \delta + \rho + g$.

a) Briefly interpret (*), (**), and (***), including the parameters.

b) Derive a phase diagram and illustrate the path the economy will follow, given some arbitrary positive initial value of $\tilde{k}$. Briefly comment on why you think the economy will follow the path you have indicated.

The model entails a simple theory of the rate of return in the long run, r^*. The model implies for instance that r^* must be above a certain value defined by the parameters ρ, λ, and g.

c) By appealing to the phase diagram you have already drawn, indicate what this value must be.

d) There are at least two respects in which the Blanchard OLG model gives different conclusions compared with a representative agent model. Briefly account for this.

Since Lawrence Summers' speech at IMF's 2013 Annual Research Conference there has been a debate and a concern about the worrying prospect of "secular stagnation" (a better name might be "lasting stagnation", in Danish "varig stagnation"). At least three simultaneous empirical circumstances have been in focus: 1) the downward trend in the demographic parameters b, m, and $n \equiv b - m$ (whereby b has gone even more down than m); 2) the downward trend in real interest rates in the last three decades; and 3) the event of a binding zero lower bound, ZLB, on the nominal policy rate coming increasingly into sight, first in Japan, then in the US, and subsequently also in the Eurozone, and the alarming duration of this state of affairs.

Let us first, in e) and f), check whether the present model envisages any connection between 1) and 2).

e) How will a lower b affect r^* in the present model? Why? You are only supposed to make a comparative analysis, considering b as a shift parameter. *Hint:* Standard curve shifting in the phase diagram does not work here, but another graphical argument is possible based on the fact that in steady state the equations for the $\dot{\tilde{k}}_t = 0$ and $\dot{\tilde{c}}_t = 0$ loci are simultaneously satisfied; elimination of $\tilde{c}$ gives an equation in $\tilde{k}$ which can be ordered so that terms involving $\tilde{k}$ appear on the left-hand side while the constant $(\lambda + b)(\rho + m)$ is isolated on the right-hand side; a graph of the left-hand side as a function of $\tilde{k}$ will be helpful.

f) Consider a simultaneous downward shift in b and m so that also n is lowered. How will such a parameter shift affect r^* in the present model? Why?

As technology growth is the result of new ideas and ideas come from human beings, there is a concern that reduced n tends to reduce g.

g) How will a lower g affect r^* in the present model? Why?

We will now interpret our non-monetary model as describing only the long-run trend of the economy. We imagine that money, sticky nominal prices and wages, and further Keynesian features are added to the model so as to open up for business cycle fluctuations to occur around the trend, primarily due to shocks to aggregate demand. We interpret our r^* as the "structural" (or "natural") rate of interest consistent with continuing "full employment" (more precisely NAIRU employment) and stationary inflation in the absence of shocks.

h) Briefly give a few examples of the kind of shocks that may trigger abrupt shifts in aggregate demand.

i) After a severe adverse demand shock, recovery requires for some time a real interest rate considerably below the structural rate. Under these circumstances, conventional monetary policy may reach a barrier where the ZLB becomes binding. Explain.

j) How does a reduced r^* affect the likelihood of the ZLB to become binding? Explain.

k) Briefly discuss what economic policy can do to take precautionary measures against a tendency to "secular stagnation".

Problem 2 Consider a given household facing uncertainty about future labor income. For simplicity, assume the household supplies one unit of labor inelastically. The household never knows for sure whether it will be able to sell that amount of labor in the next periods. Given the time horizon $T \geq 2$, the decision problem is:

$$\max E_0 U_0 = E_0[\sum_{t=0}^{T-1} u(c_t)(1+\rho)^{-t}] \quad \text{s.t.} \tag{1}$$

$$c_t \geq 0, \tag{2}$$

$$a_{t+1} = (1+r_t)a_t + w_t \ell_t - c_t, \qquad a_0 \text{ given}, \tag{3}$$

$$a_T \geq 0. \tag{4}$$

where $u' > 0$ and $u'' < 0$. Think of "period t" as the time interval $[t, t+1)$; the last period within the planning horizon T is thus period $T-1$. Real financial wealth is denoted a_t, and w_t (> 0) is the real wage, whereas ℓ_t is the amount of employment offered to the household by the market in period t, $0 \leq \ell_t \leq 1$ (ℓ_t is thus exogenous from the point of view of the household). The real rate of return on financial wealth is called r_t, and E_0 is the expectation operator, conditional on the information available in period 0. This information includes knowledge of all variables up to period 0, including that period. There is uncertainty about future values of r_t, w_t, and ℓ_t, but the household knows the stochastic processes that these variables follow.

a) Interpret (1) - (4).

b) Derive the Euler equation. *Hint:* it is convenient to consider maximization of $E_t U_t$ for $t = 0, 1, 2, ...$, where U_t is the generic expression for discounted utility as seen from an arbitrary period $t \in \{0, 1, 2, ..., T-2\}$. Interpret your result.

c) Determine the consumption in period $T-1$, given the financial wealth a_{T-1}. Comment.

d) With a CRRA utility function, $u(c) = (c^{1-\theta} - 1)/(1-\theta)$, what is the sign of u'''? Draw a graph of $u'(c)$ in the plane.

From now, assume our $u(c)$ in (1) satisfies $u''' > 0$, that is, marginal utility is strictly convex. Further, suppose there is no uncertainty about the future value of r_t, only about future labor income because future employment and real wage are uncertain.

e) Consider the decision problem as seen from period 1 and assume period 2 is the last period (i.e., $T = 3$). The consumption level chosen in period 1 will determine a_2. Let there be two possible outcomes for labor income in period 2, say y_L and y_H, each with probability $\frac{1}{2}$. Write down c_2 as a function of a_2 for each of the possible labor income outcomes.

f) Let the diagram from d) represent the situation in period 2 and enter the two possible values, c_L and c_H, of c_2 on the c_2 axis. Indicate how the expected marginal utility, $E_1 u'(c_2)$, conditional on a_2, can be found graphically.

g) How is *precautionary saving* defined? Apply the setup to show whether or not precautionary saving can arise. Comment. *Hint:* A mean-preserving spread is a convenient analytical device.

Problem 3 Short questions

a) There exist business cycle theories that rely heavily on recurrent exogenous shocks to the economy and other business cycle theories that do not. Give a brief account of the basic ideas in the two groups of theories. Make sure you identify why the first group of theories is dependent on recurrent shocks while the second is not.

b) Are there empirical regularities that support one group rather than the other? Briefly discuss (it is not presupposed that support always go in the same direction).

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