

Written exam for the M. Sc. in Economics, Winter 2016-17

Advanced Macroeconomics

Master's course

December 19, 2016

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 5 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

The weighting of the problems is:

Problem 1: 40 %, Problem 2: 40 %, Problem 3: 20 %. ¹

¹The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a closed economy with public sector. Let time be discrete. Because we shall concentrate on income taxation rather than lump-sum taxes, we consider *net* income and let Y_t denote real Net National Product (= GNP minus capital depreciation) in period $t = 0, 1, 2, \dots$. We ignore business cycle fluctuations.

Suppose Y_t grows at a given constant rate $g_Y \geq 0$, and assume the real interest rate in the economy is a constant $r > g_Y$.

Further notation is:

$$\begin{aligned} G_t &= \text{real government spending on goods and services in period } t, \\ GBD_t &= \text{real government budget deficit in period } t, \\ B_t &= \text{real public debt (all short-term) at the start of period } t. \end{aligned}$$

There are no indirect taxes and no government transfers (apart from the interest on the government debt). Suppose that all private income (whether factor income or interest income from holding government bonds) is taxed by a given constant rate $\tau \in (0, 1)$. So tax revenue is

$$T_t = \tau(Y_t + rB_t), \quad t = 0, 1, 2, \dots \quad (*)$$

Assume that any government budget deficit is exclusively financed by issuing debt (and any budget surplus by redeeming debt).

- a) Write down two equations showing how GBD_t and B_{t+1} , respectively, are determined by variables indexed by t . Also write down an equation indicating how B_{t+1} is related to GBD_t .

Let $G_t = \gamma Y_t$, $t = 0, 1, 2, \dots$, where γ is a constant, $0 < \gamma < 1$. Let the historically given initial debt be $B_0 > 0$.

Let b_t denote the debt-income ratio, defined as B_t/Y_t .

- b) Give a general definition of the concept of fiscal sustainability. Within the here described framework, express a simple operational criterion for fiscal sustainability.
- c) Given (*), derive the law of motion for the debt-income ratio.

Considering γ as given, suppose the minimum value, $\hat{\tau}$, of the income tax rate required for fiscal sustainability is such that $(1 - \hat{\tau})r > g_Y$.

- d) Determine $\hat{\tau}$. Make sure your solution is an explicit solution for $\hat{\tau}$ expressed in terms exogenous variables and parameters. *Hint:* Different approaches are possible; one focuses on the debt-income ratio and uses the fact that a difference equation $x_{t+1} = ax_t + c$, where a and c are constants, $a \neq 1$, has the solution $x_t = (x_0 - x^*)a^t + x^*$, where $x^* = c/(1 - a)$.
- e) Compare your solution with what the solution for $\hat{\tau}$ would be in case of no taxation of interest income from holding government bonds. Comment.

- f) A simple measure of the *sustainability gap* at time 0 can be expressed in terms of $\hat{\tau}$ and the actual τ . How? For given actual τ , how does this sustainability gap depend on g_Y ?

In recent times some Danish economists have debated whether a higher productivity growth rate in the Danish economy helps fiscal sustainability. No doubt, the present simplified model is far from giving an accurate picture of the Danish tax system and the issues involved in the debate. Nevertheless, from a theoretical point of view it is of interest what the model says.

- g) Given the present model, is a higher productivity growth rate then helpful for fiscal sustainability? Yes or no? Why?

Problem 2 We consider a simple short-run model for a closed economy. Private aggregate demand, D , is the sum of private consumption, C , and investment, I :

$$D(Y_t, R_t, \tau, \alpha) \equiv C(Y_t - (\tau + T(Y_t)), R_t, \alpha) + I(Y_t, R_t, \alpha), \text{ where} \\ 0 < D_Y < 1, D_R < 0, -1 < D_\tau < 0, D_\alpha > 0, \quad (*)$$

and Y is aggregate income (= aggregate output), $T(Y)$ a tax function, τ a parameter reflecting fiscal tightness, R_t is the real long-term interest rate, and α an indicator of the general “state of confidence”.

