Advanced Macroeconomics If you don't use it, you will lose it.

13.10.2016.

## Problem set for the midterm paper. Deadline Nov. 8, 2016, at 10:15

General information: You are encouraged to do the problem set together with fellow students (max four students per group). If you do not know other students, but would like to have a co-worker, you may send me an e-mail about it. Then there might be a match with somebody else in the same situation.

If you prefer (readable) handwriting, you may hand in your paper to me at the lecture Tuesday 8/11 at 10:15. The alternative is to upload an electronic version of the paper in Absalon before deadline (but not both!). The latter has the advantage that you will automatically have a kind of receipt that way - and that your paper will probably be easier to read. You may answer in English or Scandinavian as you prefer. Do not make your answers too long, that is, *write in a concise way.* For example, when you are asked to "comment", your comment should be a short remark - right to the point.[1]

The maximum *number of pages* allowed is 9 (12 if handwriting is used).

Niklas or I comment on and evaluate your paper, i.e., evaluate whether it is "accepted" (corresponding to grade $\geq$ 02) or "not accepted". To go in for the final written exam (three hours closed book) in January, it is required that the midterm paper is accepted.

### Time table

Thursday 13/10, at 15:00 a.m.: The problem set for the midterm paper is posted at the course website.

Friday 28/10 and 4/11, at 15-17: Exercise class, part of which is held as workshop on the problem set.

Tuesday 8/11, at 10:15: **deadline** for your paper.

Lectures as usual.

You should upload your paper in Absalon before deadline. If you are a group, please upload as a group.

The **front page** of the paper must contain:

1) names of all the authors,

2) date,

3) name of the course and lecturer.

Good luck!

Christian

[1] Greg Mankiw has good recommendations about how to write:
http://gregmankiw.blogspot.com/2006/10/how-to-write-well.html.

The weighting of the four problems is: Problem 1: 45 %, Problem 2: 40 %, Problem 3: 10 %, and Problem 4: 5 %.

**Problem 1** *Quantification of the long-run effect on capital and output of public debt.* Most, but not all, economists agree that in the long run public debt crowds out private capital in a closed economy. The back of an envelope calculation performed in the handbook chapter by Elmendorf and Mankiw (1999, p. 1633) is based on the assumption that $\partial K^*/\partial B = -1$ (standard notation) and concludes that decreasing the US public debt from about half GDP[2] to nil will ultimately increase net output by 3 %. As suggested at the next pages, incl. footnote 12, in the same chapter, a more rigorous calculation can be made. That is what *you* should try to show below.

Consider a Blanchard OLG model for a closed economy with a public sector, public debt, and lump-sum taxation. Time is continuous. The dynamics of the economy are described by the differential equations

$$\dot{K}_t = F(K_t, L) - \delta K_t - C_t - G, \qquad K_0 > 0, \text{ given}, \tag{1}$$

$$\dot{C}_t = (F_K(K_t, L) - \delta - \rho)C_t - m(\rho + m)(K_t + B_t), \tag{2}$$

$$\dot{B}_t = [F_K(K_t, L) - \delta]\, B_t + G - T_t, \tag{3}$$

the condition

$$\lim_{t\to\infty} B_t e^{-\int_0^t [F_K(K_s,L)-\delta]ds} \leq 0, \tag{4}$$

and a requirement that households satisfy their transversality conditions. Here, $C_t$ is aggregate private consumption, $K_t$ is physical capital, $L$ is population = labor supply, $B_t$ is public debt, $G$ is government spending on goods and services, $T_t$ is net tax revenue (= gross tax revenue−transfer payments), and $F$ is an aggregate neoclassical production function with constant returns to scale and satisfying the Inada conditions. The other symbols stand for parameters and all these are positive; $L$ and $G$ are positive constants. A dot over a variable denotes the derivative w.r.t. time $t$.

a) Briefly interpret the equations (1) - (3) and the weak inequality (4), including the parameters $\delta$, $\rho$, and $m$.

b) Assuming $B_0 > 0$ and a balanced budget for all $t \geq 0$, construct a phase diagram for the implied two-dimensional dynamics and illustrate the path the economy follows, for the given $K_0$ (as usual, "construct" means that you should indicate *why* you draw the curves and arrows in the diagram as you do). It is understood that $G$ and $B_0$ are of "modest" size relative to the production capacity of the economy, given this $K_0$ and $L$. Briefly comment on the phase diagram.

