Advanced Macroeconomics A note

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# Saddle-point stability

A concept which perplexes many is the concept of saddle-point stability.

Consider a *two-dimensional* dynamic system (two coupled first-order differential equations with two endogenous time-dependent variables). Suppose the system has a steady state which is a *saddle point* (which is the case if and only if the two eigenvalues of the associated Jacobi matrix, evaluated at the steady state, are of opposite sign). Then, so far, either presence or absence of saddle-point stability is possible. And which of the two cases occur can not be diagnosed from the two differential equations in isolation. One has to consider the boundary conditions. Here is a complete definition of (local) saddle-point stability.

DEFINITION. A steady state of a two-dimensional dynamic system is (locally) *saddle-point stable* if:

1. the steady state is a saddle point;
2. one of the two endogenous variables is predetermined while the other is a jump variable;
3. the saddle path is not parallel to the jump variable axis; and
4. there is a boundary condition on the system such that the diverging paths are ruled out as solutions.

Thus, to establish saddle-point stability, all four properties must be verified. If for instance point 1 and 2 hold but, contrary to point 3, the saddle path is parallel to the jump variable axis, then saddle-point stability does not obtain. Indeed, given that the predetermined variable initially deviated from its steady-state value, it would not be possible to find any initial value of the jump variable such that the solution of the system would converge to the steady state for $t \to \infty$.

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