Time is continuous and we have the following further equations:

$$\dot{Y}_t = \lambda(D(Y_t, R_t, \tau, \alpha) + G - Y_t), \quad \lambda > 0, \quad (1)$$

$$\frac{M_t}{P_t} = L(Y_t, i_t), \quad L_Y > 0, L_i < 0, \quad (2)$$

$$q_t = \int_t^\infty 1 \cdot e^{-R_t(s-t)} ds = \frac{1}{R_t}, \quad (3)$$

$$\frac{1 + \dot{q}_t^e}{q_t} = r_t^e, \quad (4)$$

$$r_t^e \equiv i_t - \pi_t^e, \quad (5)$$

$$\pi_t = \pi, \quad (6)$$

where the superscript e denotes subjective expectation, q_t = the real price of a consol paying one unit of output per time unit forever, G = government spending on goods and services, M_t = the monetary base (there is no private banking sector), P_t = output price, i_t = nominal short-term interest rate, r_t = real short-term interest rate, and $\pi_t \equiv \dot{P}_t/P_t$ = rate of inflation. The variables λ, τ, G, α , and π are exogenous constants. The initial values Y_0 and P_0 are pre-determined.

The central bank uses the nominal short-term interest rate as policy instrument, maintaining it at a desired level, given current circumstances. Until further notice, let the desired level equal the constant $i^* > 0$.

- a) Briefly interpret the equations of the model and explain in what way, according to the model, the central bank is able to maintain i_t at the level i^* as long as desired.

Suppose that there is no uncertainty, expectations are rational, and speculative bubbles never arise. It can be shown that this implies, in addition to (3), that

$$R_t = \frac{1}{q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_u du} ds},$$

which may sometimes help intuition.

- b) To characterize the movement over time of the economy, derive from the model a dynamic system (two coupled differential equations) in the endogenous variables Y and R . Draw the corresponding phase diagram. Indicate in the diagram the path the economy follows, given Y_0 . Comment.
- c) Determine the short-term and long-term *real* interest rates in steady state and find an implicit solution for Y in steady state. How does Y in steady state depend on the policy parameters i^* , G , and τ , respectively?

Suppose that the economy has been in steady state until time $t_1 > 0$. Then an unanticipated large decrease in the state of confidence to $\alpha' < \alpha$ occurs. After this everybody rightly expects the state of confidence to remain unchanged for a long time.

- d) Suppose further that the market participants rightly expect no change in monetary and fiscal policy. Illustrate by the same or a new phase diagram the evolution of the economy for $t \geq t_1$.
- e) Assume instead that immediately after time t_1 , the central bank responds to the confidence shock by lowering the policy rate to the constant $i^{*'} < i^*$. We imagine that the reduction in the policy rate is “moderate” relative to the size of the confidence shock. Hence an economic downturn is not avoided. Everybody rightly expects the state of confidence as well as the policy rate to remain unchanged for a long time after t_1 . Illustrate by the same or a new phase diagram the evolution of the economy for $t \geq t_1$. Provide some intuition.
- f) Assume instead that immediately after time t_1 , the central bank responds to the confidence shock by announcing that at time t_2 the policy rate will be reduced to the constant $i^{*'} < i^*$. Suppose everybody rightly expects the policy rate to behave as announced. Illustrate by the same or a new phase diagram the evolution of the economy for $t \geq t_1$. Also illustrate, by graphical time profiles, the evolution of i_t , Y_t , R_t , and r_t for $t \geq 0$. Provide some intuition. *Hint:* Be aware that in the time interval $[t_1, t_2)$, the “old dynamics” still hold; next, think about where the economy *must* be at time t_2 and onwards. Regarding intuition, recall that the long-term interest rate is a weighted average of the expected future short-term interest rates.

- g) What is the sign of the “slope of the yield curve” in the time interval (t_1, t_2) . And what is the intuition behind this sign?

Problem 3 *Short questions*

- a) “In Blanchard’s OLG model it holds that in steady state each single individual’s consumption grows faster than per capita consumption”. True or false? Why?
- b) “Price flexibility is always stabilizing”. True or false? Why?
- c) In their large econometric study Mian & Sufi (2014) explore the question: What explains the large 2007-09 drop in employment in the US? Briefly give an account of their conclusion and the main empirical basis for it.

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