[2] One half is the approximate average for $B/Y$ in the US over a period from the early 1980s to the late 1990s.

c) Suppose that at time $t_1 > 0$ the system has "settled down" in the sense that $(K_{t_1}, C_{t_1})$ is practically equal to the saddle-point stable steady state $(K^*, C^*)$. How does $K^*$ depend on the size of $B_0$? *Hint:* Calculate the long-run multiplier of $K^*$ with respect to $B_0$; regarding the sign of the key denominator, it is enough to refer to a certain inequality in Appendix B of Ch. 13 in Lecture Notes.

d) In their footnote 12, p. 1635, Elmendorf and Mankiw (1999) present a formula (from Blanchard and Fischer, 1989) for the long-run multiplier of capital w.r.t. the level of government debt in a closed economy. Rewrite their formula into the notation of this exercise problem.

e) It is possible that your result at c) at first sight still differs from the Elmendorf and Mankiw formula. Then either your formula or their formula is wrong, or both are wrong, or none of the formulas are wrong. What is your conclusion? *Hint:* The denominator in your formula can possibly be rewritten into a useful alternative form by considering the bottom of page 587 in Ch. 13 of Lecture Notes.

f) Suppose $F$ is given by $Y = K^\alpha L^{1-\alpha}$, where $\alpha = 1/3$. With one year as the time unit, let $\delta = 0.07$ and the steady-state interest rate $r^* = 0.04$. What value of $Y/K$ in steady state would you then suggest? Why?

g) In addition to the information at f), suppose that the private consumption-output ratio in steady state equals 0.55, that $m = 0.017$, and that $\rho = 0.03$. On this basis, find a numerical value for $C^*F_{KK}$ and the long-run multiplier of $K^*$ with respect to $B_0$, respectively. *Hint:* $C/K = (C/Y)(Y/K)$.

h) Briefly comment in relation to the back of an envelope calculation in Elmendorf and Mankiw (1999, p. 1633).

**Problem 2** *Can expansionary fiscal policy be self-financing?* Although the answer to this question is not decisive for whether such policy is worthwhile, it is an element in the evaluation of it. Here we shall attempt no more than a back of an envelope calculation.

Consider a closed economy. Time is discrete. Notation is:

$$\begin{aligned} G_t &= \text{real government spending on goods and services in period } t, \\ T_t &= \text{real net tax revenue ( = gross tax revenue} - \text{transfer payments) in period } t, \\ GBD_t &= \text{real government budget deficit in period } t, \\ B_t &= \text{real public debt (one-period bonds) at the start of period } t. \end{aligned}$$

Under "normal circumstances" the following holds:

- aggregate employment, $L$, is at the "full employment" level, $L = (1 - \bar{u})\bar{L}$, where $\bar{u}$ is the NAIRU and $\bar{L}$ is the aggregate labor supply, a given constant;
- real GDP, $Y_t$, equals its given trend level, $\bar{Y}_t$, and there is Harrod-neutral technical progress at a constant rate $g > 0$;

- the real rate of interest, $r_t$, is a constant, $r$; moreover, $r > g$;
- $G_t = \gamma \bar{Y}_t$, $0 < \gamma < 1$, and $T_t = \tau \bar{Y}_t$, $0 < \tau < 1$.

Assume that any government budget deficit is exclusively financed by issuing debt (and any budget surplus by redeeming debt).

a) Write down two equations showing how $GBD_t$ and $B_{t+1}$, respectively, are determined by variables indexed by $t$. Also write down an equation indicating how $B_{t+1}$ is related to $GBD_t$.

b) What is the growth rate of $Y$? Compare it with $r$.

c) Given $B_0 > 0$, find the minimum value of $\tau$, $\hat{\tau}$, consistent with fiscal sustainability. *Hint:* different approaches are possible; one focuses on the debt-income ratio and uses the fact that a difference equation $x_{t+1} = ax_t + c$, where $a$ and $c$ are constants, $a \neq 1$, has the solution $x_t = (x_0 - x^*)a^t + x^*$, where $x^* = c/(1-a)$.

d) How does $\hat{\tau}$ depend on $r$, $g$, $\gamma$, and $b_0 \equiv B_0/Y_0$, respectively?

e) What is the dynamic implication of $\tau < \hat{\tau}$, $\tau = \hat{\tau}$, and $\tau > \hat{\tau}$, respectively? Suggest a simple measure of the sustainability gap.

Now consider an alternative scenario. Suppose the above description of the economy has been valid for a sequence of periods up to and including period $t = -2$. Then, in period $t = -1$ the economy is hit by a large negative demand shock (possibly due to a financial crisis triggered by a bursting bubble) and gets into a substantial recession, henceforth denoted a slump. The central bank immediately lowers the policy rate (the short-term nominal interest rate) to near zero, but with limited effect. In response to this situation, with slack demand and $Y_{-1}$ far below "capacity", $\bar{Y}_{-1}$, the government decides an "expansionary fiscal policy" instead of "laissez-faire", where:

- "laissez-faire" means maintaining $G_t = \gamma \bar{Y}_t$, $t = 0, 1, 2, \ldots$;
- "expansionary fiscal policy" entails a discretionary increase in $G$ of size $\Delta G$, beginning in period 0 and maintained during the slump to stimulate economic activity, that is, $G_t = \gamma \bar{Y}_t + \Delta G$, where $\Delta G$ is a positive constant.

Let the tax and transfer rules in the economy imply that net tax revenue in period 0 is given by the function $T = T(Y)$. Thus, $T_0 = T(Y_0)$. Assume that under the current slump conditions marginal net tax revenue is $T'(Y) = 0.50$ whereas the spending multiplier is $\partial Y/\partial G = 1.5$. We ignore dynamic output effects (effects of $\Delta G_t$ on $Y_{t+1}$, $Y_{t+2}, \ldots$).[3]

[3] Many studies find that in a deep recession caused by deleveraging forces and monetary policy stuck at the zero-lower bound, fiscal multipliers can be "substantially above 1 early in the crisis" (Blanchard and Leigh, Growth Forecast Errors and Fiscal Multipliers, IMF Working Papers, No. 1, 2013). Under certain conditions a multiplier of 2.5 seems not unrealistic.

f) For a given $\Delta G > 0$, find expressions for the approximative effect of the expansionary fiscal policy on $GBD_0$ and $B_1$, respectively, in comparison with laissez-faire? What are the effects if $\Delta G = 8$ billion units of account?

g) For a given $r_1$, and assuming that both $\partial Y/\partial G$ and $T'(Y)$ are approximately the same in period 1 as in period 0, find an expression for the effect of the expansionary fiscal policy on $B_2$ in comparison with laissez-faire?

Suppose the slump is over in period 2 and onwards whereby $G_t = \gamma Y_t$, $t = 2, 3, \ldots$. Suppose further that compared with the expansionary fiscal policy, laissez-faire during the slump would have implied not only higher unemployment, but also more people experiencing *long-term* unemployment. As a result some workers would have become de-qualified and in effect be driven out of the effective labor force. Suppose the loss in "full employment" output from period 2 and onwards implied by laissez-faire is $\Delta Y$ per period, where $\Delta Y$ is a positive constant.[4] Finally, let the ensuing loss in net tax revenue be $\tilde{\tau} \cdot \Delta Y$ per period, where $\tilde{\tau}$ is a positive constant (possibly close to $\hat{\tau}$ from question c)).

h) With $r_t = r_2$, $t = 2, 3, \ldots$, and given $\Delta G$ and $\tilde{\tau}$, find an expression for the value of $\Delta Y$ required for the expansionary fiscal policy to "pay for itself" in period 2 and onwards in the sense that the averted loss in net tax revenue exactly offsets the extra interest payments?

i) Given $\tilde{\tau} = 0.3$, $r_1 = 0.01$, and $r_2 = 0.03$, answer again h). Comment.

**Problem 3** *Fiscal multipliers and austerity.* In IMF's *World Ec. Outlook*, Oct. 2012, p. 41-43, we read: "This box (Box 1.1) sheds light on these issues using international evidence. The main finding, based on data for 28 economies, is that the multipliers used in generating growth forecasts have been systematically too low since the start of the Great Recession, by 0.4 to 1.2, depending on the forecast source and the specifics of the estimation approach. Informal evidence suggests that the multipliers implicitly used to generate these forecasts are about 0.5. So actual multipliers may be higher, in the range of 0.9 to 1.7."

Figure 1.1.1, p. 43, is entitled "Growth Forecast Errors and Fiscal Consolidation Plans" and copied to page 7 below. Read the text in the figure carefully and give a report of the conclusion you think can be drawn from panel 1, 2, and 3 regarding fiscal multipliers and the austerity policy many countries implemented already from 2010.

Your report should be no longer than 2/3 of an A-4 page ($1\frac{1}{2}$ A-4 page if hand writing).

[4] It is theoretically possible that $\Delta Y$ is more or less constant for a long time because two offsetting effects are operative. Because of technical progress the loss of output per lost worker is growing over time. On the other hand the pool of long-term unemployed generated by the slump will over time be a decreasing share of the labor force due to exit by the old and entrance by young people in the labor force.

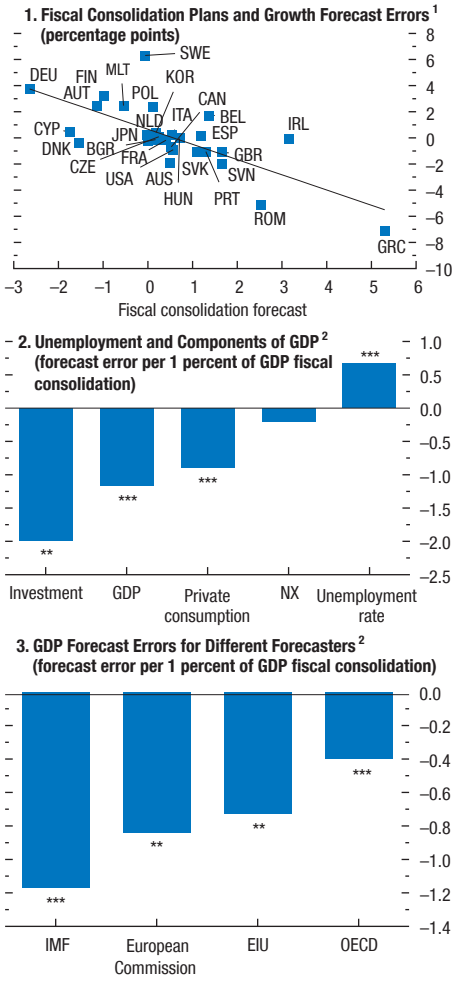
**Problem 4** *Short questions*

a) "In the Blanchard OLG model with $u(c) = \ln c$, the income effect on current consumption of a rise in the real interest rate exactly offsets the substitution effect and so a rise in the interest rate does not affect current individual consumption." True or false? Why? *Hint:* it is helpful to write down the intertemporal budget constraint and the consumption function of the individual.

b) An important aspect of macroeconomic analysis is to pose good questions in the sense of questions that are both interesting and manageable (not too big). Such questions can be raised - and discussed - in the Discussion Forum in Absalon and a good question possibly in one of the lectures or class sessions later in the semester. What question would you like to ask? Please, state the question in English.

## Figure 1.1.1. Growth Forecast Errors and Fiscal Consolidation Plans

Activity over the past few years has disappointed more in economies with more aggressive fiscal consolidation plans, suggesting that fiscal multipliers used in making growth forecasts have been systematically too low. This relationship holds for different components of GDP, the unemployment rate, and forecasts made by different institutions.




Source: IMF staff estimates.
Note: Figure identifies economies based on World Bank ISO three-letter codes (http://data.worldbank.org/node/18). NX = net exports contribution to growth. EIU = Economist Intelligence Unit.
[1]Vertical axis displays WEO forecast error for real GDP growth in 2010 and 2011 (actual forecast made in April 2010); horizontal axis displays WEO forecast of change in structural-fiscal-balance-to-GDP ratio in 2010 and 2011 (forecast made in April 2010).
[2] *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively.